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# A MATHEMATICAL MODEL FOR ESTIMATING THE RELATIONSHIP BETWEEN PROLACTIN AND GROWTH HORMONES IN THE BLOOD OF THE MILKING GOAT TO MAXIMUM LIKELIHOOD

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**Abstract:** In this paper, the rise in Growth Hormone (GH) concentration differed from that of prolactin in various ways. The time elapsed from the commencement of milking to rise in GH level was longer (2 to 7 minutes). When Growth Hormone and prolactin concentrations rise in the same series of blood samples, the levels rise independently, with the peak concentration of GH happening over a greater range (4 to 36 mins). Exponential distributions are widely utilised in the field of life-testing in Mathematical Model. The approach of the Maximum Likelihood function will be used to estimate the parameters of an exponential distribution with two components.

**Keywords and Phrases:** Growth Hormone, Maximum Likelihood Estimation, Prolactin.

**2020 Mathematics Subject Classification:** 60GXX, 62HXX, 62PXX.

## 1. Introduction

One of the most essential components of inferential statistics is parameter estimation and making conclusions based on the estimated parameters [9]. Manickam in his study used to describe about real data set in their studies. One can generally mean that data set has just one law in modelling and estimation problems

[12] [13]. As a result, high-quality exponential distributions including the Gamma, Weibull, and Generalized Exponential have been introduced into literatures [1]. The exponential distribution is a common continuous distribution. It aids in the calculation of the time between events. It's used in a variety of applications, including dependability theory, queuing theory, and physics. It has a lot in common with the Poisson distribution [11]. The time between each death can be modelled using a Poisson distribution; if the number of deaths is described using a Poisson distribution is one of the best illustration [18].

## 2. Mathematical Model

In a Poisson process, the exponential distribution is a probability distribution that represents the duration between events. Negative exponential distribution [15] is another name for it. It is a continuous Probability distribution that is used to depict the amount of time we must wait for an event to occur [10]. It is the discrete geometric distribution's constant counterpart. It also possesses [3].

If a probability density function is of the form:

$$f\left(z\right)=\frac{1}{\beta}e^{\frac{z-\alpha}{\beta}},\ z\geq\ \alpha,\ \beta\ >0 \tag{1}$$

has an exponential distribution, then the random variable Z has an exponential distribution.

Let $Z_{1:p},\ Z_{2:p},\ \cdot\ \cdot\ \cdot, Z_{p:p}$ to be an ordered random sample from an exponential distribution [8]. In exponential distributions, parameter estimation is widely explored. Parameter estimate in exponential distributions is frequently thought of in the context of a specific application situation [12]. Censored samples, reduced populations, and cases where the shift parameter is supposed to be known are all variations of this scenario [10] [16]. Exponential distributions of the form (1) are discussed here.

### 2.1. Maximum Likelihood Estimation (MLE)

If $Z_1,\ Z_2,\ \ldots..,\ Z_p$ are all independent random variables with the same probability density function, (1), then the MLE of $\alpha$ and $\beta$ are $\widehat{\alpha}_L =\ Z_{1:p}$, the minimum and $\widehat{\beta}_L = \overline{Z} -\ Z_{1:p}$ where $\overline{Z}$ is the sample mean. It can be easily shown that $E\left(\widehat{\alpha}_L\right) =\ \alpha+\frac{\beta}{p}$ and $\left(\widehat{\alpha}_L\right)=\frac{\beta^2}{p^2}$. [11].

It's to prove that

$$V\left(\widehat{\alpha}_L\right)=\frac{\beta^2}{p}(1-\frac{1}{p}) \tag{2}$$

Note the difference in $V\left(\widehat{\alpha}_L\right)$

$$V^*\left(\widehat{\alpha}_L\right) = \ \beta^2\left(\frac{1}{p}+\frac{1}{p^2}+\frac{2}{p^3}\right)$$

To demonstrate (2), let's introduce some results:
**Result (a)** For $1 \leq k \leq p$,

$$\mu_{1:p} = E(Z_{1:p}) = \ \alpha+\beta\sum_{s=1}^{k}\frac{1}{p-k+s}$$

$$\begin{aligned}
\mu_{1:p} = E(Z_{1:p}) &= \frac{p!}{(k-1)!\,(p-k)!}\int_{-\infty}^{\infty} z_i\left\{F\left(z_k\right)\right\}^{k-1}\ \left\{1-F\left(z_k\right)\right\}^{p-1}\ \mathrm{f}\left(z_k\right)dz_k \\
&= \frac{p!}{(k-1)!\,(p-k)!}.\ \frac{1}{\beta}\int_{-\infty}^{\infty} z_k\left(e^{-\frac{z_k-\alpha}{\beta}}\right)^{k-1}\ \left(1-\left(e^{-\frac{z_k-\alpha}{\beta}}\right)\right)^{p-1}\left(e^{-\frac{z_k-\alpha}{\beta}}\right)dz_k \\
&= \frac{p!}{(k-1)!\,(p-k)!}.\ \frac{1}{\beta}\sum_{s=0}^{k-1}(-1)^s\binom{k-1}{s}\ \int_{-\infty}^{\infty} z_k e^{-(p-k+1+s)\frac{z_k-\alpha}{\beta}}dz_k \\
&= \frac{p!}{(k-1)!\,(p-k)!}.\ \frac{1}{\beta}\sum_{s=0}^{k-1}(-1)^s\binom{k-1}{s}\ \frac{1}{p-k+1+s}\left(\alpha+\frac{\beta}{p-k+1+s}\right) \\
&= \ \alpha+\beta\sum_{s=1}^{k}\frac{1}{p-k+s}
\end{aligned}$$

$$\sum_{s=0}^{k-1}(-1)^s\binom{k-1}{s}\ \frac{1}{p-k+1+s} = \frac{\Gamma\left(k\right)\Gamma(p-k+1)}{\Gamma(p+1)} = \frac{(k-1)!\ \ (p-k)!}{p!}$$

$$\begin{aligned}
\sum_{s=0}^{k-1}(-1)^s\binom{k-1}{s}\ \frac{1}{\left(p-k+1+s\right)^2} &= \frac{\Gamma\left(k\right)\Gamma\left(p-k+1\right)}{\Gamma\left(p+1\right)}\left(H\left[p\right]-H\left[p-k\right]\right) \\
&= \frac{\Gamma\left(k\right)\Gamma\left(p-k+1\right)}{\Gamma\left(p+1\right)}\sum_{s=1}^{k}\ \frac{1}{p-k+s}
\end{aligned}$$

where $H[p]$ stands for the Harmonic Number

$$\sum_{s=1}^{k}\ \frac{1}{s}$$

**Result (b)** For $1 \le k \le p$, we have

$$\mu_{k:p}^{(2)} = E\left(Z_{k:p}^{2}\right)$$
$$= \alpha^2 + 2\alpha\beta \sum_{s=1}^{k} \frac{1}{p-k+s} + \beta^2 \left[ \sum_{s=1}^{k} \left( \frac{1}{p-k+s} \right)^2 + \left( \sum_{s=1}^{k} \frac{1}{p-k+s} \right)^2 \right]$$

For $1 \le k \le p$,

$$\mu_{1:p} = E(Z_{k:p}^{2} = \frac{p!}{(k-1)!\,(p-k)!} \int_{-\infty}^{\infty} z_k^2 \left\{F\left(z_k\right)\right\}^{k-1} \; \left\{1 - F\left(z_k\right)\right\}^{p-1} \; \mathrm{f}\left(z_k\right) dz_k$$
$$= \frac{p!}{(k-1)!\,(p-k)!} \cdot \frac{1}{\beta} \int_{-\infty}^{\infty} z_k^2 \left(e^{-\frac{z_k-\alpha}{\beta}}\right)^{k-1} \; \left(1 - \left(e^{-\frac{z_k-\alpha}{\beta}}\right)\right)^{p-1} \left(e^{-\frac{z_k-\alpha}{\beta}}\right) dz_k$$
$$= \frac{p!}{(k-1)!\,(p-k)!} \cdot \frac{1}{\beta} \sum_{s=0}^{k-1} (-1)^s \binom{k-1}{s} \int_{-\infty}^{\infty} z_k^2 e^{-(p-k+1+s)\frac{z_k-\alpha}{\beta}} dz_k$$
$$= \frac{p!}{(k-1)!\,(p-k)!} \cdot$$
$$\frac{1}{\beta} \sum_{s=0}^{k-1} (-1)^s \binom{k-1}{s} \left( \frac{\alpha^2}{p-k+1+s} + \frac{2\alpha\beta}{(p-k+1+s)^2} + \frac{\beta^2}{p-k+1+s} \right)$$
$$= \alpha^2 + 2\alpha\beta \sum_{s=1}^{k} \frac{1}{p-k+s} + \beta^2 \left[ \left( \sum_{s=1}^{k} \frac{1}{p-k+s} \right)^2 + \sum_{s=1}^{k} \left( \frac{1}{p-k+s} \right)^2 \right]$$

Since

$$\sum_{s=0}^{k-1} (-1)^s \binom{k-1}{s} \frac{1}{p-k+1+s} = \frac{\Gamma(k)\,\Gamma(p-k+1)}{\Gamma(p+1)} = \frac{(k-1)! \; (p-k)!}{p!}$$
$$\sum_{s=0}^{k-1} (-1)^s \binom{k-1}{s} \frac{1}{(p-k+1+s)^2} = \frac{\Gamma(k)\,\Gamma(p-k+1)}{\Gamma(p+1)} (H[p] - H[p-k])$$
$$= \frac{(k-1)!\,(p-k)!}{p!} \sum_{s=1}^{k} \frac{1}{p-k+s}$$
$$\sum_{s=1}^{k} \frac{1}{s}$$

**Result (c)** For $j-k \geq 2$,

$$\mu_{i,j:p} = E\left(Z_{k:p}Z_{j:p}\right)$$
$$= \alpha^2 + 2\alpha\beta \sum_{s=1}^{k} \frac{1}{p-k+s} + \beta^2 \left[ \sum_{s=1}^{k} \left( \frac{1}{p-k+s} \right)^2 + \left( \sum_{s=1}^{k} \frac{1}{p-k+s} \right)^2 \right]$$
$$+ \ \alpha\beta \sum_{s=1}^{j-k} \frac{k}{p-j+s} + \ \beta^2 \left( \sum_{s=1}^{j-k} \frac{k}{p-j+s} \right) \left( \sum_{s=1}^{k} \frac{k}{p-k+s} \right)$$

For $k-j \geq 2$,

$$\mu_{k,\ j:p} = E(Z_k Y_j)$$
$$= \frac{p!}{(k-1)!\,(j-k-1)!\,(p-j)!} \int_{-\infty}^{\infty} z_k \left\{F\left(z_k\right)\right\}^{k-1}$$
$$\left[ \int_{z_k}^{\infty} \left\{F\left(z_j\right) - F(z_k)\right\}^{j-k-1} z_j \left\{1 - F(z_k)\right\}^{p-j} f\left(z_j\right) dz_j \right] f\left(z_k\right) dz_k$$
$$= \frac{p!}{(k-1)!\,(j-k-1)!\,(p-j)!} \int_{-\infty}^{\infty} z_k \left\{F\left(z_k\right)\right\}^{k-1} I_1 f\left(z_k\right) dz_k$$
$$I_1 = \int_{z_k}^{\infty} \left\{F\left(z_j\right) - F(z_k)\right\}^{j-k-1} z \left\{1 - F(z_k)\right\}^{p-j} f\left(z_j\right) dz_j$$

Then

$$\mu_{k,\ j:p} = \frac{p!}{(k-1)!\,(j-k-1)!\,(p-j)!} \int_{-\infty}^{\infty} z_k \left\{F\left(z_k\right)\right\}^{k-1} I_1 f\left(z_k\right) dz_k$$
$$= \frac{p!}{(k-1)!\,(p-k)!}$$
$$\sum_{s=0}^{k-1} (-1)^s \binom{k-1}{s} \int_{-\infty}^{\infty} z_k \left( z_k + \beta \sum_{s=1}^{j-k} \frac{1}{p-j+s} \right) e^{-(p-k+1+s)\frac{z_k-\alpha}{\beta}} dz_k$$

**Result (d)** For $1 \leq k \leq p-1$,

$$\mu_{k,j:p} = \alpha^2 + \alpha\beta \sum_{s=1}^{k} \frac{1}{p-k} + 2 \sum_{s=1}^{k} \frac{1}{p-k+s}$$
$$+ \beta^2 \left[ \frac{1}{p-k} \sum_{s=1}^{k} \frac{1}{p-k+s} + \sum_{s=1}^{k} \left( \frac{1}{p-k+s} \right)^2 + \left( \sum_{s=1}^{k} \frac{1}{p-k+s} \right)^2 \right]$$

Based on standard result,

$$V\left(\widehat{\beta}_L\right) = V\left(\overline{Z} - Z_{1:p}\right) = V\left(\overline{Z}\right) - V(Z_{1:p}) - 2Cov\left(\overline{Z},\ Z_{1:p}\right)$$

where

$V\left(\overline{Z}\right) = \frac{\beta^2}{p},\ V\left(Z_{1:p}\right) = \frac{\beta^2}{p^2}$ and

$$\begin{aligned}\text{Cov}\left(\overline{Z},\ Z_{1:p}\right) &= \frac{1}{p}\text{Cov}\left(\ Z_{1:\ p}\sum_{k=1}^{p} Z_{1:p}\right)\\ &= \frac{1}{p}V\left(Z_{1:p}\right) + \frac{1}{p}\text{Cov}\left(\ Z_{1:p}Z_{2:p}\right) + \frac{1}{p}\sum_{k=3}^{p}\text{Cov}\left(\ Z_{3:p}Z_{k:p}\right)\end{aligned}$$

Since

$$\text{Cov}\left(\ Z_{1:p}Z_{2:p}\right) = E\left(\ Z_{1:p}Z_{2:p}\right) - E\left(\ Z_{1:p}\right)\ E(Z_{2:p})$$

Using result (d), we get

$$E\left(\ Z_{1:p}Z_{2:p}\right) = \alpha^2 + \alpha\beta\left(\frac{2}{p} + \frac{1}{p-1}\right) + \beta^2\left(\frac{2}{p^2} + \frac{1}{p(p-1)}\right)$$

Applying result (a), we obtain $E\left(\ Z_{2:p}\right) = \alpha + \beta\frac{2p-1}{p(p-1)}$

After simplifying, we get $\text{Cov}\left(Z_{1:p}Z_{2:p}\right) = \frac{\beta^2}{p^2}$

$$E\left(\ Z_{1:p}Z_{2:p}\right) = \alpha^2 + \alpha\beta\left(\frac{2}{p} + \sum_{s=1}^{k-1}\frac{1}{p-k+s}\right) + \beta^2\left(\frac{2}{p^2} + \sum_{s=1}^{k-1}\frac{1}{p-k+s}\right)$$

$$\text{Cov}\left(\ Z_{1:p}Z_{2:p}\right) = E\left(\ Z_{1:p}Z_{2:p}\right) - E\left(\ Z_{1:p}\right)\ E(Z_{2:p}) = \frac{\beta^2}{p^2}$$

With $\text{Cov}\left(\overline{Z},\ Z_{1:p}\right) = \frac{1}{p}\left(\frac{\beta^2}{p^2} + \frac{\beta^2}{p^2} + (p-2)\frac{\beta^2}{p^2}\right) = \frac{\beta^2}{p^2}$

We find that $V\left(\widehat{\beta}_L\right) = \frac{\beta^2}{p}(1-\frac{1}{p})$ and $\text{Cov}\left(\widehat{\alpha}_L, \widehat{\beta}_L\right) = Cov\left(\ Z_{1:p}, \overline{Z} - Z_{1:p}\ \right) = 0$

### 2.2. The Principle of Maximum Likelihood

Considering maximizing the likelihood function $L_P(\theta;\ z_1, \ldots, z_P)$ with respect to $\theta$. Since Log function is monotonically increasing, we usually maximize $\ln\text{L}P(\theta;\ z_1,\ \ldots\ ,\ z_P)$ instead [14] [17]. In this case:

$$L_P(\theta;\ z_1,\ \ldots, z_P)\ = -\theta P\ +\ \ln(\theta)\ \sum_{k=1}^{P} z_k - \ln\left(\prod_{k=1}^{P} z_k!\right)$$

$$\frac{\partial L_P(\theta;\ z_1, \ldots, z_P)}{\partial\theta} = -P + \frac{1}{\theta}\sum_{k=1}^{P} z_k$$

$$\frac{\partial L_P\,(\theta;\ z_1, \ldots, z_P)}{\partial\theta^2} = -\frac{1}{\theta^2}\sum_{k=1}^{P} z_k < 0$$

The MLE is defined as follows under proper regularity conditions:

$$\widehat{\theta} = \begin{matrix}\arg\max \\ \theta \in \mathbb{R}^+\end{matrix}\ \mathrm{lnL}_P(\theta;\ z_1,\ \ldots\ ,\ z_P)$$

$$SOC : \frac{\partial^2 L_P(\theta;\ z_1, \ldots, z_P)}{\partial\theta^2} = -\frac{1}{\theta^2}\sum_{k=1}^{P} z_k < 0$$

$\widehat{\theta}$ is a maximum

$$\widehat{\theta} \equiv \widehat{\theta}(z) = \frac{1}{P}\sum_{k=1}^{P} z_k$$

The MLE (random variable) is:

$$\widehat{\theta} = \frac{1}{P}\sum_{k=1}^{P} Z_k$$

### 2.3. Unbiased Maximum Likelihood estimation (UMLE)

The unbiased estimates which are linear functions of the UMLE mentioned [9] are $\widehat{\alpha}_U = \frac{1}{p-1}\left(pZ_{1:p} - \overline{Z}\right)$ and $\widehat{\beta}_U = \frac{1}{p-1}\left(\overline{Z} - Z_{1:p}\right)$ with $\mathrm{V}\left(\widehat{\alpha}_U\right) = \frac{\beta^2}{p(p-1)}$, $V\left(\widehat{\beta}_U\right) = \frac{\beta^2}{p-1}$ and $\mathrm{Cov}\left(\widehat{\alpha}_U,\ \widehat{\beta}_U\right) = -\frac{\beta^2}{p(p-1)}$ .

## 3. Prolactin and Growth Hormones

When it was discovered that twice-daily delivery of the milking stimulus promoted both breast development and lactation in the ovariectomized virgin goat, the potential of pituitary hormones to create undergrowth and the goat's ability to induce milk secretion was convincingly demonstrated [5].

Lactotroph cells in the pituitary gland generate and emit prolactin (PRL), a polypeptide hormone. The hypothalamus generally inhibits the release of PRL, which makes it unique among anterior pituitary hormones. PRL is a hormone produced by the pituitary gland that is important for a range of reproductive processes. Dopaminergic tracts produce dopamine (prolactin-inhibiting hormone, PIH) in the median eminence, which exerts this effect [19].

In various species, PRL has been described as a remarkable marker of both acute and chronic stress response. The use of biological matrices other than blood in the research of hormones is gaining favour due to the lower invasiveness of collection [20]. The most well-known function of PRL in goats is to stimulate mammary gland development and lactation processes. Despite this, PRL is engaged in a number of homeostatic and physiological tasks in the body, including electrolyte balance, luteal function, immune system modulation, osmoregulation, angiogenesis, and organ maintenance [4].

PRL and GH, which are produced by the pituitary gland, appear to be important in the development of breastfeeding in goats. The pituitary gland is told to stop releasing follicle-stimulating hormone (FSH) and start producing more LH as oestrogen levels rise. Higher circulating amounts of oestrogen have been suggested as a possible explanation for the increased PRL in pregnant women [6]. Immune cells, epidermis, adipose tissue, breasts, and the uterus all produce PRL hormone in addition to the pituitary gland.

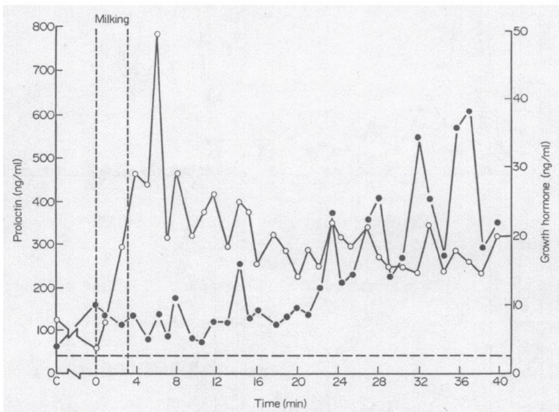


Figure 1: Prolactin (○) and growth hormone (●) concentrations in blood samples obtained from Goat 357 before, during, and after milking were detected in the same series. C stands for "control blood sample.

The concentration of PRL in man differs from that of other mammals in that it rises gradually over the last half of pregnancy till parturition [7]. Higher circulating levels of oestrogen have been proposed as a possible explanation for the elevated prolactin in pregnant women [5]. PRL is the most important hormone for reproductive health. Both males and females generate and secrete this hormone, which is produced and secreted in the front region of the pituitary gland in the brain [2].

Prolactin has a half-life of around 20 minutes because it circulates unbound to serum proteins. The circulating estrogens and progesterone levels drop sharply after the placenta is expelled at parturition. Lactation begins when oestrogen levels in the blood diminish. Researchers have confirmed that suckling frequently results in stimulation and rise in hormone concentrations in the blood [8]. Radioimmunoassay approaches for detecting prolactin in a variety of species have been developed.

## 4. Mathematical Result

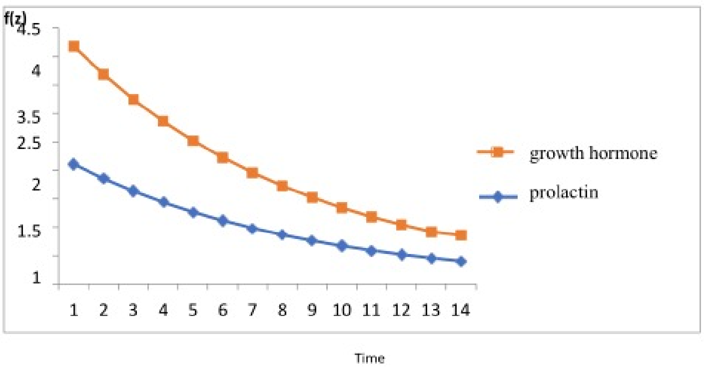


Figure 2: shows the probability density function $f(z)$.

## 5. Conclusion

According to the findings, the concentrations of GH and PRL increase in goats, and the levels of GH and prolactin rise separately. The peak level of GH is reached faster than the peak level of PRL. The probability density function was shown to be monotonic in the mathematical model. The probability density function is used to compare PRL and GH. The curves for PRL and GH are both concave upward and monotonically declining.

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