J. of Ramanujan Society of Math. and Math. Sc. **ISSN : 2319-1023**
Vol.2, No.1 (2013), pp. 21-28

Basic Analogues of Certain Multiple Series of Transformations-II

Prakriti Rai
Department of Mathematics, Amity University,
Sector-125, Noida 201313 (UP), India
Email: prakritirai.rai@gmail.com

Abstract: In this paper basic analogues of generating relations for certain multiple series with essentially arbitrary terms due to M.A. Pathan, B.B. Jaimini and Shiksha Gautam [1] and Srivastava and Pathan [9] are obtained. The importance of these results lies in obtaining new multiple series transformations and reduction formula which may be capable of yielding number theoretic and combinatorial interpretations.

1. Introduction

Generalizing Heine's series, we shall define an ${}_r\phi_s$ basic hypergeometric series by

$$ {}_r\phi_s(a_1, a_2, \ldots, a_r; b_1, b_2, \ldots, b_s; q, z) \equiv {}_r\phi_s \left[\begin{array}{l} a_1, a_2, \ldots, a_r; q, z \\ b_1, b_2, \ldots, b_s \end{array} \right] $$

$$ = \sum_{n=0}^{\infty} \frac{(a_1;q)_n (a_2;q)_n \ldots (a_r;q)_n}{(q;q)_n (b_1;q)_n \ldots (b_s;q)_n} [(-1)^n q^{n(n-1)/2}]^{1+s-r} z^n \tag{1.1} $$

$$ (\lambda)_n \equiv (\lambda;q)_n = \left\{ \begin{array}{l} 1, \text{if } n=0 \\ (1-\lambda)(1-\lambda q)(1-\lambda q^2)\ldots(1-\lambda q^{n-1}) \end{array} \right\} \tag{1.2} $$

$$ (\lambda;q)_{n+m} = (\lambda;q)_n (\lambda q^n;q)_m \tag{1.3} $$

$$ (\lambda;q)_{-n} = \frac{(-\lambda)^{-n} q^{n(n+1)/2}}{(q/\lambda;q)_n} \tag{1.4} $$

$$ (\lambda;q^2)_n = (\sqrt{\lambda};q)_n(-\sqrt{\lambda};q)_n \tag{1.5} $$

$$ (\lambda;q)_{2n} = (\lambda;q^2)_n(\lambda q;q^2)_n \tag{1.6} $$

$$ \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} A(k,n) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} A(k, n-k) \tag{1.7} $$

$$ \sum_{n=0}^{\infty} \sum_{k_1 \ldots k_r=0}^{M \leq n} \varphi(k_1, k_2 \ldots k_r; n) = \sum_{n=0}^{\infty} \sum_{k_1 \ldots k_r=0}^{\infty} \phi(k_1 \ldots k_r; n+M) \tag{1.8} $$

Where $M = \sum_{i=1}^{r} m_i k_i$.

In 2008, M.A. Pathan, B.B. Jaimini and Shiksha Gautam [1] established new classes of bilateral generating relations for functions of several variables. The two general multivariable theorems given by them are stated as follows:

Theorem A. *Let*

$$\phi(t_1^{m_1}x_1,\ldots,t_1^{m_r}x_r;t_2\ldots t_s) = \sum_{n=0}^{\infty}\frac{(-t_1)^n}{n!(\lambda+n)_n}\sum_{k_1\ldots k_r=0}^{M\leq n}(\lambda+n)_M(-n)_M$$

$$\Lambda(k_1\ldots k_r)x_1^{k_1}\ldots x_r^{k_r}\sum_{l_1\ldots l_s=0}^{\infty}\frac{\Omega_{n+l_1}^{l_2\ldots l_s}}{(\lambda+2n+1)_{l_1}}\frac{t_1^{l_1}}{l_1!}\frac{t_2^{l_2}}{l_2!}\cdots\frac{t_s^{l_s}}{l_s!} \tag{1.9}$$

Where $M = \sum_{i=1}^{r} m_i k_i$, provided that for every complex number $\lambda \neq 0, -1, -2, \ldots$ the result in (1.9) exists.

Theorem B. *Let*

$$\phi(t_1^{m_1}x_1,\ldots,t_1^{m_r}x_r;t_2\ldots t_s)$$

be defined by (2.1), *then for arbitrary* α *and* β, $\beta \neq 0$;

$$\phi(t_1^{m_1}x_1,\ldots,t_1^{m_r}x_r;t_2\ldots t_s) = \sum_{n=0}^{\infty}\frac{(-t_1)^n}{n!}\sum_{k_1\ldots k_r=0}^{M\leq n}\frac{(-n)_M}{(\beta-\alpha n)_M}$$

$$\Lambda(k_1\ldots k_r)x_1^{k_1}\ldots x_r^{k_r}\frac{[(1-\alpha)M+\beta]}{[M-\alpha n+\beta]}\sum_{l_1\ldots l_s=0}^{\infty}(\beta-\alpha n)_{n+l_1}\Omega_{n+l_1}^{l_2\ldots l_s}\frac{t_1^{l_1}}{l_1!}\frac{t_2^{l_2}}{l_2!}\cdots\frac{t_s^{l_s}}{l_s!} \tag{1.10}$$

Where $M = \sum_{i=1}^{r} m_i k_i$, provided that the result in (1.10) exists.

Theorem C. *Let* $\{A_n\}$, $\{B_n\}$ *and* $\{C_n\}$ *be sequences of arbitrary complex numbers, and let* $\Lambda(m_2,\ldots,m_r)$, $r \geq 2$ *denote a multiple sequence. Suppose also let the complex parameter* α *and* β *be independent of* n, $\beta \neq 0$, *and set* $M = m_1 + \ldots + m_r$ *for all* $m_i \in \{0, 1, 2, \ldots\}$, $i = 1, \ldots, r$.

Then

$$\sum_{n=0}^{\infty}\frac{(x_1)^n}{n!}\sum_{k=0}^{[n/N]}\left\{\frac{(1-\alpha)_{Nk}+\beta}{Nk-\alpha n+\beta}\right\}\frac{(-n)_{Nk}}{(\beta-\alpha_n)_{Nk}}C_k\frac{w^k}{k!}$$

$$\sum_{m_1,\ldots,m_r=0}^{\infty}(\beta-\alpha_n)_{n+m_1}A_{n+M}B_{n+m_1}\Lambda(m_2,\ldots,m_r)\frac{x_1^{m_1}}{m_1!}\ldots\frac{x_r^{m_r}}{m_r!} \tag{1.11}$$

$$=\sum_{m_1,\ldots,m_r=0}^{\infty}A_{(N-1)m_1+M}B_{n+m_1}C_{m_1}\Lambda(m_2,\ldots,m_r)\frac{(wx_1^N)^{m_1}}{m_1!}\frac{x_2^{m_2}}{m_2!}\ldots\frac{x_r^{m_r}}{m_r!}$$

for every integer $N \geq 1$ provided that both members of Theorem C exist.

In this paper, we derive q-analogues of the above three theorems in section 2. In section 3, we give a simple application of our theorems.

2. Main Theorems

For bounded complex coefficients $\Lambda(k_1\ldots k_r)$ and $\Omega_n^{l_2\ldots l_s}$ for all $n, k_i \in (0,1,2,\ldots)$, $l_j \in (0,1,2,\ldots)$, $i=1,\ldots,r$, $j=2,\ldots,s$.

Let

$$\phi(t_1^{m_1}x_1,\ldots,t_1^{m_r}x_r;t_2\ldots t_s)$$

$$=\sum_{k_1\ldots k_r=0}^{\infty}\sum_{l_2\ldots l_s=0}^{\infty}\Lambda(k_1\ldots k_r)\Omega_M^{l_2\ldots l_s}(x_1t_1^{m_1})^{k_1}\ldots(x_rt_1^{m_r})^{k_r}\frac{t_2^{l_2}}{(q)_{l_2}}\ldots\frac{t_s^{l_s}}{(q)_{l_s}};$$

$$M=\sum_{i=1}^{r}m_ik_i \tag{2.1}$$

Theorem 1. *Let* $\phi(t_1^{m_1}x_1,\ldots,t_1^{m_r}x_r;t_2\ldots t_s)$ *be defined by* (2.1), *then*

$$\phi(t_1^{m_1}x_1,\ldots,t_1^{m_r}x_r;t_2\ldots t_s)=\sum_{n=0}^{\infty}\frac{(-1)^nq^{n(n-1)/2}}{(q)_n(\lambda q^n)_n}\sum_{k_1\ldots k_r=0}^{M\leq n}q^M(q^{-n})_M(\lambda q^n)_M$$

$$\Lambda(k_1\ldots k_r)x_1^{k_1}\ldots x_r^{k_r}\sum_{l_1\ldots l_s=0}^{\infty}\frac{\Omega_{n+l_1}^{l_2\ldots l_s}}{(\lambda q^{2n+1})_{l_1}}\frac{t_1^{l_1+n}}{(q)_{l_1}}\frac{t_2^{l_2}}{(q)_{l_2}}\ldots\frac{t_s^{l_s}}{(q)_{l_s}} \tag{2.2}$$

Where $M=\sum\limits_{i=1}^{r}m_ik_i$, provided that for every complex number $\lambda \neq 0,-1,-2,\ldots$ the result in (2.2) exists.

Proof: Let Δ_1 denote the following series of multiple basic hyper geometric sum: Then $(\Delta_1)\equiv\phi(t_1^{m_1}x_1,\ldots t_1^{m_r}x_r;t_2\ldots t_s)$

$$\Delta_1=\sum_{n=0}^{\infty}\frac{(-1)^nq^{n(n-1)/2}}{(q)_n(\lambda q^n)_n}\sum_{k_1\ldots k_r=0}^{M\leq n}q^M(q^{-n})_M(\lambda q^n)_M$$

$$\times\Lambda(k_1\ldots k_r)x_1^{k_1}\ldots x_r^{k_r}\sum_{l_1\ldots l_s=0}^{\infty}\frac{\Omega_{M+l_1}^{l_2\ldots l_s}}{(\lambda q^{2n+1})_{l_1}}\frac{t_1^{l_1+n}}{(q)_{l_1}}\frac{t_2^{l_2}}{(q)_{l_2}}\ldots\frac{t_s^{l_s}}{(q)_{l_s}}.$$

Now on making use of series rearrangements (1.8) and (1.7) respectively, it reduces to:

$$\Delta_1=\sum_{l_1\ldots l_s=0}^{\infty}\sum_{k_1\ldots k_r=0}^{\infty}\Lambda(k_1\ldots k_r)(x_1t_1^{m_1})^{k_1}\ldots(x_rt_1^{m_r})^{k_r}\Omega_{M+l_1}^{l_2\ldots l_s}\frac{t_2^{l_2}}{(q)_{l_2}}\cdots\frac{t_s^{l_s}}{(q)_{l_s}}$$

$$\times\sum_{n=0}^{l_1}\frac{(-1)^n(\lambda)_{n+2M}(\lambda q)_{2n+2m}q^{n(n-1)/2}t_1^{l_1}}{(q)_n(q)_{l_1-n}(\lambda)_{2n+2M}(\lambda q)_{2n+2M+l_1-n}} \tag{2.3}$$

On applying (1.3) and (1.4) respectively and then on using the formula (1.6) and (1.5) therein we have

$$\Delta_1=\sum_{l_1\ldots l_s=0}^{\infty}\sum_{k_1\ldots k_r=0}^{\infty}\Lambda(k_1\ldots k_r)(x_1t_1^{m_1})^{k_1}\ldots(x_rt_1^{m_r})^{k_r}\Omega_{M+l_1}^{l_2\ldots l_s}\frac{t_2^{l_2}}{(q)_{l_2}}\cdots\frac{t_s^{l_s}}{(q)_{l_s}}\frac{1}{(\lambda q^{2M+1})_{l_1}}$$

$$\times\,{}_4\phi_3\left[\begin{array}{ccccc}\lambda q^{2M} & q^{1+M}\sqrt{\lambda} & -q^{1+2M}\sqrt{\lambda} & q^{-l_1}; & q^{l_1}\\ & q^M\sqrt{\lambda} & -q^M\sqrt{\lambda} & \lambda q^{1+l_1+2M}; & \end{array}\right] \tag{2.4}$$

The above basic hypergeometric ${}_4\phi_3$ series is well poised and therefore by applying q-analogue of Dixon's theorem.

$${}_4\phi_3\left[\begin{array}{ccccc}\lambda q^{2M} & q^{1+M}\sqrt{\lambda} & -q^{1+2M}\sqrt{\lambda} & q^{-l_1}; & q^{l_1}\\ & q^M\sqrt{\lambda} & -q^M\sqrt{\lambda} & \lambda q^{1+l_1+2M}; & \end{array}\right]=\left\{\begin{array}{l}1 \text{ as } l_1=0\\ 0 \text{ for all } l_1=1,2,3...\end{array}\right\}$$

We at once arrive at the desired result (2.2).

We shall now prove the following basic analogue of Theorem B for the complex parameter α is taken to be zero.

Theorem 2. *Let* $\phi(t_1^{m_1}x_1,\ldots t_1^{m_r}x_r;t_2\ldots t_s)$ *be defined by* (2.1)*, then for arbitrary* α *and* β*,* $\beta\neq 0$*;*

$$\phi(t_1^{m_1}x_1,\ldots,t_1^{m_r}x_r;t_2\ldots t_s)$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^nq^{n(n-1)/2}}{(q)_n}\sum_{k_1\ldots k_r=0}^{M\leq n}\frac{q^M(q^{-n})_M}{(\beta)_M}$$

$$\times\Lambda(k_1\ldots k_r)x_1^{k_1}\ldots x_r^{k_r}\sum_{l_1\ldots l_s=0}^{\infty}(\beta)_{n+l_1}\Omega_{n+l_1}^{l_2\ldots l_s}\frac{t_1^{l_1+n}}{(q)_{l_1}}\frac{t_2^{l_2}}{(q)_{l_2}}\cdots\frac{t_s^{l_s}}{(q)_{l_s}} \tag{2.5}$$

Where $M = \sum_{i=1}^{r} m_i k_i$, provided that the result (2.5) exists.

Proof: To prove the Theorem 2 we denote the R.H.S. of (2.5) by Δ_3, i.e.

$$\Delta_3 = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n(n-1)/2}}{(q)_n} \sum_{k_1 \ldots k_r = 0}^{M \le n} \frac{q^M (q^{-n})_M}{(\beta)_M}$$

$$\times \Lambda(k_1 \ldots k_r) x_1^{k_1} \ldots x_r^{k_r} \sum_{l_1 \ldots l_s = 0}^{\infty} (\beta)_{n+l_1} \Omega_{n+l_1}^{l_2 \ldots l_s} \frac{t_1^{l_1+n}}{(q)_{l_1}} \frac{t_2^{l_2}}{(q)_{l_2}} \cdots \frac{t_s^{l_s}}{(q)_{l_s}} .$$

Now on making use of series rearrangements (1.8) and (1.7) respectively, it reduces to:

$$\Delta_3 = \sum_{l_1 \ldots l_s = 0}^{\infty} \sum_{k_1 \ldots k_r = 0}^{\infty} \Lambda(k_1 \ldots k_r) x_1^{k_1} \ldots x_r^{k_r} t_1^{l_1+M} \Omega_{M+l_1}^{l_2 \ldots l_s} \frac{t_2^{l_2}}{(q)_{l_2}} \cdots \frac{t_s^{l_s}}{(q)_{l_s}}$$
$$\times \sum_{n=0}^{l_1} \frac{(-1)^n q^{n(n-1)/2} (\beta q^M)_{l_1}}{(q)_n (q)_{l_1 - n}} .$$

On applying (1.3) and (1.4) we have

$$\Delta_3 = \sum_{l_1 \ldots l_s = 0}^{\infty} \sum_{k_1 \ldots k_r = 0}^{\infty} \Lambda(k_1 \ldots k_r) x_1^{k_1} \ldots x_r^{k_r} \Omega_{M+l_1}^{l_2 \ldots l_s} (\beta q^M)_{l_1} \frac{t_1^{l_1+M}}{(q)_{l_1}} \frac{t_2^{l_2}}{(q)_{l_2}} \cdots \frac{t_s^{l_s}}{(q)_{l_s}}$$
$$\times \sum_{n=0}^{l_1} \frac{(q^{-l_1})_n}{(q)_n} q^{l_1 n} . \tag{2.6}$$

Now in (2.6) the sum of $1\phi_0(q^{-l_1}; q^{l_1})$, becomes zero if $l_1 > 0$. Therefore, we must take $l_1 = 0$ in (2.6). We then get the R.H.S. of Theorem B on replacing k by l_1.

We shall now prove q-generalization of another Theorem C of Srivastava and Pathan [9, Theorem 2] for the case when $N = 1$ and the complex parameter α is taken to be zero.

Theorem 3. *Let* $\{A_n\}$, $\{B_n\}$, $\{C_n\}$ *and* $\Lambda(m_2, \ldots, m_r)$, $r \ge 2$, *be arbitrary complex sequences, and let* $M = m_1 + m_2 + \ldots + m_r$. *Then*

$$\sum_{n=0}^{\infty} \frac{(-x_1)^n q^{n(n-1)/2}}{(q)_n} \sum_{k=0}^{n} \frac{(q^{-n})_k}{(\beta)_k} \frac{(wq)^k}{(q)_k} C_k$$

$$\left\{\sum_{m_1,\ldots,m_r=0}^{\infty}(\beta)_{n+m_1}A_{n+M}B_{n+m_1}\Lambda(m_2,\ldots,m_r)\frac{x_1^{m_1}}{(q)_{m_1}}\frac{x_2^{m_2}}{(q)_{m_2}}\cdots\frac{x_r^{m_r}}{(q)_{m_r}}\right\}$$

$$=\sum_{m_1,\ldots,m_r=0}^{\infty}A_M B_{m_1}C_{m_1}\Lambda(m_2,\ldots,m_r)\frac{(wx_1)^{m_1}}{(q)_{m_1}}\frac{x_2^{m_2}}{(q)_{m_2}}\cdots\frac{x_r^{m_r}}{(q)_{m_r}}, \tag{2.7}$$

provided that each side has a meaning.

Proof: Let $M(\beta,q)$ denote the following series of multiple basic hypergeometric sums:

$$M(\beta,q)=\sum_{n=0}^{\infty}\frac{(-x_1)^n q^{n(n-1)/2}}{(q)_n}\sum_{k=0}^{n}\frac{(q^{-n})_k}{(\beta)_k}\frac{(wq)^k}{(q)_k}C_k$$

$$\times\sum_{m_1,\ldots m_r=0}^{\infty}(\beta)_{n+m_1}A_{n+M}B_{n+m_1}\Lambda(m_2,\ldots,m_r)\frac{x_1^{m_1}}{(q)_{m_1}}\frac{x_2^{m_2}}{(q)_{m_2}}\cdots\frac{x_r^{m_r}}{(q)_{m_r}}$$

After some simplification and then taking $n+k$ for n, we get

$$M(\beta,q)=\sum_{m_1,\ldots m_r,k,n=0}^{\infty}\frac{(-1)^n q^{n(n-1)/2}}{(q)_n}\frac{w^k}{(q)_k}C_k(\beta q^k)_{n+m_1}$$

$$\times\ A_{n+k+M}B_{n+k+m_1}\Lambda(m_2,\ldots,m_r)\frac{x_1^{m_1+n+k}}{(q)_{m_1}}\frac{x_2^{m_2}}{(q)_{m_2}}\cdots\frac{x_r^{m_r}}{(q)_{m_r}},$$

on using the series transformation (1.7).

As in the case of Theorem 1, let us now replace m_1 by m_1-n and use the transformation (1.7) in the other direction. We then get

$$M(\beta,q)=\sum_{m_1,\ldots m_r=0}^{\infty}A_{k+M}B_{k+m_1}\frac{w^k}{(q)_k}C_k\Lambda(m_2,\ldots,m_r)$$

$$\times\frac{x_1^{m_1+k}}{(q)_{m_1}}\frac{x_2^{m_2}}{(q)_{m_2}}\cdots\frac{x_r^{m_r}}{(q)_{m_r}}\left\{\sum_{n=0}^{m_1}\frac{(q^{-m_1})_n}{(q)_n}(\beta q^k)_{m_1}q^{m_1 n}\right\},$$

or

$$M(\beta,q)=\sum_{m_1,\ldots m_r=0}^{\infty}A_{k+M}B_{k+m_1}\frac{W^k}{(q)_k}C_k\Lambda(m_2,\ldots m_r)(\beta q^k)_{m_1}$$

$$\times\frac{x_1^{m_1+k}}{(q)_{m_1}}\frac{x_2^{m_2}}{(q)_{m_{21}}}\cdots\frac{x_r^{m_r}}{(q)_{m_r}}\ {}_1\phi_0(q^{-m_1};q^{m_1}) \tag{2.8}$$

The sum of ${}_1\Phi_0$-series in (2.8) viz., $\frac{(l)_{m_1}}{(q^{m_1})_{m_1}}$, becomes zero if $m_1 > 1$. Therefore, we must taken $m_1 = 0$ in (2.8). We then get the right-hand of Theorem 3 on replacing k by m_1.

If we take $x_3 = \ldots = x_r = 0$ and $\Lambda(n, 0, \ldots, 0) \equiv D_n$, $n \geq 0$ in (3.1) above, we get the following q-analogue of Srivastava and Pathan's theorem [9, Theorem 2] for the case when $N = 1$ and $\alpha = 0$, namely

$$= \sum_{n=0}^{\infty} \frac{(-x)^n q^{n(n-1)/2}}{(q)_n} \sum_{k=0}^{n} \frac{(q^{-n})_k}{(\beta)_k} \frac{(wq)^k}{(q)_k} C_k$$

$$\times \sum_{l,m=0}^{\infty} (\beta)_{n+1} A_{l+m+n} B_{n+1} D_m \frac{x^l}{(q)_1} \frac{y^m}{(q)_m}$$

$$= \sum_{l,m=0}^{\infty} A_{l+m} B_l C_1 D_m \frac{(wx)^1}{(q)_1} \frac{y^m}{(q)_m},$$

provided that each side has a meaning.

3. Applications

(1) If we take $r = 1$ in (2.2) and set $\Lambda(k_1, 0, \ldots 0) \to \frac{C_k}{(q)_k}$,
$\Omega_{n+l_1}^{l_2 \ldots l_s} = A_{n+l_1+l_2+\ldots+l_s} B_{n+l_1} \Lambda(l_2, \ldots, l_s)$; $\{A_n\}$, $\{B_n\}$, $\{C_n\}$ and $\Lambda(m_2, \ldots, m_r)$, $r \geq 2$, be arbitrary complex sequences, then on replacing x_i by Ω. These reduces to the known results [10, (2.1)] which in turn at $s = 2$ provides the know results [10, (2.6)].

(2) On similar setting at $r = 1$, as in (1) the result in (2.5) at $s = 2$ reduces to the above result (2.7) and (2.7).

(3)If we take $x_3 = \ldots = x_r = 0$ and $\Lambda(n, 0, \ldots, 0) \equiv D_n$, $n \geq 0$ in (2.7),
we get the following q-analogue of Srivastava and Pathan's theorem [9, Theorem 2] for the case when $N = 1$ and $\alpha = 0$, namely

$$= \sum_{n=0}^{\infty} \frac{(-x)^n q^{n(n-1)/2}}{(q)_n} \sum_{k=0}^{n} \frac{(q^{-n})_k}{(\beta)_k} \frac{(wq)^k}{(q)_k} C_k$$

$$\times \sum_{l,m=0}^{\infty} (\beta)_{n+1} A_{l+m+n} B_{n+1} D_m \frac{x^l}{(q)_1} \frac{y^m}{(q)_m}$$

$$= \sum_{l,m=0}^{\infty} A_{l+m} B_l C_1 D_m \frac{(wx)^1}{(q)_1} \frac{y^m}{(q)_m},$$

provided that each side has a meaning.

Acknowledgement

The author expresses her deep sense of gratitude to Prof. M.A. Pathan for their help and useful suggestions.

References

[1] Pathan, M.A., Jaimini, B.B. and Gautam, Shiksha, Certain bilateral generating Functions involving multiple series with arbitrary terms,Scientia, 16 (2008), 78–86.

[2] Rainville, E.D.,Special Functions, Macmillan, New York (1960).

[3] Slater, L.J., Generalised Hypergeometric Series, Cambridge University Press, Cambridge (1966).

[4] Srivastava, H.M., Some polynomial expension for functions of several variables,*IMA J. Appl.Math.*, 27 (1982), 299–306.

[5] Srivastava, H.M. and Karisson, P.W.,Multiple Gaussian Hyprgeometric Series, Ellis Horwood Limited, Chichester, 1985.

[6] Srivastava, H.M. and Manocha, H.L., A Treatise on Generating Functions, Ellis Horwood Limited, Chichester, 1984.

[7] Srivastava, H.M. and Panda, R., Some expansions of hypergeometric functions in series of hypergeometric functions, Glasgow Math. J., 17 (1976), 17–21.

[8] Srivasvata, H.M. and Pathan, M.A., Some bilateral generating functions for the extended Jacobi polynomials, Comment. Math. Univ. St. Paul., 28 (1979), 23–30.

[9] Srivasvata, H.M. and Pathan, M.A., Some bilateral generating functions for the extended Jacobi polynomials-II, Comment. Math. Univ. St. Paul., 29 (1980), 105–114.

[10] Rai, Prakriti, Basic analogues of certain multiple series of transformations, South East Asian J. Math. and Math. Sc., 8 (1) (2009), 5–8.