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THE STRUCTURE OF THE UNIT GROUP OF A GROUP ALGEBRA OF A GROUP OF ORDER 37

Nikita Srivastava, Harish Chandra and Suchi Bhatt*

Department of Mathematics and Scientific Computing,
M. M. M. University of Technology, Gorakhpur - 273016, U.P., INDIA

E-mail : nikitasrivastava566@gmail.com, hcmsc@mmmut.ac.in

*Department of Mathematics,
Institute of Applied Sciences and Humanities,
G. L. A. University, Mathura - 281406, U.P., INDIA

E-mail : 1995suchibhatt@gmail.com

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Abstract: Let FG be the group algebra of a group G over a finite field F of characteristic $p > 0$ with $q = p^n$ elements. In this paper, a complete characterization of the unit group $U(FC_{37})$ of the group algebra FC_{37} for the group C_{37} of order 37, over a finite field of characteristic $p > 0$ has been obtained.

Keywords and Phrases: Group algebras, Unit groups, Jacobson radical.

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1. Introduction

Let FG be the group algebra of a group G over a field F, for a given normal subgroup H of G, we can extend any group homomorphism G to G/H, to an F-algebra homomorphism from FG onto $F[G/H]$. The homomorphism is defined as:

$$\sum_{g\in G} a_g g \mapsto \sum_{g\in G} a_g gH, \text{ for } a_g \in F.$$

It can be written as $\frac{FG}{\omega(H)} \cong F[\frac{F}{H}]$, where $\omega(H)$ is the kernal of $F-$algebra homomorphism. Also,

$$\omega(H) = \omega(FH)FG = FG\omega(FH),$$

where $\omega(FG)$ is called the augmentation ideal of FG. It is easy to see that, if $U(FG)$ is the unit group of FG and $V(FG)$ is the normalized unit group of FG, then

$$U(FG) \cong V(FG) \times F^*.$$

Let G be a finite p-group. Then $V(FG)$ is a finite p-group of order $|F|^{|G|-1}$. If F is a finite field of characteristic $p > 0$, then element $g \in G$ is called a **p-regular element** if, $(p, o(g)) = 1$. Let m be the L.C.M. of all the p-regular elements of G and ξ be a primitive mth root of unity over the field F. Now define a multiplicative group T of integers modulo m as $T = \{t : \xi \to \xi^t$ is an automorphism of $F(\xi)$ over $F\}$. Any two p-regular elements $g, h \in G$ are said to be F-conjugate if $g^t = x^{-1}hx$, for some $x \in G$ and $t \in T$, which gives an equivalence relation which partitions the p-regular elements of G onto p-regular F-conjugacy classes. Our main results depend on the Witt-Berman Theorem [14, Ch.17, Theorem 5.3], which says the number of non-isomorphic simple FG-modules is equal to number of F-conjugacy classes of p-regular elements of G.

The problem of determining the unit group structure of group algebras is a classical problem. Many researchers have shown interest in characterizing the structures of unit groups of FG. Some of the interesting results can be seen in [2-5, 7-13, 26, 27]. Sharma, Srivastava and Khan in [15, 23-25] established the structure of the unit groups of the group algebras for the finite groups D_{10}, S_3, A_4 and S_4. Makhijani, Sharma and Srivastava [16-18] characterized the unit group of the group algebras for some dihedral groups. Further, Sahai and Ansari [1, 20-22], characterized the unit group of group algebras for the abelian groups of order up to 24. Recently, Bhatt and Chandra [6] characterized the unit group of group algebras for the abelian groups of order 32. In this paper we have classified the complete structure of unit group of the group algebra for the group of order 37, namely C_{37}.

2. Preliminaries

Lemma 2.1. [20, Lemma 2.3] *Let F be a finite field of characteristic p with $|F| = q = p^n$. Then*

$$U(FC_{p^k}) \cong \begin{cases} C_p^{n(p-1)} \times C_{p^n-1} & \text{if } k = 1; \\ \prod_{s=1}^{k} C_{p^s}^{h_s} \times C_{p^n-1}, & \text{otherwise}, \end{cases}$$

where $h_k = n(p-1)$ and $h_s = np^{k-s-1}(p-1)^2$ for all s, $1 \le s < k$.

Lemma 2.2. [19] *Let G be a group and R be a commutative ring. Then the set of all finite class sums forms an R-basis of $Z(RG)$, the center of RG.*

Lemma 2.3. [19] *Let FG be a semi-simple group algebra. If G' denotes the* *commutator subgroup of G, then*

$$FG = FG_{e_{G'}} \oplus \Delta(G, G')$$

where $FG_{e_{G'}} \cong F(G/G')$ *is the sum of all commutative simple components of* FG *and* $\Delta(G, G')$ *is the sum of all the others.*

3. Main Results

Theorem 3.1. *Let F be a finite field of characteristic p with* $|F| = q = p^n$ *and G* $\cong C_{37}$.
(A) *Let* $p = 37$. *Then,* $U(FC_{37}) \cong C_{37}^{36n} \times C_{37^n-1}$.
(B) *Let* $p \neq 37$. *Then, we have:*

1. *If* $q \equiv 1$ *mod 37, then* $U(FC_{37}) \cong C_{p^n-1}^{37}$.
2. *If* $q \equiv -1$ *mod 37, then* $U(FC_{37}) \cong C_{p^{2n}-1}^{18} \times C_{p^n-1}$.
3. *If* $q \equiv \pm 2,\ -3,\ \pm 5, -11,\ -13, \pm 15,\ \pm 17,\ \pm 18$ *mod 37, then* $U(FC_{37}) \cong C_{p^{36n}-1} \times C_{p^n-1}$.
4. *If* $q \equiv 3,\ 4,\ -7,\ -9,\ -12,\ 13,\ -16$ *mod 37, then* $U(FC_{37}) \cong C_{p^{18n}-1}^{2} \times C_{p^n-1}$.
5. *If* $q \equiv -4,\ 7,\ 9,\ 12,\ 16$ *mod 37, then* $U(FC_{37}) \cong C_{p^{9n}-1}^{4} \times C_{p^n-1}$.
6. *If* $q \equiv \pm 6$ *mod 37, then* $U(FC_{37}) \cong C_{p^{4n}-1}^{9} \times C_{p^n-1}$.
7. *If* $q \equiv \pm 8$ *mod 37, then* $U(FC_{37}) \cong C_{p^{12n}-1}^{3} \times C_{p^n-1}$.
8. *If* $q \equiv -10,\ 11$ *mod 37, then* $U(FC_{37}) \cong C_{p^{6n}-1}^{6} \times C_{p^n-1}$.
9. *If* $q \equiv 10$ *mod 37, then* $U(FC_{37}) \cong C_{p^{3n}-1}^{12} \times C_{p^n-1}$.
10. *If* $q \equiv \pm 14$ *mod 37, then* $U(FC_{37}) \cong C_{p^{12n}-1}^{3} \times C_{p^n-1}$.

Proof. The Group C_{37} is given by:

$$C_{37} = < r \mid r^{37} = 1 > .$$

(A) If $p = 37$, then $|F| = q = (37)^n$ and $G \cong C_{37}$ thus using Lemma 2.1, we get

$$U(FC_{37}) \cong C_{37}^{36n} \times C_{(37)^n-1}.$$

(B) If $p \neq 37$, then p does not divides $|C_{37}|$, therefore FC_{37} is semisimple over F. Now using Wedderburn decomposition theorem and by Lemma 2.3, we have

$FC_{37} \cong \oplus_{i=1}^{r} M(n_i, K_i))$, where for each i, $n_i \geq 1$ and $K_i's$ are finite field extension of F. Since group is abelian, therefore dimension constraint gives $n_i = 1$, for every i. It is clear that C_{37} has 37 conjugacy classes. Here $x^{q^k} = x$, for all $x \in Z(FC_{37})$, for any $k \in \mathbb{N}$ iff $\mathscr{C}_i^{q^t} = \mathscr{C}_i$, for every $1 \leq i \leq 37$, where $\mathscr{C}_i$ denotes the conjugacy class of C_{37}. This holds if and only if $37|q^s - 1$ or $37|q^s + 1$. Now if $k_i^* =< y_i >$, for all i , $1 \leq i \leq r$, then $x^{q^s} = x$, for all $x \in Z(FC_{37})$ if and only if $y_i^{q^s} = 1$, which satisfied if and only if $[K_i : F]|s$, for all $1 \leq i \leq r$. Therefore the least number t,

$$t = l.c.m.\{[K_i : F]|1 \leq i \leq r\}.$$

Therefore all conjugacy classes of C_{37} are p-regular hence $m = 37$ as described in introduction section. By observation we have following possibilities for q:

1. If $q \equiv 1 \ mod \ 37$, then t=1;
2. If $q \equiv -1 \ mod \ 37$, then t=2;
3. If $q \equiv 2 \ mod \ 37$, then t=36;
4. If $q \equiv -2 \ mod \ 37$, then t=36;
5. If $q \equiv 3 \ mod \ 37$, then t=18;
6. If $q \equiv -3 \ mod \ 37$, then t=36;
7. If $q \equiv 4 \ mod \ 37$, then t=18;
8. If $q \equiv -4 \ mod \ 37$, then t=9;
9. If $q \equiv 5 \ mod \ 37$, then t=36;
10. If $q \equiv -5 \ mod \ 37$, then t=36;

11. If $q \equiv 6 \ mod \ 37$, then t=4;

12. If $q \equiv -6 \ mod \ 37$, then t=9;

13. If $q \equiv 7 \ mod \ 37$, then t=9;

14. If $q \equiv -7 \ mod \ 37$, then t=18;

15. If $q \equiv 8 \ mod \ 37$, then t=12;

16. If $q \equiv -8 \ mod \ 37$, then t=12;

17. If $q \equiv 9 \ mod \ 37$, then t=9;

18. If $q \equiv -9 \ mod \ 37$, then t=18;

19. If $q \equiv 10 \ mod \ 37$, then t=3;

20. If $q \equiv -10 \ mod \ 37$, then t=6;

21. If $q \equiv 11 \ mod \ 37$, then t=6;

22. If $q \equiv -11 \ mod \ 37$, then t=36;

23. If $q \equiv 12 \ mod \ 37$, then t=9;

24. If $q \equiv -12 \ mod \ 37$, then t=18;

25. If $q \equiv 13 \; mod \; 37$, then t=18;

26. If $q \equiv -13 \; mod \; 37$, then t=36;

27. If $q \equiv 14 \; mod \; 37$, then t=12;

28. If $q \equiv -14 \; mod \; 37$, then t=12;

29. If $q \equiv 15 \; mod \; 37$, then t=36;

30. If $q \equiv -15 \; mod \; 37$, then t=36;

31. If $q \equiv 16 \; mod \; 37$, then t=9;

32. If $q \equiv -16 \; mod \; 37$, then t=18;

33. If $q \equiv 17 \; mod \; 37$, then t=36;

34. If $q \equiv -17 \; mod \; 37$, then t=36;

35. If $q \equiv 18 \; mod \; 37$, then t=36;

36. If $q \equiv -18 \; mod \; 37$, then t=36;

Now we will find T and the number of p-regular $F-$ conjugacy classes denoted by c. Using Lemma 2.2, we have $dim_F(Z(FC_{37})) = 37$, therefore $\sum_{i=1}^{r}[K_i : F] = 37$ and it gives the following cases:

1. If $q \equiv 1 \, mod \; 37$, then T=$\{1\}$ mod 37. Thus $p-$ regular $F-$ conjugacy classes are the conjugacy classes of C_{37} and c=37. Hence $FC_{37} \cong F^{37}$.

2. If $q \equiv -1$ *mod* 37, then T=$\{-1, 1\}$ *mod* 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r^{\pm 1}\}, \{r^{\pm 2}\}, \{r^{\pm 3}\}, \{r^{\pm 4}\}, \{r^{\pm 5}\}, \{r^{\pm 6}\}, \{r^{\pm 7}\}, \{r^{\pm 8}\}, \{r^{\pm 9}\}$, $\{r^{\pm 10}\}, \{r^{\pm 11}\}, \{r^{\pm 12}\}, \{r^{\pm 13}\}, \{r^{\pm 14}\}, \{r^{\pm 15}\}, \{r^{\pm 16}\}, \{r^{\pm 17}\}, \{r^{\pm 18}\}$. and $c = 19$. Hence, if F_k denotes the finite field of order q^k, then $FC_{37} \cong F_2^{18} \oplus F$.

3. If $q \equiv 2$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$ *mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

4. If $q \equiv -2$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

5. If $q \equiv 3$ *mod* 37, then T= $\{1, 3, 4, 7, 9, 10, 11, 12, 16, 21, 25, 26, 27, 28, 30, 33, 34, 36\}$ *mod* 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r^{\pm 1}, r^{\pm 3}, r^{\pm 4}, r^{\pm 7}, r^{\pm 9}, r^{\pm 10}, r^{\pm 11}, r^{\pm 12}, r^{\pm 16},\}, \{r^{\pm 2}, r^{\pm 6}, r^{\pm 8}, r^{\pm 14}, r^{\pm 18}, r^{\pm 5}, r^{\pm 13}, r^{\pm 15}, r^{\pm 17}\}$ and $c = 3$. Hence $FC_{37} \cong F_{18}^2 \oplus F$.

6. If $q \equiv -3$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$, and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

7. If $q \equiv 4$ *mod* 37, then T= $\{1, 3, 4, 7, 9, 10, 11, 12, 16, 21, 25, 26, 27, 28, 30, 33, 34, 36\}$ *mod* 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r^{\pm 1}, r^{\pm 3}, r^{\pm 4}, r^{\pm 7}, r^{\pm 9}, r^{\pm 10}, r^{\pm 11}, r^{\pm 12}, r^{\pm 16},\}, \{r^{\pm 2}, r^{\pm 6}, r^{\pm 8}, r^{\pm 14}, r^{\pm 18}, r^{\pm 5}, r^{\pm 13}, r^{\pm 15}, r^{\pm 17}\}$ and $c = 3$. Hence $FC_{37} \cong F_{18}^2 \oplus F$.

8. If $q \equiv -4$ *mod* 37, then T=$\{1, 7, 9, 10, 12, 16, 26, 33, 34\}$ *mod* 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r, r^7, r^9, r^{-17}, r^{10}, r^{12}, r^{16}, r^{-11}, r^{-4}, r^{-3}\}$, $\{r^2, r^{14}, r^{18}, r^{-17}, r^{-13}, r^{-5}, r^{15}, r^{-8}, r^{-6},\}, \{r^3, r^{-16}, r^{-10}, r^{-7}, r^{-1}, r^{11}, r^4, r^{-12}, r^{-9}\}$, $\{r^5, r^{-2}, r^8, r^{13}, r^{-14}, r^6, r^{-18}, r^{17}, r^{-15}\}$ and $c = 5$. Hence $FC_{37} \cong F_9^4 \oplus F$.

9. If $q \equiv 5$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

10. If $q \equiv -5$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

11. If $q \equiv 6 \; mod \; 37$, then T=$\{1, 6, 31, 36\}$ mod 37. Thus $p-$regular $F-$conjugacy classes are $\{1\}$,$\{r^{\pm1}, r^{\pm6}\}$, $\{r^{\pm2}, r^{\pm12}\}$, $\{r^{\pm3}, r^{\pm18}\}$, $\{r^{\pm4}, r^{\pm13}\}$, $\{r^{\pm5}, r^{\pm7}\}$, $\{r^{\pm8}, r^{\pm11}\}$, $\{r^{\pm9}, r^{\pm17}\}$, $\{r^{\pm10}, r^{\pm14}\}$, $\{r^{\pm15}, r^{\pm16}\}$ and $c = 10$. Hence $FC_{37} \cong F_4^9 \oplus F$.

12. If $q \equiv -6 \; mod \; 37$, then T=$\{1, 7, 9, 10, 12, 16, 26, 33, 34\}$ mod 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r, r^7, r^9, r^{-17}, r^{10}, r^{12}, r^{16}, r^{-11}, r^{-4}, r^{-3}\}$, $\{r^2, r^{14}, r^{18}, r^{-17}, r^{-13}, r^{-5}, r^{15}, r^{-8}, r^{-6}, \}$, $\{r^3, r^{-16}, r^{-10}, r^{-7}, r^{-1}, r^{11}, r^4, r^{-12}, r^{-9}\}$, $\{r^5, r^{-2}, r^8, r^{13}, r^{-14}, r^6, r^{-18}, r^{17}, r^{-15}\}$ and $c = 5$. Hence $FC_{37} \cong F_9^4 \oplus F$.

13. If $q \equiv 7 \; mod \; 37$, then T=$\{1, 7, 9, 10, 12, 16, 26, 33, 34\}$ mod 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r, r^7, r^9, r^{-17}, r^{10}, r^{12}, r^{16}, r^{-11}, r^{-4}, r^{-3}\}$, $\{r^2, r^{14}, r^{18}, r^{-17}, r^{-13}, r^{-5}, r^{15}, r^{-8}, r^{-6}, \}$, $\{r^3, r^{-16}, r^{-10}, r^{-7}, r^{-1}, r^{11}, r^4, r^{-12}, r^{-9}\}$, $\{r^5, r^{-2}, r^8, r^{13}, r^{-14}, r^6, r^{-18}, r^{17}, r^{-15}\}$ and $c = 5$. Hence $FC_{37} \cong F_9^4 \oplus F$.

14. If $q \equiv -7 \; mod \; 37$, then T=$\{1, 2, 3, 5, 6, 7, 11, 12, 13, 17, 21, 24, 26, 27, 28, 30, 31, 33\}$ mod 37. Thus $p-$regular $F-$conjugacy classes are $\{1\}$, $\{r, r^2, r^3, r^{-4}, r^5, r^{\pm6}, r^{\pm7}, r^{-9}, r^{-10}, r^{\pm11}, r^{12}, r^{\pm13}, r^{-16}, r^{17}\}$, $\{r^{-1}, r^{-2}, r^{-3}, r^4, r^{-5}, r^{\pm8}, r^9, r^{10}, r^{-12}, r^{\pm14}, r^{\pm15}, r^{16}, r^{-17}, r^{\pm18}\}$ and $c = 3$. Hence $FC_{37} \cong F_{18}^2 \oplus F$.

15. If $q \equiv 8 \; mod \; 37$, then T=$\{1, 6, 8, 10, 11, 14, 23, 26, 27, 29, 31, 36\}$ mod 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r^{\pm1}, r^{\pm6}, r^{\pm8}, r^{\pm10}, r^{\pm11}, r^{\pm14}\}$, $\{r^{\pm2}, r^{\pm9}, r^{\pm12}, r^{\pm15}, r^{\pm16}, r^{\pm17}\}$, $\{r^{\pm3}, r^{\pm4}, r^{\pm5}, r^{\pm7}, r^{\pm13}, r^{\pm18}\}$ and $c = 4$. Hence $FC_{37} \cong F_{12}^3 \oplus F$.

16. If $q \equiv -8 \; mod \; 37$, then T=$\{1, 6, 8, 10, 11, 14, 23, 26, 27, 29, 31, 36\}$ mod 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r^{\pm1}, r^{\pm6}, r^{\pm8}, r^{\pm10}, r^{\pm11}, r^{\pm14}\}$, $\{r^{\pm2}, r^{\pm9}, r^{\pm12}, r^{\pm15}, r^{\pm16}, r^{\pm17}\}$, $\{r^{\pm3}, r^{\pm4}, r^{\pm5}, r^{\pm7}, r^{\pm13}, r^{\pm18}\}$ and $c = 4$. Hence $FC_{37} \cong F_{12}^3 \oplus F$.

17. If $q \equiv 9 \; mod \; 37$, then T=$\{1, 7, 9, 10, 12, 16, 26, 33, 34\}$ mod 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r, r^7, r^9, r^{-17}, r^{10}, r^{12}, r^{16}, r^{-11}, r^{-4}, r^{-3}\}$, $\{r^2, r^{14}, r^{18}, r^{-17}, r^{-13}, r^{-5}, r^{15}, r^{-8}, r^{-6}, \}$, $\{r^3, r^{-16}, r^{-10}, r^{-7}, r^{-1}, r^{11}, r^4, r^{-12}, r^{-9}\}$, $\{r^5, r^{-2}, r^8, r^{13}, r^{-14}, r^6, r^{-18}, r^{17}, r^{-15}\}$ and $c = 5$. Hence $FC_{37} \cong F_9^4 \oplus F$.

18. If $q \equiv -9 \; mod \; 37$, then T=$\{1, 2, 3, 5, 6, 7, 11, 12, 13, 17, 21, 24, 26, 27, 28, 30, 31, 33\}$ mod 37. Thus $p-$regular $F-$conjugacy classes are $\{1\}$, $\{r, r^2, r^3, r^{-4}, r^5, r^{\pm6}, r^{\pm7}, r^{-9}, r^{-10}, r^{\pm11}, r^{12}, r^{\pm13}, r^{-16}, r^{17}\}$, $\{r^{-1}, r^{-2}, r^{-3}, r^4, r^{-5}, r^{\pm8}, r^9, r^{10}, r^{-12}, r^{\pm14}, r^{\pm15}, r^{16}, r^{-17}, r^{\pm18}\}$ and $c = 3$. Hence $FC_{37} \cong F_{18}^2 \oplus F$.

19. If $q \equiv 10 \; mod \; 37$, then T=$\{1, 10, 26\} \; mod \; 37$. Thus $p-$regular $F-$ conjugacy classes are $\{1\}$, $\{r, r^{10}, r^{-11}\}$, $\{r^2, r^{-17}, r^{15}\}$, $\{r^3, r^{-7}, r^4\}$, $\{r^5, r^{13}, r^{-18}\}$, $\{r^6, r^{-14}, r^8\}$, $\{r^7, r^{-4}, r^{-3}\}$, $\{r^9, r^{16}, r^{12}\}$, $\{r^{11}, r^{-1}, r^{-10}\}$, $\{r^{14}, r^{-8}, r^{-6}\}$, $\{r^{17}, r^{-15}, r^{-2}\}$, $\{r^{18}, r^{-5}, r^{-13}\}$, $\{r^{-16}, r^{-12}, r^{-9}\}$ and $c = 13$. Hence $FC_{37} \cong F_3^{12} \oplus F$.

20. If $q \equiv -10 \; mod \; 37$, then T=$\{1, 10, 11, 26, 27, 36\} \; mod \; 37$. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r^{\pm 1}, r^{\pm 10}, r^{\pm 11}, \}$, $\{r^{\pm 2}, r^{\pm 15}, r^{\pm 17}\}$, $\{r^{\pm 3}, r^{\pm 4}, r^{\pm 7}\}$, $\{r^{\pm 5}, r^{\pm 13}, r^{\pm 18}, \}$, $\{r^{\pm 6}, r^{\pm 8}, r^{\pm 14}\}$, $\{r^{\pm 9}, r^{\pm 12}, r^{\pm 16}\}$ and $c = 7$. Hence $FC_{37} \cong F_6^6 \oplus F$.

21. If $q \equiv 11 \; mod \; 37$, then T=$\{1, 10, 11, 26, 27, 36\} \; mod \; 37$. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r^{\pm 1}, r^{\pm 10}, r^{\pm 11}, \}$, $\{r^{\pm 2}, r^{\pm 15}, r^{\pm 17}\}$, $\{r^{\pm 3}, r^{\pm 4}, r^{\pm 7}\}$, $\{r^{\pm 5}, r^{\pm 13}, r^{\pm 18}, \}$, $\{r^{\pm 6}, r^{\pm 8}, r^{\pm 14}\}$, $\{r^{\pm 9}, r^{\pm 12}, r^{\pm 16}\}$ and $c = 7$. Hence $FC_{37} \cong F_6^6 \oplus F$.

22. If $q \equiv -11 \; mod \; 37$, then T= $\{j \mid 1 \leq j \leq 36, \, j \in \mathbb{N}\} mod \; 37$. Thus $p-$regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i}, \, 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

23. If $q \equiv 12 \; mod \; 37$, T=$\{1, 7, 9, 10, 12, 16, 26, 33, 34\} \; mod \; 37$. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r, r^7, r^9, r^{-17}, r^{10}, r^{12}, r^{16}, r^{-11}, r^{-4}, r^{-3}\}$, $\{r^2, r^{14}, r^{18}, r^{-17}, r^{-13}, r^{-5}, r^{15}, r^{-8}, r^{-6}, \}$, $\{r^3, r^{-16}, r^{-10}, r^{-7}, r^{-1}, r^{11}, r^4, r^{-12}, r^{-9}\}$, $\{r^5, r^{-2}, r^8, r^{13}, r^{-14}, r^6, r^{-18}, r^{17}, r^{-15}\}$ and $c = 5$. Hence $FC_{37} \cong F_9^4 \oplus F$.

24. If $q \equiv -12 \; mod \; 37$, then T=$\{1, 2, 3, 5, 6, 7, 11, 12, 13, 17, 21, 24, 26, 27, 28, 30, 31, 33\} \; mod \; 37$. Thus $p-$regular $F-$conjugacy classes are $\{1\}$, $\{r, r^2, r^3, r^{-4}, r^5, r^{\pm 6}, r^{\pm 7}, r^{-9}, r^{-10}, r^{\pm 11}, r^{12}, r^{\pm 13}, r^{-16}, r^{17}\}$, $\{r^{-1}, r^{-2}, r^{-3}, r^4, r^{-5}, r^{\pm 8}, r^9, r^{10}, r^{-12}, r^{\pm 14}, r^{\pm 15}, r^{16}, r^{-17}, r^{\pm 18}\}$ and $c = 3$. Hence $FC_{37} \cong F_{18}^2 \oplus F$.

25. If $q \equiv 13 \; mod \; 37$, then T=$\{1, 2, 3, 5, 6, 7, 11, 12, 13, 17, 21, 24, 26, 27, 28, 30, 31, 33\}$ $mod \; 37$. Thus $p-$regular $F-$conjugacy classes are $\{1\}$, $\{r, r^2, r^3, r^{-4}, r^5, r^{\pm 6}, r^{\pm 7}, r^{-9}, r^{-10}, r^{\pm 11}, r^{12}, r^{\pm 13}, r^{-16}, r^{17}\}$, $\{r^{-1}, r^{-2}, r^{-3}, r^4, r^{-5}, r^{\pm 8}, r^9, r^{10}, r^{-12}, r^{\pm 14}, r^{\pm 15}, r^{16}, r^{-17}, r^{\pm 18}\}$ and $c = 3$. Hence $FC_{37} \cong F_{18}^2 \oplus F$.

26. If $q \equiv -13 \; mod \; 37$, then T= $\{j \mid 1 \leq j \leq 36, \, j \in \mathbb{N}\} mod \; 37$. Thus $p-$regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i}, \, 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

27. If $q \equiv 14 \; mod \; 37$, then T=$\{1, 6, 8, 10, 11, 14, 23, 26, 27, 29, 31, 36\} \; mod \; 37$. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r^{\pm 1}, r^{\pm 6}, r^{\pm 8}, r^{\pm 10}, r^{\pm 11}, r^{\pm 14}\}$,

$\{r^{\pm 2}, r^{\pm 9}, r^{\pm 12}, r^{\pm 15}, r^{\pm 16}, r^{\pm 17}\}$, $\{r^{\pm 3}, r^{\pm 4}, r^{\pm 5}, r^{\pm 7}, r^{\pm 13}, r^{\pm 18}\}$ and $c = 4$. Hence $FC_{37} \cong F_{12}^3 \oplus F$.

28. If $q \equiv -14$ *mod* 37, then T=$\{1, 6, 8, 10, 11, 14, 23, 26, 27, 29, 31, 36\}$ *mod* 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r^{\pm 1}, r^{\pm 6}, r^{\pm 8}, r^{\pm 10}, r^{\pm 11}, r^{\pm 14}\}$, $\{r^{\pm 2}, r^{\pm 9}, r^{\pm 12}, r^{\pm 15}, r^{\pm 16}, r^{\pm 17}\}$, $\{r^{\pm 3}, r^{\pm 4}, r^{\pm 5}, r^{\pm 7}, r^{\pm 13}, r^{\pm 18}\}$ and $c = 4$. Hence $FC_{37} \cong F_{12}^3 \oplus F$.

29. If $q \equiv 15$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

30. If $q \equiv -15$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$, and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

31. If $q \equiv 16$ *mod* 37, then T=$\{1, 7, 9, 10, 12, 16, 26, 33, 34\}$ *mod* 37. Thus $p-$ regular $F-$ conjugacy classes are $\{1\}$, $\{r, r^7, r^9, r^{-17}, r^{10}, r^{12}, r^{16}, r^{-11}, r^{-4}, r^{-3}\}$, $\{r^2, r^{14}, r^{18}, r^{-17}, r^{-13}, r^{-5}, r^{15}, r^{-8}, r^{-6}, \}$, $\{r^3, r^{-16}, r^{-10}, r^{-7}, r^{-1}, r^{11}, r^4, r^{-12}, r^{-9}\}$, $\{r^5, r^{-2}, r^8, r^{13}, r^{-14}, r^6, r^{-18}, r^{17}, r^{-15}\}$ and $c = 5$. Hence $FC_{37} \cong F_9^4 \oplus F$.

32. If $q \equiv -16$ *mod* 37, then T=$\{1, 2, 3, 5, 6, 7, 11, 12, 13, 17, 21, 24, 26, 27, 28, 30, 31, 33\}$ *mod* 37. Thus $p-$regular $F-$conjugacy classes are $\{1\}$, $\{r, r^2, r^3, r^{-4}, r^5, r^{\pm 6}, r^{\pm 7}, r^{-9}, r^{-10}, r^{\pm 11}, r^{12}, r^{\pm 13}, r^{-16}, r^{17}\}$, $\{r^{-1}, r^{-2}, r^{-3}, r^4, r^{-5}, r^{\pm 8}, r^9, r^{10}, r^{-12}, r^{\pm 14}, r^{\pm 15}, r^{16}, r^{-17}, r^{\pm 18}\}$ and $c = 3$. Hence $FC_{37} \cong F_{18}^2 \oplus F$.

33. If $q \equiv 17$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

34. If $q \equiv -17$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

35. If $q \equiv 18$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

36. If $q \equiv -18$ *mod* 37, then T= $\{j \mid 1 \leq j \leq 36,\ j \in \mathbb{N}\}$*mod* 37. Thus $p-$ regular $F-$conjugacy classes are $\{1\}$, $\{r^{\pm i},\ 1 \leq i \leq 18\}$ and $c = 2$. Hence $FC_{37} \cong F_{36} \oplus F$.

References

[1] Ansari, S. F. and Sahai, M., Unit groups of group algebras of groups of order 20, Quaes. Math., (2020), 1-9.

[2] Balogh, Zs., The structure of the unit group of some group algebras, Miskolc Math. Notes, 21(2) (2020), 615–620.

[3] Balogh, Zs. and Bovdi, A., Group algebras with unit group of class p, Publ. Math. Debrecen, 65, no. 3-4 (2004), 261–268.

[4] Balogh, Zs. and Bovdi, A., On units of group algebras of 2-groups of maximal class, Comm. Algebra, 32, no. 8 (2004), 3227–3245.

[5] Bhatt, S. and Chandra, H., Structure of unit group of $F_{p^n}D_{60}$, Asian-Eur. J. Math, (2020), https://doi.org/10.1142/S1793557121500753.

[6] Bhatt, S. and Chandra, H., Unit Groups of group algebras of abelian groups of order 32, Proy. Jr. Math. Accepted (2021).

[7] Bovdi, V. and Salim, M., Group algebras whose groups of normalized units have exponent 4, Czechoslovak Math. J., 68(143), no. 1 (2018), 141–148.

[8] Bovdi, Adalbert, The group of units of a group algebra of characteristic p, Publ. Math. Debrecen, 52, no. 1-2 (1998), 193-244.

[9] Creedon, L., The unit group of small group algebras and the minimum counter example to the isomorphism problem, Int. J. Pure. Appl. Math., 49 (2008), 531-537.

[10] Creedon, L. and Gildea, J., The Structure of the unit group of the group algebra $F_{3^k}D_6$, Int. J. Pure. Appl. Math., 45(2) (2008), 315-320.

[11] Ferraz, R. A., Simple components of the center of $FG/J(FG)$, Commun. Algebra, 36(9) (2008), 3191–3199.

[12] Gildea, J. and O'Malley, S., Units of the Group Algebra of the Group $C_n \times D_8$ over Any Finite Field of Characteristic 2, Malays. J. Math. Sci., 15(2) (2021), 303-312.

[13] Gildea, J., Units of the Group Algebra $F_3^k(C_3^2 \rtimes C_2)$, Acta Sci. Math. (Szeged), 77(1-2) (2011), 63-72.

[14] Karpilovsky, G., Group Representations, Vol(1) Introduction to Group Representations and Characters, North Holland Mathematics Studies, Amsterdam, North Holland Publishing Co., (175), 1992.

[15] Khan, M., Structure of the unit group of FD_{10}, Serdica Math. J., 35(1) (2009), 15-24.

[16] Makhijani, N., Sharma R. K. and Srivastava J. B., Unit in $F_{2^k}D_{2n}$, Int. J. Group Theory, 3(3) (2014), 25-34.

[17] Makhijani, N., Sharma R. K. and Srivastava J. B., The unit group of $F_q[D_{30}]$, Serdica Math. J., 41 (2015).

[18] Makhijani, N., Sharma R. K. and Srivastava J. B., A Note on Units in $F_{p^m}D_{2p^n}$, Acta Math. Acad. Paedagog. Nyi'r, 30 (2014), 17-25.

[19] Milies, C. P. and Sehgal, S. K., An introduction to Group Rings. Algebras and Applications, Bol. Soc. Brasil. Mat., 1 (2002).

[20] Sahai, M. and Ansari, S. F., Unit groups of finite group algebras of abelian groups of order at most 16, Asi. Eur. J. Math., (2020) 2150030.

[21] Sahai, M. and Ansari, S. F., Unit groups of group algebras of groups of order 18, Comm. Alg., (2021), https://doi.org/10.1080/00927872.2021.1893740.

[22] Sahai, M. and Ansari, S. F., Group of Units of Finite Group Algebras of Groups of Order 24, (2020), http://arxiv.org/abs/2005.05014v1.

[23] Sharma, R. K., Srivastava, J. B. and Khan, M., The unit group of FA_4, Pub. Math. Debrecen, 71 (2007), 21-26.

[24] Sharma, R. K., Srivastava, J. B. and Khan, M., The unit group of FS_3, Acta Math. Acad. Paed. Ny´ıregyh´azi., 23(2) (2007), 129-142.

[25] Sharma, R. K., Srivastava, J. B. and Khan, M., The Unit Group of FS_4, Acta. Math. Hungar., 118(1-2) (2008), 105-113.

[26] Tang, G. and Gao, Y., The unit group of FG of group with order 12, Int. J. Pure Appl. Math., 73(1) (2011), 143–158.

[27] Tang, G., Wei, Y. and Li, Y., Unit groups of group algebras of some small groups, Czech. Math. J., 64 (2014), 149–157.