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HALL CURRENT EFFECT ON DOUBLE DIFFUSIVE CONVECTION OF COUPLE-STRESS FERROMAGNETIC FLUID IN THE PRESENCE OF VARYING GRAVITATIONAL FIELD AND HORIZONTAL MAGNETIC FIELD THROUGH A POROUS MEDIA

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Abstract: The effect of hall current on a couple-stress ferromagnetic fluid heated and soluted from below in the presence of varying gravitational field and horizontal magnetic field through a porous media is considered. A linearized hypothesis and normal mode procedure are utilized to get dispersion relation. For the case of stationary convection, stable solute gradient has a stabilizing effect on the system. Medium permeability and couple-stress both have stabilizing and destabilizing effects under specific conditions. Additionally, magnetic field and hall current have both stabilizing as well as a destabilizing effect on the system under some conditions. It is likewise discovered that in the absence of stable solute gradient, magnetization has a stabilizing effect on the system. Oscillatory modes are introduced in the system in the presence of magnetic field (hence hall current) and stable solute gradient, though in their nonappearance, the principle of exchange of stabilities is satisfied in the system. Graphs also have been plotted by giving some numerical values to the parameters.

Keywords and Phrases: Double diffusive convection, ferromagnetic fluid, couple-stress fluid, hall current, porous media.

2020 Mathematics Subject Classification: 76A05, 76A10, 76D05, 76E25,

76M25, 76W05.

1. Introduction

There are various stability problems on couple-stress fluid and furthermore on ferromagnetic fluids. The convective instability also referred to as Bénard convection (Chandrasekhar [1]) is one among the stability problems of couple-stress and ferromagnetic fluids. Another stability problem on these fluids is double-diffusive (or thermosolutal) convection. There are many researchers, who have done a lot of work on these stability problems. The idea of couple-stress fluids was initiated by Stokes [23, 24]. Thermal convection of magneto compressible couple-stress fluid saturated in a porous medium with hall current has been discussed by Mehta et al. [9]. Kumar [6] has examined the effect of hall currents on the thermal instability of compressible dusty viscoelastic fluid saturated in a porous medium. Hall current effect on thermosolutal instability in a viscoelastic fluid through a porous medium has been studied by Kumar and Hari Mohan [7]. Kumar and Kumar [8] have discussed the thermosolutal convection in a viscoelastic dusty fluid with hall currents in a porous medium. Double diffusive convection of synovial (couple-stress) fluid in the presence of hall current through a porous medium has been discussed by Singh [22]. The effect of hall current on thermal convection has also been discussed by several authors (Sunil et al. [18], Gupta et al. [4], [5]). A definite introduction to magnetic liquids has been given by Rosensweig [15] in his monograph. The convective instability of ferromagnetic fluid for a layer heated from below in the presence of a uniform vertical magnetic field has been studied by Finlayson [2]. The Bénard convection in ferromagnetic fluids was mentioned by several authors (Siddheswar [20, 21], Venkatasubramaniam and Kaloni [32]). Sekar and Vaidyanathan [17] studied the convective instability of a magnetized ferrofluid in a rotating porous medium. Also, the convective instability of a layer of a ferromagnetic fluid rotating about a vertical axis was discussed by Gupta and Gupta [3]. The thermosolutal convection in a ferromagnetic fluid was examined by Sharma et al. [26]. Sunil et al. [27] discussed the effect of rotation on a ferromagnetic fluid heated and soluted from below saturating a porous medium. Also, the effect of the magnetic field-dependent viscosity on the thermosolutal convection in a ferromagnetic fluid saturating a porous medium was examined by Sunil et al. [28]. The effect of rotation on the double-diffusive convection in a magnetized ferrofluid with internal angular momentum was studied by Mahajan et al. [29]. The thermosolutal convection in a ferromagnetic fluid saturating a porous medium was investigated with the aid of using the results of Sunil et al. [30]. Also, Sunil et al. [31] studied the effect of the magnetic-field-dependent viscosity on the thermosolutal convection of a rotating ferromagnetic fluid saturating a porous medium. The magneto-rotational convection for ferromagnetic fluids in the presence of compressibility and heat source via a porous medium was studied with the aid of using the results of Sharma et al. [19]. Exploring magnetic dipole contribution on ferromagnetic nanofluid flow over a stretching sheet: An application of Stefan blowing was studied by Gowda et al. [14]. Kumar et al. [33] discussed the comparative study of ferromagnetic hybrid (manganese zinc ferrite, nickle zinc ferrite) nanofluids with velocity slip and convective conditions. A comprehensive study of thermophoretic diffusion deposition velocity effect on heat and mass transfer of ferromagnetic fluid flow along a stretching cylinder was discussed by Prasannakumara et al. [13]. Sudha et al. [16] discussed the hydrodynamic effects of secant slider bearings lubricated with second-order fluids. Unsteady flow of blood through a stenosed artery under the influence of transverse magnetic field was studied by Sujatha and Karthikeyan [25]. Recently, Nadian et al. [10], [11], [12]) discussed some stability problems about couple-stress ferromagnetic fluid. In the current investigation, we have talked about the effect of hall current on thermosolutal convection of a layer of couple-stress ferromagnetic fluid in the presence of varying gravitational field and horizontal magnetic field through porous media. We have assumed that gravity is varying as $g = \lambda g_0$, where g_0 is the value of g at the Earth's surface, which is always positive and λ can be positive or negative as gravity increases or decreases upwards from its value g_0. The outcome of this investigation will contribute a better understanding of the onset criterion for double-diffusive convection in a couple-stress ferromagnetic fluid soaked isotropic and homogeneous porous medium subjected to hall current effect, which is an often encountered phenomenon in different systems and industries.

2. Mathematical Formulation of the Problem

Consider an infinite, incompressible, electrically non-conducting, and thin layer of couple-stress ferromagnetic fluid which is bounded by the planes $z = 0$ and $z = d$. The fluid layer is heated from below so that a uniform temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained within the fluid. The whole system is acted upon by a uniform vertical magnetic field $\mathbf{H_m}(0, 0, H_m)$ and variable gravity field $\mathbf{g}(0, 0, -g)$, where $g = \lambda g_0$. Furthermore, the couple-stress ferromagnetic fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ϵ. The equation of conservation of momentum, continuity, temperature, concentration and equation of density for the above model are as follows:

$$\frac{1}{\epsilon}\left[\frac{\partial \mathbf{q_f}}{\partial t} + \frac{1}{\epsilon}(\mathbf{q_f}.\nabla)\mathbf{q_f}\right] = -\frac{1}{\rho_0}\nabla p + \mathbf{g}\left(1 + \frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\nu\mathbf{q_f} + \frac{1}{\rho_0}\mathbf{M}.\nabla\mathbf{H_m}$$

$$+\left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\mathbf{q_f} + \frac{\mu_e}{4\pi\rho_0}\left[(\nabla\times\mathbf{H_m})\times\mathbf{H_m}\right], \tag{1}$$

$$\nabla.\mathbf{q_f} = 0, \tag{2}$$

$$E_p\frac{\partial T}{\partial t} + (\mathbf{q_f}.\nabla)T = \kappa_T\nabla^2 T, \tag{3}$$

$$E'_p\frac{\partial C}{\partial t} + (\mathbf{q_f}.\nabla)C = \kappa_S\nabla^2 C, \tag{4}$$

$$\rho_d = \rho_0\left[1-\alpha(T-T_0)+\alpha'(C-C_0)\right], \tag{5}$$

where, $\mathbf{q_f}(u_1,u_2,u_3)$ is the fluid velocity, p is the fluid pressure, ρ_d is the fluid density, ρ_0 is the reference density, T is the temperature, T_0 is the reference temperature, α is the thermal coefficient of expansion, C is the solute concentration, C_0 is the solute concentration at reference level, μ_e is the magnetic permeability, μ' is the couple-stress viscosity, ν is the kinematic viscosity, κ_T is the thermal conductivity, κ_S is the solute conductivity, $E_p = \epsilon + (1-\epsilon)\frac{\rho_s c_s}{\rho_0 c_v}$ (where, ρ_s, c_s, c_v denote the density of the solid (porous) material, heat capacity of the solid (porous) material and heat capacity of the fluid at constant volume), E'_p is a constant analogous to E_p, $\mathbf{M}$ is the magnetization, $\nabla\mathbf{H_m}$ is the magnetic field gradient.
In presence of Hall current, the Maxwell's equations are given by,

$$\epsilon\frac{\partial\mathbf{H_m}}{\partial t} = \nabla\times(\mathbf{q_f}\times\mathbf{H_m}) + \epsilon\eta\nabla^2\mathbf{H_m} - \frac{\epsilon}{4\pi Ne}\nabla\times\left[(\nabla\times\mathbf{H_m})\times\mathbf{H_m}\right], \tag{6}$$

$$\text{and } \nabla.\mathbf{H_m} = 0. \tag{7}$$

Let the magnetization be independent of magnetic field intensity but depends upon the temperature and concentration so that $M = M(T,C)$. Considering as a first approximation,

$$M = M_0\left[1-K_1(T-T_0)+K_2(C-C_0)\right], \tag{8}$$

where, $K_1 = \frac{1}{M_0}\left(\frac{\partial M}{\partial T}\right)_{H_m}$, $K_2 = \frac{1}{M_0}\left(\frac{\partial M}{\partial C}\right)_{H_m}$ and M_0 is the magnetization at $T = T_0$ with T_0 being the reference temperature.

3. Basic State and Perturbation Equations

The basic state is characterized by,

$$\mathbf{q_f} = (0,0,0), p = p(z), \rho_d = \rho(z) = \rho_0(1-\alpha\beta z+\alpha'\beta' z), T = T_0 - \beta z,$$
$$\mathbf{H_m} = (0,0,H_m), M = M_0(1-K_1\beta z + K_2\beta' z), \mathbf{M} = M(z) \tag{9}$$

Here, β and β' may be either positive or negative.

Assuming small perturbations around the basic state and let $\mathbf{q_f}(u_1, u_2, u_3)$, $\mathbf{h}(h_x, h_y, h_z)$, θ, $\delta\rho$, δp and δM denote respectively the perturbations in fluid velocity, magnetic field, temperature, density, pressure and magnetization. Hence, the perturbed flow may be represented as,

$$\begin{aligned} \mathbf{q_f} = (0,0,0) + (u_1, u_2, u_3), \mathbf{h} &= (0,0,H_m) + (h_x, h_y, h_z), T = T(z) + \theta, \\ \rho &= \rho(z) + \delta\rho, p = p(z) + \delta p, M = M(z) + \delta M \end{aligned} \tag{10}$$

Linearizing the equations in perturbation and reading the perturbation into normal modes, the perturbation quantities are of the form,

$$(u_3, \theta, \zeta, \xi, h_z) = [W(z), \Theta(z), Z(z), X(z), K(z)] \, .e^{\{i(k_x x + k_y y) + nt\}}, \tag{11}$$

where, k_x and k_y are wavenumbers in x and y directions respectively and $k = \left(k_x^2 + k_y^2\right)^{1/2}$ is the resultant wavenumber of the disturbance and n is the frequency of any arbitrary disturbance.
Eliminating the physical quantities using the non-dimensional parameters $a = kd$, $\sigma = \dfrac{nd^2}{\nu}$, $p_1 = \dfrac{\nu}{\kappa_T}$, $p_2 = \dfrac{\nu}{\eta}$, $F = \dfrac{\mu'}{\rho_0 d^2 \nu}$, $P_1 = \dfrac{k_1}{d^2}$, $D^* = dD$. Also dropping (*) for convenience to obtain,

$$\begin{aligned} &(D^2 - a^2)\left[\left(\frac{\sigma}{\epsilon} + \frac{1}{P_1}\right) + F(D^2 - a^2)^2 - (D^2 - a^2)\right] W \\ &+ \frac{\lambda \alpha a^2 d^2}{\nu}\left(g_0 - \frac{K_1 M_0 \nabla H_m}{\rho_0 \alpha \lambda}\right)\Theta - \frac{\lambda \alpha' a^2 d^2}{\nu}\left(g_0 - \frac{K_2 M_0 \nabla H_m}{\rho_0 \alpha' \lambda}\right)\Gamma \\ &- \frac{\mu_e H_m d}{4\pi \rho_0 \nu}(D^2 - a^2) DK = 0, \end{aligned} \tag{12}$$

$$\left[\left(\frac{\sigma}{\epsilon} + \frac{1}{P_1}\right) + F(D^2 - a^2)^2 - (D^2 - a^2)\right] Z = \frac{\mu_e H_m d}{4\pi \rho_0 \nu} DX, \tag{13}$$

$$(D^2 - a^2 - \sigma p_2) X = -\frac{H_m d}{\epsilon \eta} DZ - \frac{H_m}{4\pi N e \eta d}(D^2 - a^2) DK, \tag{14}$$

$$(D^2 - a^2 - \sigma p_2) K = -\frac{H_m d}{\epsilon \eta} DW + \frac{H_m d}{4\pi N e \eta} DX, \tag{15}$$

$$(D^2 - a^2 - E_p \sigma p_1)\Theta = -\frac{\beta d^2}{\kappa_T} W, \tag{16}$$

$$(D^2 - a^2 - E'_p q \sigma)\Gamma = -\frac{\beta' d^2}{\kappa_S} W. \tag{17}$$

Now, eliminating X, Θ, Γ, K, Z among Eqs. (12) - (17), the stability governing equation is,

$$\lambda R_f a^2 W = (D^2-a^2)\left\{\left(\frac{\sigma}{\epsilon}+\frac{1}{P_1}\right)+F(D^2-a^2)^2-(D^2-a^2)\right\}(D^2-a^2-E_p\sigma p_1)W$$
$$+\lambda a^2 S_f\left\{\frac{(D^2-a^2-E_p\sigma p_1)}{(D^2-a^2-E'_p\sigma q)}\right\}W$$
$$+\frac{\frac{Q}{\epsilon}D^2(D^2-a^2)\left[\frac{Q}{\epsilon}D^2+(D^2-a^2-\sigma p_2)\left\{\left(\frac{\sigma}{\epsilon}+\frac{1}{P_1}\right)+F(D^2-a^2)^2-(D^2-a^2)\right\}\right](D^2-a^2-E_p\sigma p_1)W}{\left[(D^2-a^2-\sigma p_2)\left[\frac{Q}{\epsilon}D^2+(D^2-a^2-\sigma p_2)\left\{\left(\frac{\sigma}{\epsilon}+\frac{1}{P_1}\right)+F(D^2-a^2)^2-(D^2-a^2)\right\}\right]\right.}$$
$$\left.+M_h D^2(D^2-a^2)\left\{\left(\frac{\sigma}{\epsilon}+\frac{1}{P_1}\right)+F(D^2-a^2)^2-(D^2-a^2)\right\}\right] \quad (18)$$

where, $R_f=\dfrac{\alpha\beta d^4}{\nu\kappa_T}\left(g_0-\dfrac{K_1M_0\nabla H_m}{\rho_0\alpha\lambda}\right)$ is the Rayleigh number for ferromagnetic fluid, $S_f=\dfrac{\alpha'\beta' d^4}{\nu\kappa_S}\left(g_0-\dfrac{K_2M_0\nabla H_m}{\rho_0\alpha'\lambda}\right)$ is the solute Rayleigh number for ferromagnetic fluid, $Q=\dfrac{\mu_e H_m^2 d^2}{4\pi\rho_0\nu\eta}$ is the Chandrasekhar number and $M_h=\left(\dfrac{H_m}{4\pi Ne\eta}\right)^2$ is the Hall current parameter.

Now, the appropriate boundary conditions (when considering the case of two free boundaries) are,

$$W=0, D^2W=0, D^4W=0, \Theta=0, Z=0, X=0, DZ=0, DX=0, DK=0$$
$$\text{at } z=0 \text{ and } z=1 \quad (19)$$

The solution of Eq. (18) characterizing the lowest mode is,

$$W=W_0\sin\pi z, \quad \text{where } W_0 \text{ is constant} \quad (20)$$

Now, using Eq. (20) in Eq. (18) to get,

$$R_1=\frac{(1+x)}{\lambda x}\left\{\left(\frac{i\sigma_1}{\epsilon}+\frac{1}{P}\right)+F_1(1+x)^2+(1+x)\right\}(1+x+E_pi\sigma_1p_1)+S_1\left\{\frac{(1+x+i\sigma_1E_pp_1)}{(1+x+i\sigma_1E'_pq)}\right\}$$
$$+\frac{Q_1}{\lambda x\epsilon}\frac{(1+x)\left[\frac{Q_1}{\epsilon}+(1+x+i\sigma_1p_2)\left\{\left(\frac{i\sigma_1}{\epsilon}+\frac{1}{P}\right)+F_1(1+x)^2+(1+x)\right\}\right](1+x+E_pi\sigma_1p_1)}{(1+x+i\sigma_1p_2)\left[\frac{Q_1}{\epsilon}+(1+x+i\sigma_1p_2)\left\{\left(\frac{i\sigma_1}{\epsilon}+\frac{1}{P}\right)+F_1(1+x)^2+(1+x)\right\}\right]}$$
$$+M_h(1+x)\left\{\left(\frac{i\sigma_1}{\epsilon}+\frac{1}{P}\right)+F_1(1+x)^2+(1+x)\right\} \quad (21)$$

where, $x=\dfrac{a^2}{\pi^2}$, $i\sigma_1=\dfrac{\sigma}{\pi^2}$, $F_1=\pi^2F$, $R_1=\dfrac{R_f}{\pi^4}$, $Q_1=\dfrac{Q}{\pi^2}$, $P=\pi^2P_1$, $S_1=\dfrac{S_f}{\pi^4}$.

4. Analytical Discussion

4.1. Stationary Convection

When stability sets in at stationary convection, the marginal state will be characterized by $\sigma_1 = 0$. So, put $\sigma_1 = 0$ in Eq. (21) to get,

$$R_1 = \frac{(1+x)^2}{\lambda x}\left\{\frac{1}{P} + F_1(1+x)^2 + (1+x)\right\} + S_1$$
$$+ \frac{Q_1}{\lambda x \epsilon}\frac{(1+x)\left\{\frac{Q_1}{\epsilon} + \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}}{\left\{\frac{Q_1}{\epsilon} + \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\} + M_h\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}}. \tag{22}$$

This relation expresses the modified Rayleigh number R_1 as a function of the parameters P, F_1, S_1, Q_1, M_h and dimensionless wave number x. Now, to have a look at the effect of medium permeability parameter, couple-stress parameter, solute gradient parameter, magnetic field parameter and Hall current parameter, examine the behavior of $\frac{dR_1}{dP}$, $\frac{dR_1}{dF_1}$, $\frac{dR_1}{dS_1}$, $\frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dM_h}$.

So, by Eq. (22),

$$\frac{dR_1}{dP} = \frac{(1+x)^2}{\lambda x P^2}\left[\frac{M_h Q_1^2}{\epsilon^2\left[\frac{Q_1}{\epsilon} + (1+M_h)\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\right]^2} - 1\right], \tag{23}$$

which indicates that medium permeability has a stabilizing effect on the system under the condition,

$$\lambda > 0, M_h Q_1^2 > \epsilon^2\left[\frac{Q_1}{\epsilon} + (1+M_h)\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\right]^2$$

and

$$\lambda < 0, M_h Q_1^2 < \epsilon^2\left[\frac{Q_1}{\epsilon} + (1+M_h)\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\right]^2.$$

Also, medium permeability has a destabilizing effect on the system under the condition,

$$\lambda > 0, M_h Q_1^2 < \epsilon^2\left[\frac{Q_1}{\epsilon} + (1+M_h)\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\right]^2$$

and

$$\lambda < 0, M_h Q_1^2 > \epsilon^2 \left[\frac{Q_1}{\epsilon} + (1 + M_h) \left\{ \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2 \right\} \right]^2.$$

In the absence of magnetic field, Eq. (23) becomes,

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{\lambda x P^2}, \tag{24}$$

which indicates that medium permeability has a stabilizing effect for $\lambda < 0$ and destabilizing effect for $\lambda > 0$.

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{\lambda x} \left[1 - \frac{M_h Q_1^2}{\epsilon^2 \left[\frac{Q_1}{\epsilon} + (1 + M_h) \left\{ \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2 \right\} \right]^2} \right], \tag{25}$$

which indicates that couple-stress has a stabilizing effect on the system under the condition,

$$\lambda > 0, M_h Q_1^2 < \epsilon^2 \left[\frac{Q_1}{\epsilon} + (1 + M_h) \left\{ \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2 \right\} \right]^2$$

and

$$\lambda < 0, M_h Q_1^2 > \epsilon^2 \left[\frac{Q_1}{\epsilon} + (1 + M_h) \left\{ \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2 \right\} \right]^2.$$

Also, couple-stress has a destabilizing effect on the system under the condition,

$$\lambda > 0, M_h Q_1^2 > \epsilon^2 \left[\frac{Q_1}{\epsilon} + (1 + M_h) \left\{ \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2 \right\} \right]^2$$

and

$$\lambda < 0, M_h Q_1^2 < \epsilon^2 \left[\frac{Q_1}{\epsilon} + (1 + M_h) \left\{ \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2 \right\} \right]^2.$$

In the absence of magnetic field or hall current, Eq. (25) becomes,

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{\lambda x}, \tag{26}$$

which clearly indicates that couple-stress has a stabilizing effect for $\lambda > 0$ and destabilizing effect for $\lambda < 0$.

$$\frac{dR_1}{dS_1} = 1, \tag{27}$$

clearly, the solute gradient has a stabilizing effect on the system.

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{\lambda \epsilon x} \left[\frac{\frac{M_h Q_1}{\epsilon}\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\} + \left\{\frac{Q_1}{\epsilon} + \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\left[\frac{Q_1}{\epsilon} + (1+M_h)\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\right]}{\left[\frac{Q_1}{\epsilon} + (1+M_h)\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\right]^2} \right], \tag{28}$$

which clearly indicates that the magnetic field has a stabilizing effect on the system for $\lambda > 0$ and destabilizing effect for $\lambda < 0$.

$$\frac{dR_1}{dM_h} = -\frac{Q_1(1+x)}{\lambda \epsilon x}\left\{\frac{Q_1}{\epsilon} + \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\} \frac{\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}}{\left[\frac{Q_1}{\epsilon} + (1+M_h)\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\right]^2} \tag{29}$$

which clearly indicates that Hall current has a stabilizing effect on the system for $\lambda < 0$ and destabilizing effect for $\lambda > 0$.

To see the effect of magnetization, examine the effect of $\frac{dR}{dM_0}$ analytically.

$$\frac{dR}{dM_0} = \left[\frac{\pi^4(1+x)^2}{\lambda x}\left\{\frac{(1)}{P} + F_1(1+x)^2 + (1+x)\right\} + \frac{Q_1\pi^4}{\lambda x \epsilon} \frac{(1+x)\left\{\frac{Q_1}{\epsilon} + \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}}{\left\{\frac{Q_1}{\epsilon} + \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\} + M_h\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}}\right] \left(1 - \frac{\gamma M_0 \nabla H_m}{\rho_0 \alpha g_0 \lambda}\right)^{-2} \cdot \left(\frac{\gamma \nabla H_m}{\rho_0 \alpha g_0 \lambda}\right). \tag{30}$$

which indicates that magnetization has a stabilizing effect on the system for both $\lambda > 0$ and $\lambda < 0$.

4.2. The case of oscillatory modes

Multiplying Eq. (12) by the conjugate of W i.e. W^* and integrate over the range of z and making use of Eqs. (13) - (17) together with boundary conditions (19) to get,

$$\left(\frac{\sigma}{\epsilon}+\frac{1}{P_1}\right)I_1+I_2+FI_3+d^2\left\{\left(\frac{\sigma^*}{\epsilon}+\frac{1}{P_1}\right)I_4+I_5+FI_6+\frac{\mu_e\epsilon\eta}{4\pi\rho_0\nu}(I_7+p_2\sigma I_8)\right\}$$
$$+\frac{\mu_e\epsilon\eta}{4\pi\rho_0\nu}(I_9+p_2\sigma^* I_{10})-\frac{\lambda\alpha a^2\kappa_T}{\beta\nu}\left(g_0-\frac{K_1M_0\nabla H_m}{\rho_0\alpha\lambda}\right)(I_{11}+E_pp_1\sigma^* I_{12})$$
$$+\frac{\lambda\alpha' a^2\kappa_S}{\beta'\nu}\left(g_0-\frac{K_2M_0\nabla H_m}{\rho_0\alpha'\lambda}\right)(I_{13}+E'_pq\sigma^* I_{14})=0, \tag{31}$$

where,

$$I_1=\int\left(|DW|^2+a^2|W|^2\right)dz,\ I_2=\int\left(|D^2W|^2+2a^2|DW|^2+a^4|W|^2\right)dz,$$
$$I_3=\int\left(|D^3W|^2+3a^2|D^2W|^2+3a^4|DW|^2+a^6|W|^2\right)dz,\ I_4=\int\left(|Z|^2\right)dz,$$
$$I_5=\int\left(|DZ|^2+a^2|Z|^2\right)dz,\ I_6=\int\left(|D^2Z|^2+2a^2|DZ|^2+a^4|Z|^2\right)dz,$$
$$I_7=\int\left(|DX|^2+a^2|X|^2\right)dz,\ I_8=\int\left(|X|^2\right)dz,$$
$$I_9=\int\left(|D^2K|^2+2a^2|DK|^2+a^4|K|^2\right)dz,\ I_{10}=\int\left(|DK|^2+a^2|K|^2\right)dz,$$
$$I_{11}=\int\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\ I_{12}=\int\left(|\Theta|^2\right)dz,$$
$$I_{13}=\int\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz,\ I_{14}=\int\left(|\Gamma|^2\right)dz.$$

All these integrals from I_1 to I_{14} are positive definite.
Now, putting $\sigma=i\sigma_i$ (where σ_i is real) in Eq. (31) and equating the imaginary part to get,

$$\sigma_i\left\{\frac{1}{\epsilon}I_1-\frac{d^2}{\epsilon}I_4+\frac{\mu_e\epsilon\eta d^2}{4\pi\rho_0\nu}p_2I_8-\frac{\mu_e\epsilon\eta}{4\pi\rho_0\nu}p_2I_{10}+\left(\frac{\lambda\alpha a^2\kappa_T}{\beta\nu}\right)\right.$$
$$\left.\left(g_0-\frac{K_1M_0\nabla H_m}{\rho_0\alpha\lambda}\right)E_pp_1I_{12}-\left(\frac{\lambda\alpha' a^2\kappa_S}{\beta'\nu}\right)\left(g_0-\frac{K_2M_0\nabla H_m}{\rho_0\alpha'\lambda}\right)E'_pqI_{14}\right\}=0. \tag{32}$$

In the absence of magnetic field (hence hall current), Eq. (32) becomes,

$$\sigma_i \left\{ \frac{1}{\epsilon} I_1 + \frac{\mu_e \epsilon \eta d^2}{4\pi\rho_0\nu} p_2 I_8 + \left(\frac{\lambda\alpha a^2 \kappa_T}{\beta\nu}\right)\left(g_0 - \frac{K_1 M_0 \nabla H_m}{\rho_0 \alpha\lambda}\right) E_p p_1 I_{12} \right.$$
$$\left. - \left(\frac{\lambda\alpha' a^2 \kappa_S}{\beta'\nu}\right)\left(g_0 - \frac{K_2 M_0 \nabla H_m}{\rho_0\alpha'\lambda}\right) E'_p q I_{14} \right\} = 0. \tag{33}$$

If $\lambda g_0 \geq \dfrac{K_1 M_0 \nabla H_m}{\rho_0 \alpha}$ and $\lambda g_0 \leq \dfrac{K_2 M_0 \nabla H_m}{\rho_0 \alpha'}$, then the terms in bracket are positive definite, which implies that $\sigma_i = 0$. Therefore, oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied if $\lambda g_0 \geq \dfrac{K_1 M_0 \nabla H_m}{\rho_0 \alpha}$ and $\lambda g_0 \leq \dfrac{K_2 M_0 \nabla H_m}{\rho_0 \alpha'}$.

5. Numerical Discussion

The dispersion relation (22) is analyzed numerically also. The numerical value of thermal Rayleigh number R_1 is decided for numerous values of medium permeability parameter P, couple-stress parameter F_1, magnetic field parameter Q_1, Hall current parameter M_h and magnetization parameter M_0. Also, graphs have been plotted between R_1 and these parameters as shown in Figures (1) - (16).

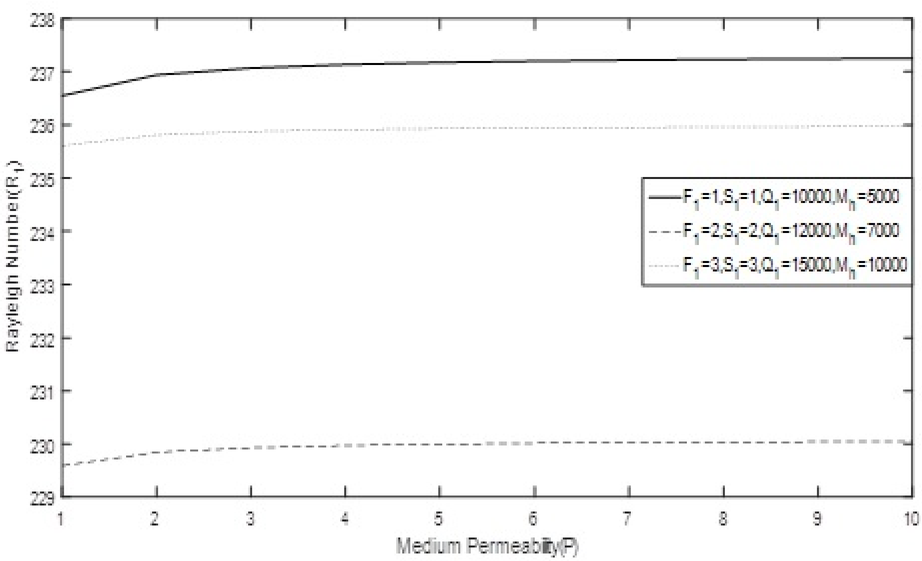

Fig. 1. R_1 vs P for $\lambda > 0 (\lambda = 50)$

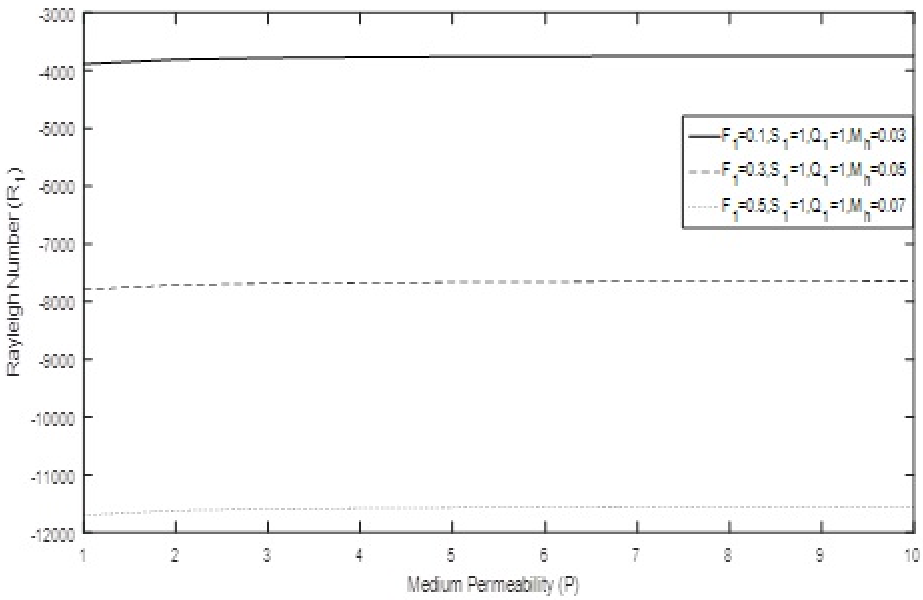

Fig. 2. R_1 vs P for $\lambda < 0 (\lambda = -5)$

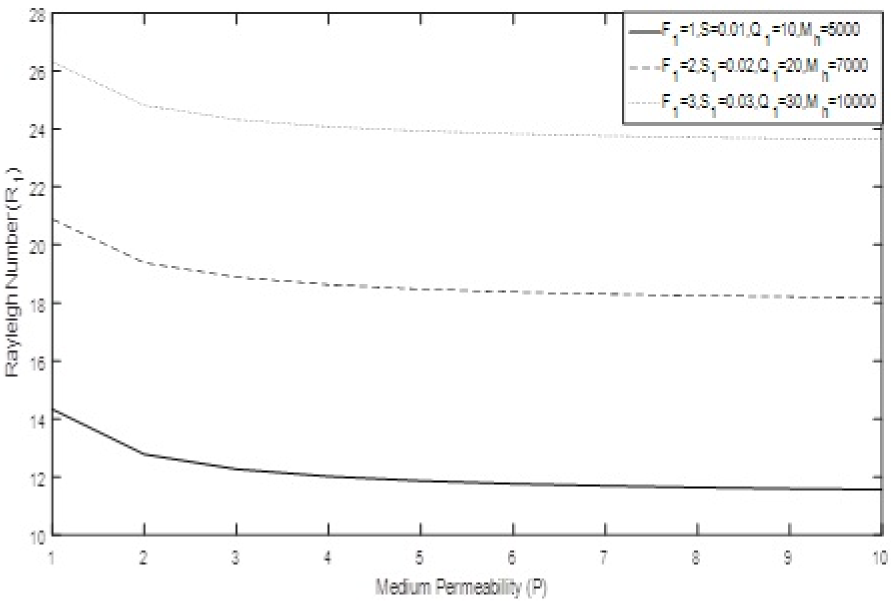

Fig. 3. R_1 vs P for $\lambda > 0 (\lambda = 2)$

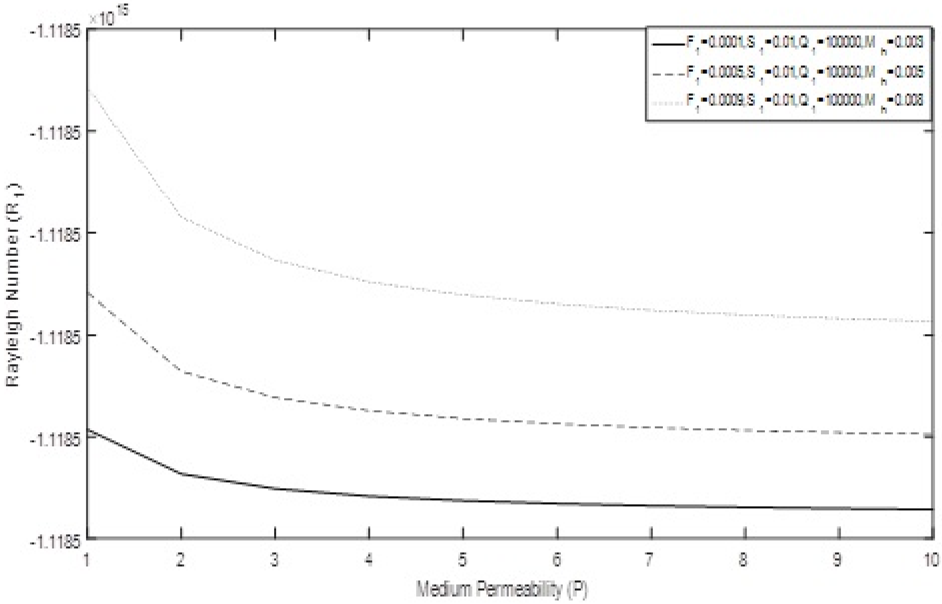

Fig. 4. R_1 vs P for $\lambda < 0 (\lambda = -0.00001)$

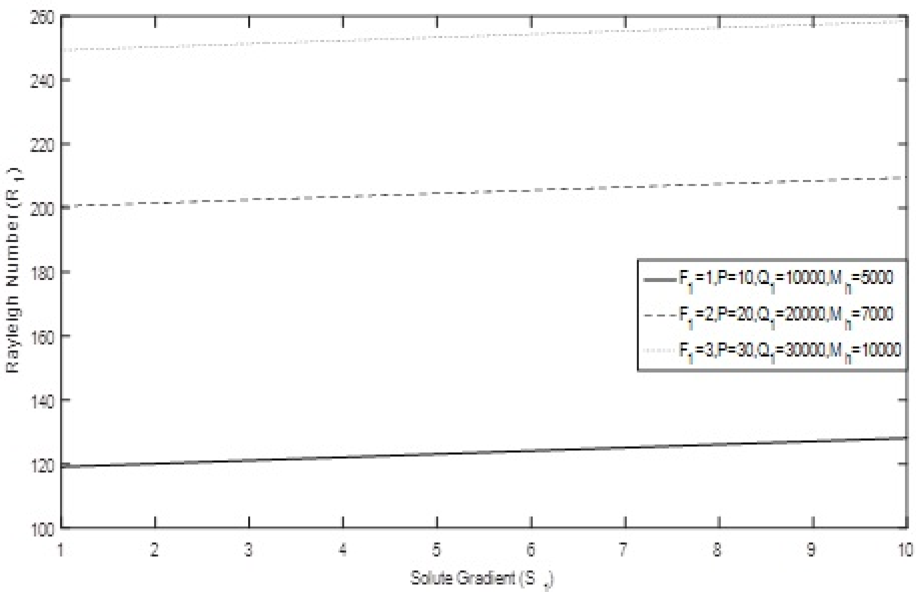

Fig. 5. R_1 vs S_1 for $\lambda > 0 (\lambda = 100)$

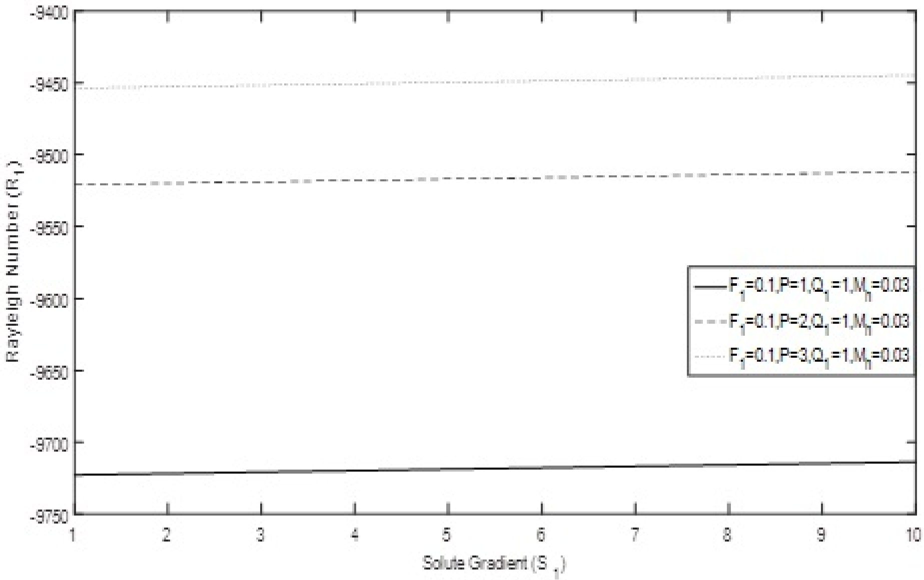

Fig. 6. R_1 vs S_1 for $\lambda < 0(\lambda = -2)$

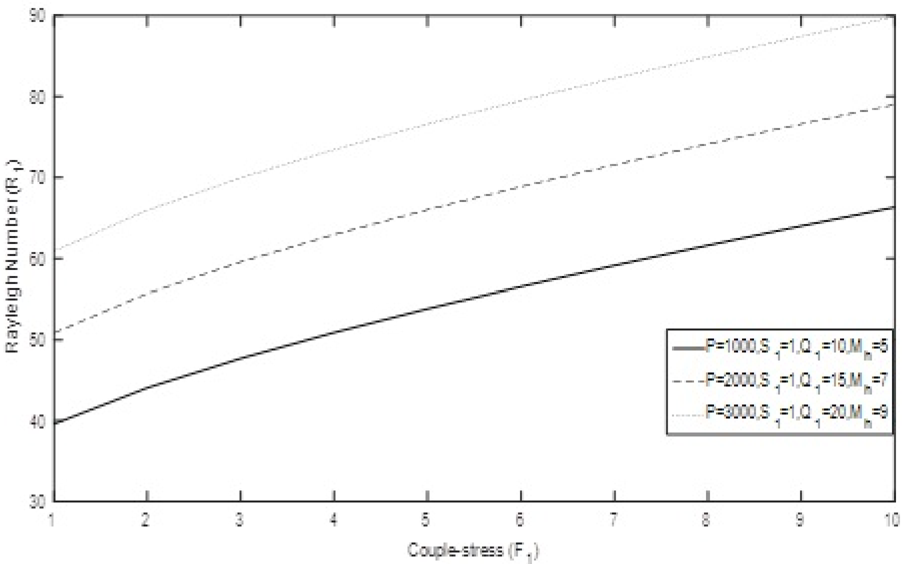

Fig. 7. R_1 vs F_1 for $\lambda > 0(\lambda = 5)$

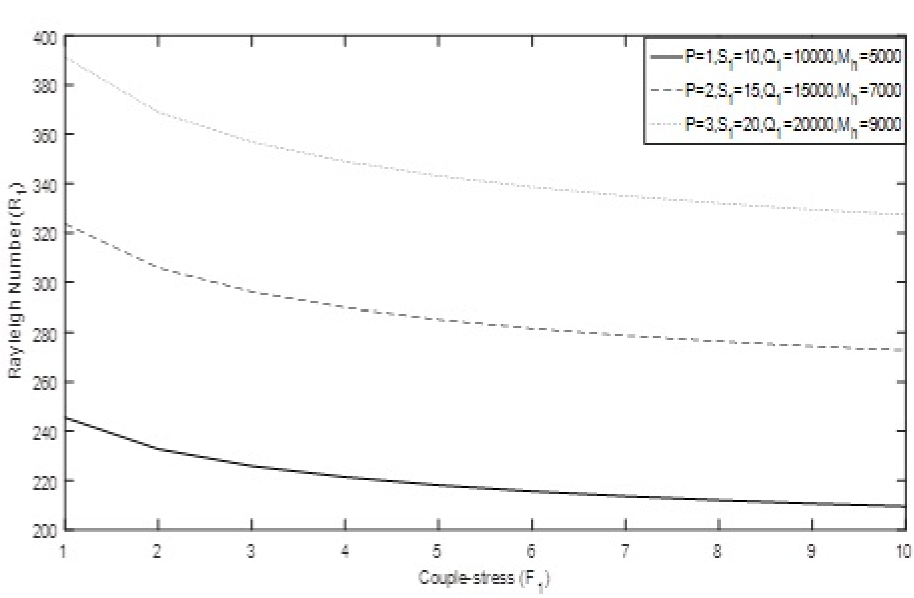

Fig. 8. R_1 vs F_1 for $\lambda > 0(\lambda = 50)$

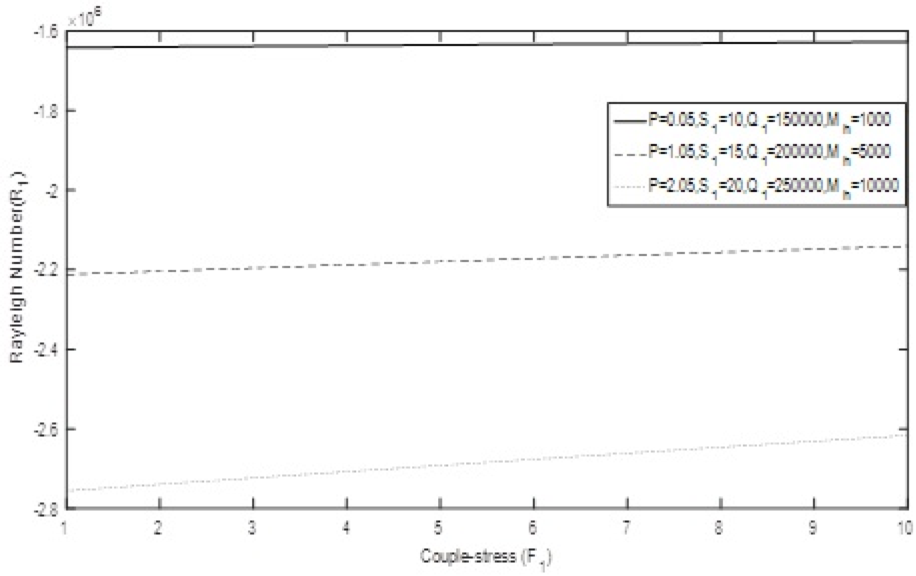

Fig. 9. R_1 vs F_1 for $\lambda < 0(\lambda = -1000)$

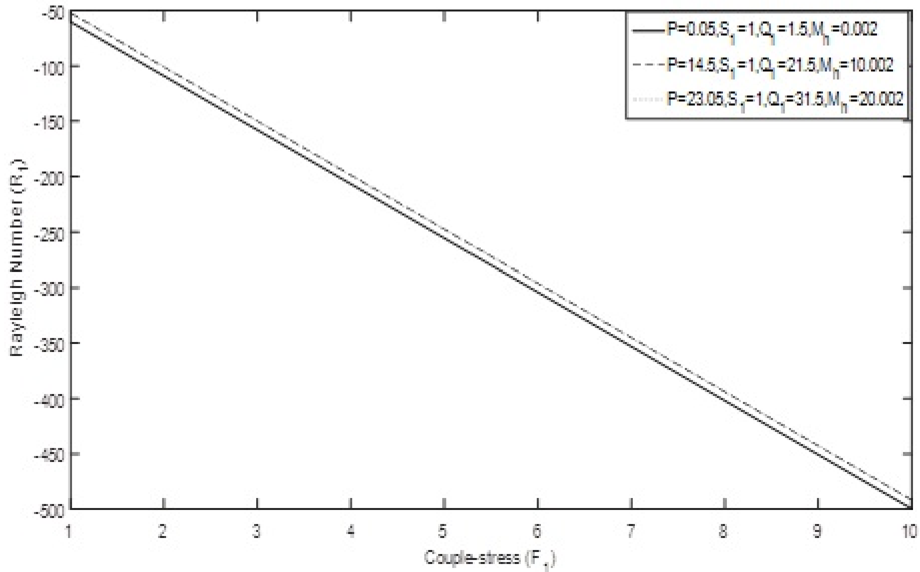

Fig. 10. R_1 vs F_1 for $\lambda < 0 (\lambda = -2000)$

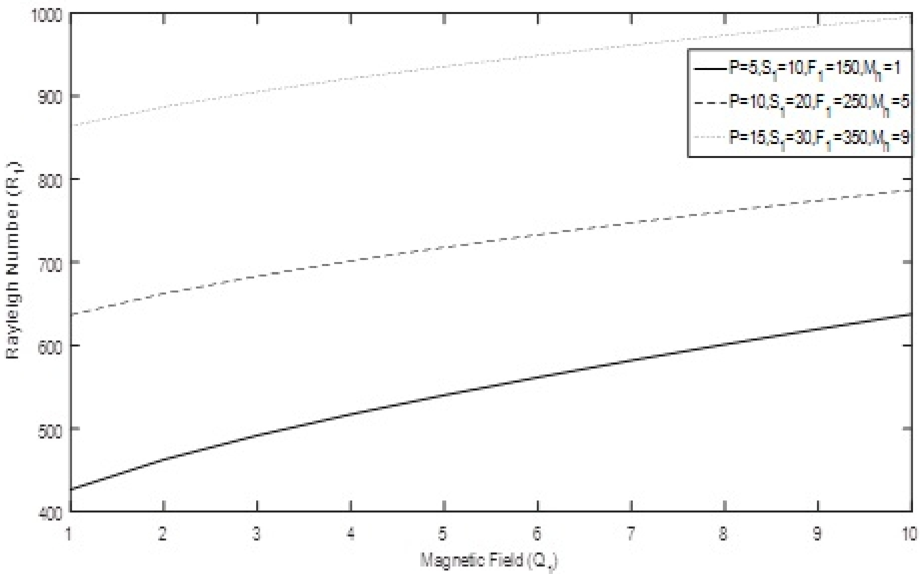

Fig. 11. R_1 vs Q_1 for $\lambda > 0 (\lambda = 3)$

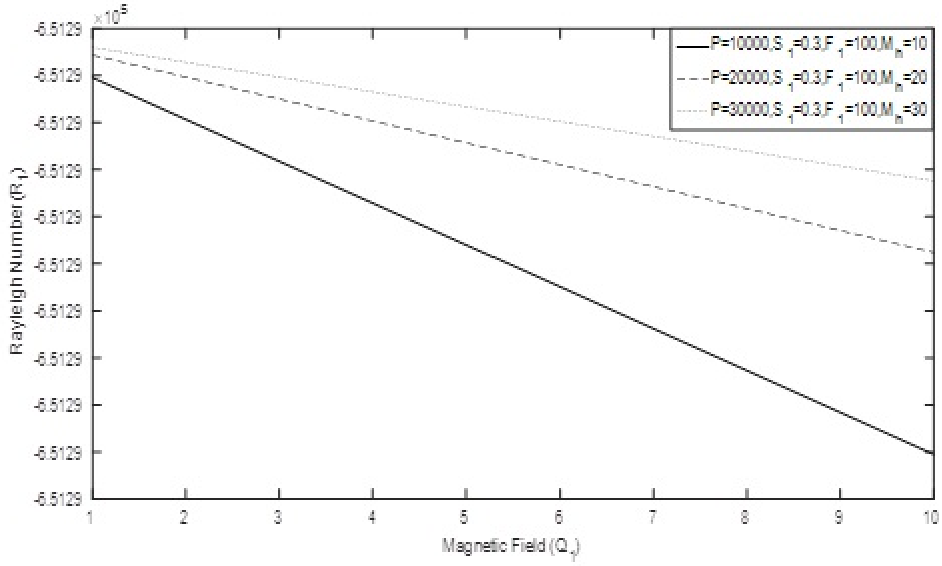

Fig. 12. R_1 vs Q_1 for $\lambda < 0(\lambda = -15)$

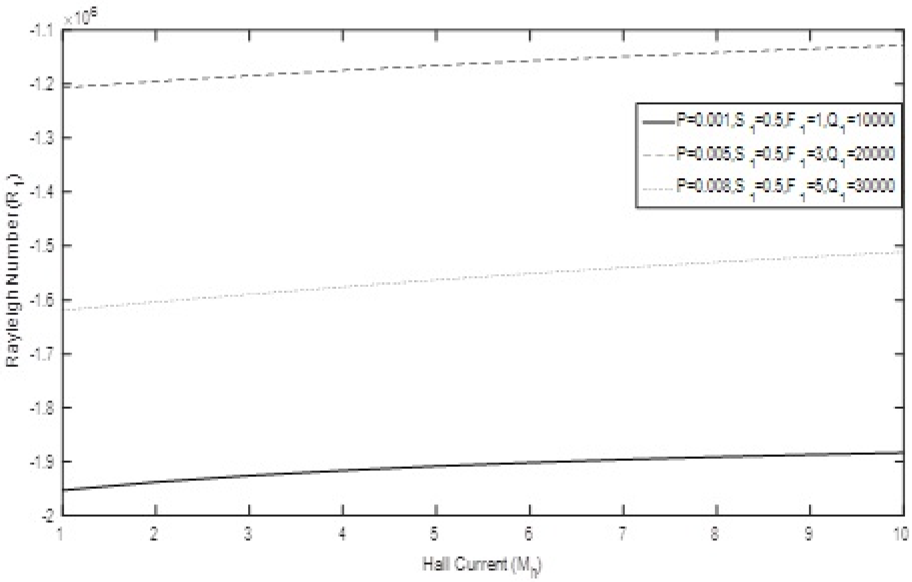

Fig. 13. R_1 vs M_h for $\lambda < 0(\lambda = -0.5)$

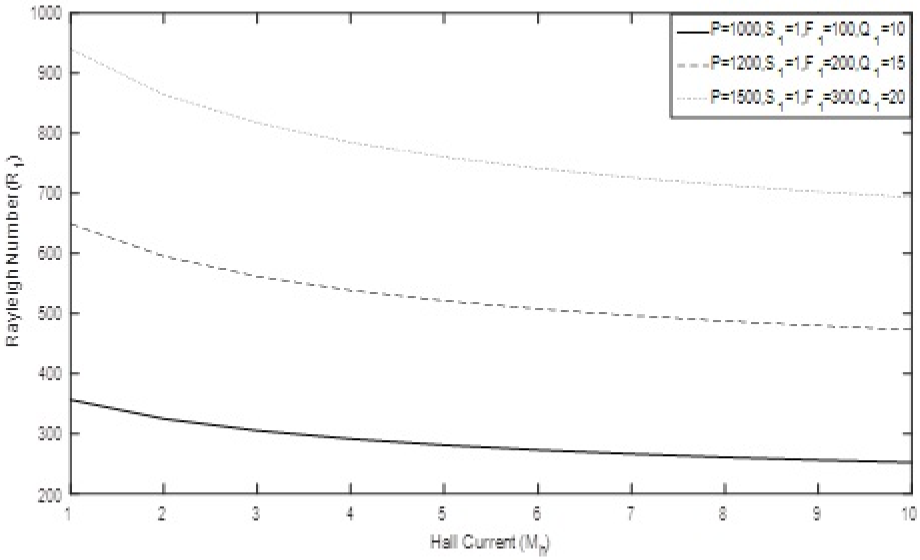

Fig. 14. R_1 vs M_h for $\lambda > 0(\lambda = 4)$

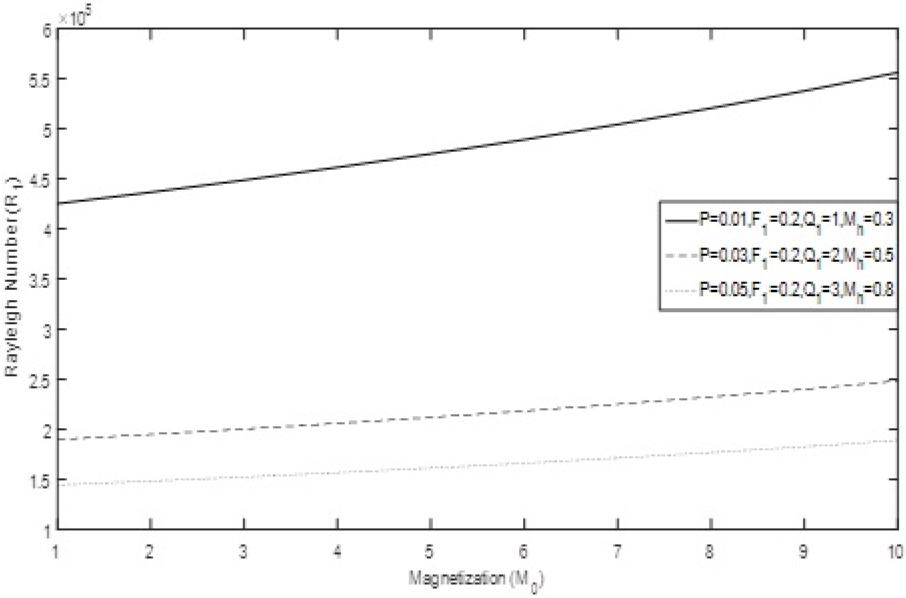

Fig. 15. R_1 vs M_0 for $\lambda > 0(\lambda = 0.2)$

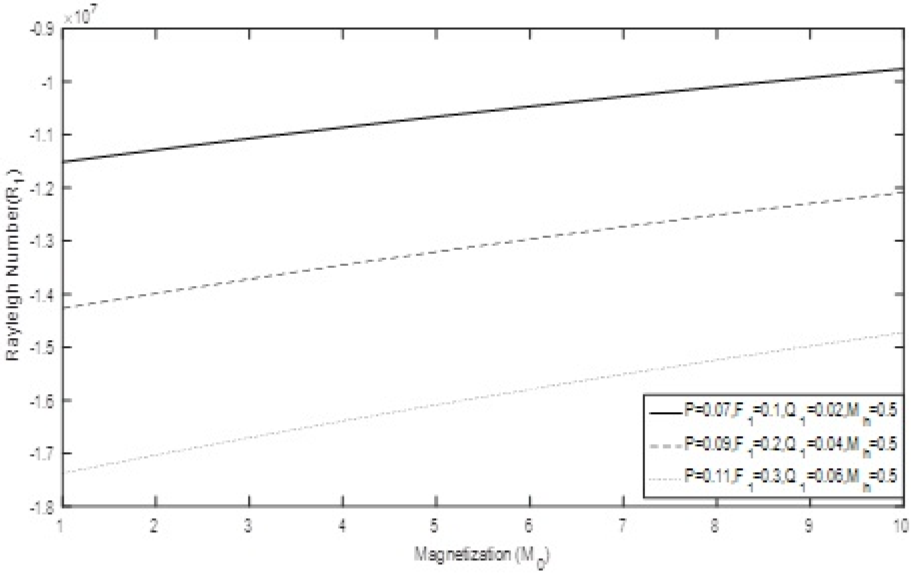

Fig. 16. R_1 vs M_0 for $\lambda < 0(\lambda = -0.25)$

These are some graphs plotted between critical Rayleigh number and the parameters (medium permeability, stable solute gradient, couple-stress parameter, magnetic field, hall current and magnetization) for the different values of these parameters.

6. Results and Discussion

In the present paper, the effect of hall currents on the thermosolutal convection of couple-stress ferromagnetic fluid in the presence of varying gravitational field and horizontal magnetic field through porous media is discussed. A linearized theory and normal mode technique are used to attain the dispersion relation. The main results from the evaluation of the present paper are as below:

1. Medium permeability has both stabilizing and destabilizing effects on the system for $\lambda > 0$ and $\lambda < 0$ under certain conditions. Furthermore, in the absence of a magnetic field, medium permeability has a stabilizing effect on the system for $\lambda < 0$ and destabilizing effect for $\lambda > 0$.

2. Stable solute gradient has a stabilizing effect on the system.

3. Couple-stress has both stabilizing and destabilizing effects on the system for $\lambda > 0$ and $\lambda < 0$ under certain conditions. Furthermore, in the absence of a magnetic field, couple-stress has a stabilizing effect on the system for $\lambda > 0$ and destabilizing effect for $\lambda < 0$.

4. The magnetic field has a stabilizing effect on the system for $\lambda > 0$ and destabilizing effect for $\lambda < 0$.

5. Hall current has a stabilizing effect on the system for $\lambda < 0$ and destabilizing effect for $\lambda > 0$.

6. Magnetization has a stabilizing effect on the system for both $\lambda > 0$ and $\lambda < 0$ (in the absence of stable solute gradient).

7. The principle of exchange of stabilities is not valid for the present problem under consideration, whereas in the absence of a magnetic field (hence hall current), it is valid for the present problem if
$\lambda g_0 \geq \dfrac{K_1 M_0 \nabla H_m}{\rho_0 \alpha}$ and $\lambda g_0 \leq \dfrac{K_2 M_0 \nabla H_m}{\rho_0 \alpha'}$.

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