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A CRYPTOGRAPHIC SCHEME WITH LAPLACE-CARSON TRANSFORM AND SANDIP'S METHOD

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Abstract: In this paper, authors have used Laplace-Carson transform of exponential, hyperbolic and algebraic functions and their various combinations for Cryptography with Sandip's method. Sandip's method is the procedure given to Encryption and Decryption of message. In Sandip's method two keys are provided that gives high security to the message. Various examples with different values of variables are given to show applicability of Laplace-Carson transform with Sandip's method.

Keywords and Phrases: Mahgoub Transform, Laplace-Carson transform, Cryptography, Laplace transform, Sandip transform.

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1. Introduction

Now a days the world has accepted digitalization in all sectors like banking, digital payments, google account etc. everywhere it is in need to secure our transaction or data (information). Cryptography is most widely technique used to secure our data. It passes our massage to the receiver and third person do not understand it. G. Naga Laxmmi, et.al [10] gave Cryptography scheme with Laplace transform of exponential function. A. Hiwarekar [5-6] generalized the concept with hyperbolic function. A. Chinde [1] uses Natural transform with hyperbolic function to write message in cipher text. The integral transform method is widely used in

Engineering, many applications of Laplace transform was given by Debnath [2], Vasishta[10]. Laplace-Carson transform is redefined by Mahgoub [9] in 2016 and apply it to solve differential equations. Kiwne and Sonawane [8] gave properties and applications of Laplace-Carson transform. As Laplace-Carson transform has unit preserving property it is more applicable than other transforms. Sonawane and Kiwne [12] introduced Sandip transform and gave its properties with the help of H-function.

Cryptography contains three steps, first one is plain text string means original massage, second is cipher set means encryption data and the last step is decryption of cipher set. Now a days, when we open an Email account we receive, one time password and we see Cap-cha on computer screen, both are related with some mathematical method so that our account become secure. We called this one time password as key and Cap-cha as cipher set in our new method. In this paper we use Laplace-Carson transform with Sandip's method to from cipher set of plain text. We have given the method with two keys that can be send through mobile and Email and the Cap-cha will display on screen for public. To convert plain text string into cipher set, we have used series expansion of algebraic, exponential and hyperbolic function with its Laplace-Carson transform and Inverses. The procedure used in this paper, we named it as Sandip's method.
Let us start with definition Laplace-Carson transform,

Definition 1.1. Laplace-Carson transform [9] *Let $\theta(w)$, be exponentially ordered function then the Laplace-Carson transform is,*

$$M\left[\theta(w)\right] = v_1 \int_0^\infty e^{-wv_1}\theta(w)dw \ = \Theta(v_1) \tag{1}$$

here v_1 is real or complex number. The Inverse Laplace-Carson transform is,

$$M^{-1}[\Theta(v_1)] = \theta(w)$$

1.1. Some Results on Laplace-Carson Transform

Some properties of Laplace-Carson transform are [8],
$M[e^{aw}] = \frac{v_1}{v_1 - a}$
$M[w^n] = \frac{n!}{v_1^n}, n \geq 0$ and n is integer
$M[\cosh aw] = \frac{v_1^2}{v_1^2 - a^2}$
$M[\theta^n(w)] = v_1^n \theta(v_1) - \sum\limits_{k=0}^{n-1} v_1^{n-k}\theta^k(0)$

Theorem 1.2. (for more detail [8])
If $\theta(w) = \sum_{j=0}^{\infty} c_j w^j$ *and* $|c_j| \leq \frac{Bk^j}{j!}$ *for sufficiently large* k *and* $B, j > 0$ *then,*

$$M[\theta(w)] = \sum_{j=0}^{\infty} c_j \frac{j!}{v_1^j}$$

2. Sandip's Method for Cryptography

Consider the function

$$\theta(w) = \frac{w}{(1-w)}, \quad |w| < 1$$

then we can write it in series expansion as,

$$\theta(w) = \sum_{j=1}^{\infty} w^j$$

and its Laplace-Carson transform using theorem (1.2)

$$M[\theta(w)] = \sum_{j=1}^{\infty} \frac{j!}{v_1^j}$$

2.1. Procedure for Encryption

1. Consider alphabets A to Z with numbers 0 to A, 1 to B and so on 25 to Z and denote then as F_i^1.

2. Find F_i^2, using $F_i^1 = F_i^2 (mod\ b_1)$ with $k_{1i} = \frac{F_i^1 - F_i^2}{b_1}$ called key one, here $i = 1, 2, ...$ Also find $F_i^3 = \frac{F_i^2}{10}$, so that we get numbers between 0 and 1.

3. Use these values as a coefficients in the series expansion of $\theta(w)$ and apply Laplace-Carson transform to the series, we get new coefficients say F_i^4.

4. Find F_i^5 using $F_i^4 * 10 = F_i^5 (mod\ b_2)$ with $k_{2i} = \frac{F_i^4 * 10 - F_i^5}{b_2}$ called key second, here $i = 1, 2, ...$

5. Finally, we get Encrypted message with two keys.

2.2. Procedure for Decryption

1. Using Encrypted massage find $F_i'^1$ and calculate $F_i'^2 = \frac{F_i'^1 + k_{2i} * b_2}{10}$, using key second $k_{2i}, i = 1, 2, ...$
2. Use these values as a coefficients in the series expansion of $M[\theta(w)]$ and apply Inverse Laplace-Carson transform to the series, we get new coefficients say $F_i'^3$.
3. Find $F_i'^4 = F_i'^3 * 10$ and with help of key one obtain $F_i'^5 = F_i'^4 + b_1 k_{2i}, i = 1, 2, ...$
4. Finally we get original message with $F_i'^5 = F_i^5, i = 1, 2, ...$

Example 2.1. Encryption: Let the given message in plain text string be YOU . Given message can written as,

$$24 \ 14 \ 20$$

Using Sandip's method with $b_1 = 9$, $b_2 = 10$ we get F_1 and key one k_1 as,

$$F_1 = \ 6 \ 5 \ 2 \text{ with } k_1 = \ 2 \ 1 \ 2$$

Use values of F_1 in the series expansion of $\theta(w)$ by dividing 10 and take its Laplace-Carson transform, we get

$$F\theta(w) = 0.6w + 0.5w^2 + 0.2w^3$$

$$M[F\theta(w)] = \frac{0.6}{v_1} + \frac{1}{v_1^2} + \frac{1.2}{v_1^3}$$

Take coefficients by multiplying 10 and find F_1', with key second,

$$F_1' = \ 6 \ 0 \ 2 \text{ with } k_2 = \ 0 \ 1 \ 1$$

The cipher text is GAC.
We send one key by mobile, second key by Email and cipher text in public.
Decryption:
Take the cipher text GAC with key second $k_2 = \ 0 \ 1 \ 1$, we get

$$F_1' = \ 6 \ 0 \ 2 \text{ and } F_2' = \ 6 \ 10 \ 12$$

Use $F_3' = \frac{F_2'}{10}$ as coefficients in series expansion of Laplace-Carson transform of $M\left[\frac{Fw}{1-w}\right]$, we write

$$M[F\theta(w)] = \frac{0.6}{v_1} + \frac{1}{v_1^2} + \frac{1.2}{v_1^3}$$

Taking Inverse Laplace-Carson transform, we may get

$$F\theta(w) = 0.6w + 0.5w^2 + 0.2w^3$$

Take coefficients by multiplying 10 and using key one $k_1 =$ 2 1 2, we get numbers,

$$F_1 = \text{ 24 14 20 with plain text } YOU$$

Theorem 2.2. *The message given using plain text string in terms of F_i, $i = 1, 2, ...$ such that F_i are the coefficients of the series expansion of $F\theta(w) = F\frac{w}{1-w}$ can be converted in to F_i' using Laplace-Carson transform and Sandip's Method for encryption, where,*

$$F_i' = g_i * 10 + k_{2i} * b_2,\ i = 1, 2, ... \tag{2}$$

and

$$g_i = s_i * i!,\ s_i = \frac{F_i - k_{1i} * b_1}{10},\ i = 1, 2, ... \tag{3}$$

with key one $k_{1i} = \frac{s_i - F_i}{b_1}$ and key second $k_{2i} = \frac{F_i' - g_i * 10}{b_2}$, $i = 1, 2, ...$

Theorem 2.3. *The message given using cipher text string in terms of F_i', $i = 1, 2, ...$ such that F_i are the coefficients of the series expansion of*

$$M[\theta(w)] = M\left[\frac{w}{1-w}\right] = \sum_{i=1}^{\infty} \frac{s_i'}{v_1^i} \tag{4}$$

can be converted in to F_i' using Inverse Laplace-Carson transform and Sandip's Method for decryption, where,

$$F_i = s_i' - k_{1i} * b_1,\ i = 1, 2, ... \tag{5}$$

and

$$s_i' = \frac{g_i'}{i!} * 10,\ F_i' = \frac{g_i' + k_{2i} * b_2}{10},\ i = 1, 2, ... \tag{6}$$

2.3. General Concept

In the above example if we take $b_1 = b_2 = 9$ then with the same key one and key second the plain text YOU is converted in to cipher text GBD.
We can generalized this method by taking the function

$$F\left(\frac{wx^t}{1 - wx^t}\right), t = 1, 2, ...$$

As the value of x and t changes, we get new cipher text and different key one and key second.

Example 2.4. In Example [2.1] Let $b_1 = b_2 = 9$ and $x = 2,\ t = 1$
Given message can written as,

$$24\ 14\ 20$$

Lets find F_1 and key one k_1 using $(mod\ 9)$ we get,

$$F_1 =\ 6\ 5\ 2 \text{ with } k_1 =\ 2\ 1\ 2$$

and using Sandip's method we get,

$$F_1' =\ 3\ 4\ 6 \text{ with } k_2 =\ 0\ 4\ 10$$

The cipher text is DEG.

Example 2.5. In Example [2.1] Let $b_1 = 9, b_2 = 10$ and $x = 2,\ t = 1$
Given message can written as,

$$24\ 14\ 20$$

Lets find F_1 and key one k_1 using $(mod\ 9)$ we get,

$$F_1 =\ 6\ 5\ 2 \text{ with } k_1 =\ 2\ 1\ 2$$

and using Sandip's method we get,

$$F_1' =\ 2\ 0\ 6 \text{ with } k_2 =\ 1\ 4\ 9$$

The cipher text is CAG.

Example 2.6. In Example [2.1] Let $b_1 = 9, b_2 = 10$ and $x = 3,\ t = 1$
The cipher text is IAE.

$$k_1 =\ 2\ 1\ 2 \text{ with } k_2 =\ 1\ 9\ 32$$

Example 2.7. Let the plain string be $MOBILE$ and $b_1 = 9, b_2 = 10$ and $x = 1,\ t = 1$
Given message can written as,

$$12\ 14\ 1\ 8\ 11\ 4$$

Lets find F_1 and key one k_1 using $(mod\ 9)$ we get,

$$F_1 = 3\ 5\ 1\ 8\ 2\ 4 \text{ with } k_1 =\ 1\ 1\ 0\ 0\ 1\ 0$$

and using Sandip's method we get,

$$F_1' = 3\ 0\ 6\ 2\ 0\ 0 \text{ with } k_2 = \ 0\ 1\ 0\ 19\ 24\ 288$$

The cipher text is $DAGCAA$.

2.4. Use of exponential and Hyperbolic function

Let

$$e^w = 1 + w + \frac{w^2}{2!} + ...$$

and

$$coshw = 1 + \frac{w^2}{2!} + \frac{w^4}{4!} + ...$$

then

$$e^w coshw = 1 + w + w^2 + \frac{8w^3}{3!} + \frac{8w^4}{4!} + \frac{16w^5}{5!} + ...$$

Here we observe that the series pattern is not fixed, if we take this series expansion with F_i and x, we get

$$Fwe^{xw}coshxw = F_1 w + F_2 w^2 x + F_3 w^3 x^2 + F_4 \frac{8w^4x^3}{3!} + F_5 \frac{8w^5x^4}{4!} + F_6 \frac{16w^6x^5}{5!} + ... \quad (7)$$

Use equation (7) as a function in the Sandip's method, we get new cipher text string.

Example 2.8. Let the plain string be $MOBILE$ and $b_1 = 5, b_2 = 10$ and $x = 1,\ t = 1$

Given message can written as,

$$12\ 14\ 1\ 8\ 11\ 4$$

Lets find F_1 and key one k_1 using $(mod\ 5)$ we get,

$$F_1 = 2\ 4\ 1\ 3\ 1\ 4 \text{ with } k_1 = \ 2\ 2\ 0\ 1\ 2\ 0$$

and using Sandip's method with equation (7) we get,

$$M[Fwe^{xw}coshxw] = \frac{0.2}{v_1} + \frac{0.8}{v_1^2} + \frac{0.6}{v_1^3} + \frac{9.6}{v_1^4} + \frac{4}{v_1^5} + \frac{38.4}{v_1^6}$$

we get,

$$F_1' = 2\ 8\ 6\ 6\ 0\ 4 \text{ with } k_2 = \ 0\ 0\ 0\ 9\ 4\ 38$$

The cipher text is $CIGGAE$.

3. Conclusion

The method used in this article for Cryptography scheme is can be applied in banking sector for verifying password of online account or any online transaction. As it generate two keys so not easy to decode for the third person.

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