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A METHOD FOR SOLVING ASSIGNMENT PROBLEM USING RANKING OF CENTROID OF CENTROIDS IN TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER

K. Dhilipkumar and C. Praveenkumar*

Department of Mathematics,
SSM College of Arts and Science,
Komarapalayam - 638183, Tamil Nadu, INDIA

E-mail : dhilipkumarmaths@gmail.com

*PG and Research Department of Mathematics,
Bharathidasan College of Arts and Science,
Erode - 638116, Tamil Nadu, INDIA

E-mail : praveenmukunth@gmail.com

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Abstract: In this paper, intuitionistic fuzzy assignment problem with ordering of trapezoidal intuitionistic fuzzy number (TIFN) is considered. The centroid of centroids in trapezoidal intuitionistic fuzzy numbers(TIFN's) is evaluated using a modern ranking technique. Furthermore, the proposed ranking method was used to find the best solution to the intuitionistic fuzzy assignment problem. Finally, a numerical illustration of the method is shown.

Keywords and Phrases: Intuitionistic fuzzy number - trapezoidal intuitionistic fuzzy number - triangular intuitionistic fuzzy number- ranking of trapezoidal intuitionistic fuzzy number.

2020 Mathematics Subject Classification: 03E72, 03F55, 90B06, 90B99.

1. Introduction

The notion of fuzzy sets was introduced by Zadeh [14] and generalization of the fuzzy sets into intuitionistic fuzzy sets which includes the non-membership function was proposed by Atanassov [3]. Ranking of fuzzy numbers plays a vital role in fuzzy set theory and intuitionistic fuzzy numbers (IFNs) seems proper for interpreting the uncertainty followed by the generalized fuzzy numbers. Grzegorzewski P. [5] proposed the process of ordering triangular intuitionistic fuzzy numbers (T_rIFN's) by accepting the statistical outlook and thereby explaining each IFN's as a group of ordinary fuzzy numbers.

Based on membership and non-membership function ranking [1], triangular intuitionistic fuzzy numbers(T_rIFN's) are attained by converting two related triangular fuzzy numbers(T_rFN's) [4]. Then a new defuzzification for the acquired TFN's utilizing their values and vagueness were recommended by Salahshour et.al. [9]. Seikh et al. [10] gave another example of using a modified perspective to choose investment options, as well as the primary arithmetic operations of generalised triangular intuitionistic fuzzy numbers (GT_rIFN's). Centroid of an intuitionistic fuzzy number was established by Annie Varghesel et.al. [2] and B. Pardhasaradhi et.al [8] introduced the idea of centroid of centroids in TIFN. The method of ranking of TIFN with centroid of centroids index uses the geometric centre of a TIFN. The Geometric center corresponds to $\bar{x}(\tilde{A})$ value on the horizontal axis and $\bar{y}(\tilde{A})$ vertical axis.

The assignment problem (AP) is a particular case of a linear programming problem that deals with the allocation of various resources for various activities on a one-one basis. It does so in such a manner that the profit or sale involved in the process is maximum and cost or time is minimum. Therefore, to solve the AP under an intuitionistic fuzzy environment in this paper, the author proposes the concept of ranking of centroid of centroids.

The PSK method for solving fully intuitionistic fuzzy assignment problems with some software tools was introduced by Kumar, P. S [6]. Suresh et.al. [12, 13] pioneered the idea of solving intuitionistic fuzzy linear programming by using ranking function and assignment problem. A.Nagoorgani and V. M. Mohamed [7] developed the algorithm for solving assignment problems having ordering generalized trapezoidal intuitionistic fuzzy numbers as costs. B. Srinivas and G. Ganesan [11] used a branch and bound technique for solving intuitionistic fuzzy assignment problem.

Furthermore, while there are several ways for solving assignment problems with intuitionistic fuzzy costs in the literature, none of them used the ordering centroid of centroids ranking method for intuitionistic trapezoidal fuzzy costs. In this paper, the ranking of centroid of centroids in trapezoidal intuitionistic fuzzy number is entrenched and a numerical example is presented to the proposed method.

2. Preliminaries

In this section, we observed the primary concepts of ordering trapezoidal intuitionistic fuzzy numbers and discussed the arithmetic operations of such numbers.

Definition 2.1. *(Intuitionistic Fuzzy Set). Let X be the universal set and the membership function, non-membership functions defined on X. Then an intuitionistic fuzzy set(IFS) is defined as*

$$\hat{A} = \{\langle x, \alpha_{\hat{A}}(x), \beta_{\hat{A}}(x)\rangle / x \in X\}$$

where the $\alpha_{\hat{A}} : X \to [0,1]$ is the membership function and $\beta_{\hat{A}} : X \to [0,1]$ is the non-membership function of the element $x \in X$ respectively and for every $x \in X$, $0 \leq \alpha_{\hat{A}} + \beta_{\hat{A}} \leq 1$.
The value of non-determinacy or uncertainty of the element $x \in X$ is defined as

$$\pi_{\hat{A}}(x) = 1 - \alpha_{\hat{A}}(x) - \beta_{\hat{A}}(x)$$

Definition 2.2. *(Intuitionistic Fuzzy Number). An intuitionistic fuzzy set $\hat{A} = \{\langle x, \alpha_{\hat{A}}(x), \beta_{\hat{A}}\rangle / x \in X\}$ is said to be an intuitionistic fuzzy number (IFN) if*

1. *$\hat{A}$ is intuitionistic fuzzy normal*
2. *$\hat{A}$ is intuitionistic fuzzy convex*
3. *The set $\hat{A} = \{x \in X / \beta_{\hat{A}}(x) < 1\}$ is bounded.*

Definition 2.3. *(Generalized Trapezoidal Intuitionistic Fuzzy and Triangular Intuitionistic fuzzy number). A trapezoidal intuitionistic fuzzy number (TIFN) with parameters $c_1 \leq d_1 \leq c_2 \leq d_2 \leq d_3 \leq c_3 \leq d_4 \leq c_4$ is represented by $\hat{A} = (c_1, d_1, c_2, d_2, d_3, c_3, d_4, c_4)$ with the membership and non-membership function is defined as*

$$\alpha_{\hat{A}}(x) = \begin{cases} 0 & x < d_1 \\ \frac{x-d_1}{d_2-d_1} & d_1 \leq x < d_2 \\ 1 & d_2 \leq x \leq d_3 \\ \frac{x-d_4}{d_3-d_4} & d_3 \leq x < d_4 \\ 0 & d_4 < x \end{cases}$$

$$\beta\hat{A}(x) = \begin{cases} 0 & x < c_1 \\ \frac{x-c_2}{c_1-c_2} & c_1 \leq x < c_2 \\ 0 & c_2 \leq x \leq c_3 \\ \frac{x-d_4}{d_3-d_4} & c_3 \leq x < c_4 \\ 1 & c_4 < x \end{cases}$$

Here, if $c_1 \leq d_1 \leq (c_2 = d_2 = d_3 = c_3) \leq d_4 \leq c_4$ *then the trapezoidal intuitionistic fuzzy number becomes Triangular intuitionistic fuzzy number is represented as* $\hat{A} = (c_1, d_1, c_2, d_4, c_4)$

Definition 2.4. *(Arithmetic Operations of* $TIFN$*). If* $\hat{A} = (c_1, d_1, c_2, d_2, d_3, c_3, d_4, c_4 : u_1, u_2)$ *and* $\hat{B} = (\bar{c_1}, \bar{d_1}, \bar{c_2}, \bar{d_2}, \bar{d_3}, \bar{c_3}, \bar{d_4}, \bar{c_4} : \bar{u_1}, \bar{u_2})$ *then*

- $$\hat{A} + \hat{B} = \begin{pmatrix} c_1 + \bar{c_1}, d_1 + \bar{d_1}, c_2 + \bar{c_2}, d_2 + \bar{d_2}, d_3 + \bar{d_3}, c_3 + \bar{c_3}, d_4 + \bar{d_4}, c_4 + \bar{c_4}; \max\{u_1, u_2, \bar{u_1}, \bar{u_2}\} \\ 0 < u_1, u_2, \bar{u_1}, \bar{u_2} \leq 1 \end{pmatrix}$$

- $$\hat{A} \times \hat{B} = \begin{pmatrix} c_1\bar{c_1}, d_1\bar{d_1}, c_2\bar{c_2}, d_2\bar{d_2}, d_3\bar{d_3}, c_3\bar{c_3}, d_4\bar{d_4}, c_4\bar{c_4}; \\ 0 < u_1, u_2, \bar{u_1}, \bar{u_2} \leq 1 \end{pmatrix}$$

- *If* $\hat{A} = (c_1, d_1, c_2, d_2, d_3, c_3, d_4, c_4 : u_1, u_2)$ *then*

$$\lambda\hat{A} = \begin{pmatrix} \lambda c_1, \lambda d_1, \lambda c_2, \lambda d_2, \lambda d_3, \lambda c_3, \lambda d_4, \lambda c_4; \\ 0 < u_1, u_2, \bar{u_1}, \bar{u_2} \leq 1 \end{pmatrix}$$

Definition 2.5. *(Ranking of Centroid of Centroids in* $TIFN$*). An ordering trapezoidal intuitionistic fuzzy number with parameters* $c_1 \leq d_1 \leq c_2 \leq d_2 \leq d_3 \leq c_3 \leq d_4 \leq c_4$ *is represented by* $\hat{A} = (c_1, d_1, c_2, d_2, d_3, c_3, d_4, c_4 : u_1, u_2)$ *then the ranking of centroid of centroids in trapezoidal intuitionistic fuzzy number is*

$$R(\hat{A}) = \left(\frac{(d_1 + c_1) + 5(d_3 + c_2) + 2(d_2 + c_3) + (d_4 + c_4)}{18}\right)\left(\frac{4u_1 + 5u_2}{18}\right)$$

3. Mathematical Formulation of Intuitionistic Fuzzy Assignment Problems

In this section, a mathematical formulation of intuitionistic fuzzy assignment problem is provided.

Suppose there are n jobs to be carry out and n persons are accessible for performing the jobs. Assume that each person can do each job at a time, depending on their efficiency to do the job. Let ξ_{ij} be the intuitionistic fuzzy cost if the $i-th$ person is assigned the $j-th$ job. The objective is to minimize the total intuitionistic fuzzy cost of assigning all the jobs to the available persons (one job to one person).

The intuitionistic fuzzy assignment problem can be expressed in the form of an $n \times n$ cost matrix $[\xi_{ij}]$ of intuitionistic fuzzy numbers is stated as follows

Persons	Jobs				
	1	2	3	...	N
1	ξ_{11}	ξ_{12}	ξ_{13}	$\cdots$	ξ_{1n}
2	ξ_{21}	ξ_{22}	ξ_{23}	$\cdots$	ξ_{2n}
.	.	.	.	.	.
N	ξ_{n1}	ξ_{n2}	ξ_{n3}	...	ξ_{nn}

Mathematical formulation of the intuitionistic fuzzy assignment problem is given by
Minimize $\hat{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \xi_{ij} x_{ij}$
Subject to $\sum_{j=1}^{n} x_{ij}, i = 1, 2, \ldots n; \sum_{i=1}^{n} x_{ij}, j = 1, 2, \ldots n$

$$x_{ij} = \begin{cases} 1, & \text{if the ith person is assigned to the jth job} \\ 0, & \text{otherwise} \end{cases}$$

is the decision variable denoting the assignment of the person i to job j. ξ_{ij} is the cost of assigning the $j-th$ job to the $i-th$ person.

4. Algorithm for Proposed Method

Step 1: Test whether the given intuitionistic fuzzy cost matrix is a balanced one or not. If the given cost matrix is balanced one then go to the step 2 otherwise introduce the dummy columns (rows) with zero cost to form the cost matrix as a balanced one.

Step 2: Determine the rank for each element ξ_{ij} of the given intuitionistic fuzzy cost matrix by applying centroid of centroids ranking method. Using all these values, obtain the reduced cost matrix.

Step 3: Find the optimal assignment schedule using hungarian method in the reduced cost matrix and thus we get one marked zero in each row and each column of $R[\xi_{ij}]$.

Step 4: Find the optimum fuzzy cost by adding the intuitionistic fuzzy numbers corresponding to the cells having a marked zero.

5. Numerical Example

In this section, a numerical computation is presented to the proposed algorithm.

Consider the assignment problem whose elements are ordering of intuitionistic fuzzy number as follows

works \ jobs	1	2	3	4
I	(1,3,5,7,9,11, 13,15 : 0.7,0.2)	(2,6,10,14,18,22, 26,30 : 0.6,0.2)	(4,4,5,6,7,8,9,10 : 1,0)	(3,6,9,12,15,18, 21,24 : 0.5,0.4)
II	(2,6,10,14,18,22, 26,30 : 0.4,0.3)	(-3,-1,1,3,5,7, 9,11 : 0.6,0.2)	(8,9,10,11,12,13, 14,15: 0.5,0.3)	(1,3,5,7,9,11,13, 15 : 0.7,0.2)
III	(0.5,1,1.5,2,2.5, 3, 3.5,4 :0.7,0.1)	(8,11,14,17,20,23, 26,29 :0.8,0.1)	(6,7,8,9,10,11, 12, 13:0.5,0.3)	(-2,2,6,10,14,18, 22,26 : 0.4,0.3)
IV	(4,4,5,6,7,8,9, 10 : 1,0)	(1,1,3,3,5,7,9,11 : 0.8,0.1)	(2,2,4,5,6,7, 8,8:0.4,0.3)	(12,13,14,15,16, 17,18,19:0.4,0.2)

Solution

Step 1: The number of persons is equal to number of jobs in the given problem. So, the problem is balanced.

Step 2:

$$R(\xi_{11}) = \left(\frac{3+1+5(5+9)+2(7+11)+(11+13)}{18}\right)\left(\frac{4(0.7)+5(0.2)}{18}\right)$$
$$= 7.667 * 0.211 = 1.618$$

Similarly we have the other ranks as follows

$$R(\xi_{12}) = 2.898; R(\xi_{13}) = 1.418; R(\xi_{14}) = 2.886$$

$$R(\xi_{21}) = 2.637; R(\xi_{22}) = 0.693; R(\xi_{23}) = 2.199; R(\xi_{24}) = 1.618$$
$$R(\xi_{31}) = 0.397; R(\xi_{32}) = 3.708; R(\xi_{33}) = 1.811; R(\xi_{34}) = 1.632$$
$$R(\xi_{41}) = 1.418; R(\xi_{42}) = 0.939; R(\xi_{43}) = 3.327; R(\xi_{44}) = 0.898$$

Step 3: From the above rankings, the reduced matrix is

$$\begin{pmatrix} 1.618 & 2.898 & 1.418 & 2.886 \\ 2.637 & 0.639 & 2.119 & 1.618 \\ 0.397 & 3.708 & 1.811 & 1.632 \\ 1.418 & 0.937 & 0.898 & 3.327 \end{pmatrix}$$

$$\begin{pmatrix} 0.2 & 1.48 & 0 & 1.468 \\ 1.944 & 0 & 1.426 & 0.925 \\ 0 & 3.311 & 1.771 & 1.235 \\ 0.520 & 0.039 & 0 & 2.429 \end{pmatrix}$$

$$\begin{pmatrix} 0.2 & 1.48 & 0 & 1.468 \\ 1.944 & 0 & 1.426 & 0.925 \\ 0 & 3.311 & 1.771 & 1.235 \\ 0.520 & 0.039 & 0 & 2.429 \end{pmatrix}$$

$$\begin{pmatrix} 0.161 & 1.441 & \boxed{0} & 0.504 \\ 1.944 & 0 & 1.465 & \boxed{0} \\ \boxed{0} & 3.311 & 1.810 & 0.310 \\ 0.481 & \boxed{0} & 0 & 1.465 \end{pmatrix}$$

The optimum Assignment Schedule is $I \to 3; II \to 4; III \to 1; IV \to 2$.

Step 4: The Optimum fuzzy cost $= \xi_{13}+\xi_{24}+\xi_{31}+\xi_{42} = (4,4,5,6,7,8,9,10:1,0)+ (1,3,5,7,9,11,13,15:0.7,0.2) + (0.5,1,1.5,2,2.5,3,3.5,4:0.7,0.1) + (1,1,3,3,5,7,9,11:0.8,0.1) = (6.5,9,14.5,18,23,33,34.5,40:1,0) = 4.37$

6. Conclusion

In this paper, a new ranking method for $T_rIFN's$ and $TIFN's$ based on the centroid is proposed. Many decision-making and optimization problems of uncertain nature can be solved using this ranking. The optimal solution for ordering trapezoidal intuitionistic fuzzy numbers is very simple to understand using the proposed method. The proposed method is more time consuming than existing methods, and it can be used for both $T_rIFN's$ and $TIFN's$, and it accurately reflects the uncertainty. The proposed centroid based ranking technique is adaptable to the researchers ranking index of their attitudinal analysis.

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