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AN INVESTIGATION ON VERTICAL POROUS PLATE IN A CONDUCTING FLUID WITH MULTIPLE BOUNDARY LAYER FLOW OF CASSON FLUID

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Abstract: The present investigation generates an analytical solution of multiple boundary layer flow of Casson fluid past over a vertical plate through porous medium in a conducing fluid in the presence of a uniform transverse magnetic field. In this investigation the effects of radiation, heat generation/absorption, radiation absorption and homogeneous chemical reaction are considered. The coupled nonlinear partial equations are turned to ordinary equations by super imposing solutions with steady and time dependent transient part. Finally, the set of ordinary differential equations are solved with a perturbation method to meet the inadequacy of boundary condition. The effect of different parameters on the flow is described with the help of graphs and tables. The most interesting observation found from this investigation is the fluctuation of velocity appears near the plate due to the presence of sink and presences of elastic element as well heat source reduce the skin friction.

Keywords and Phrases: Casson fluid, Porous plate, Thermal radiation, Chemical reaction, Heat and mass transfer, Radiation absorption.

2020 Mathematics Subject Classification: 35Q35, 35Q40, 80A21, 80A30, 76S05.

1. Introduction

An important class of two-dimensional time dependent flow problem dealing with the response of boundary layer to external unsteady fluctuations of the free stream velocity about a mean value attracted the attention of many researchers. Besides, the convective flow through porous medium has applications in geothermal energy recovery, thermal energy storage, oil extraction, and flow through filtering devices. Nowadays Magneto hydrodynamics is very much attracting the attention of the many authors due to its applications in geophysics and engineering. MHD flow with heat and mass transfer has been a subject of interest of many researchers because of its varied application in science and technology. Such phenomena are observed buoyancy induced motions in the atmosphere, in water bodies, quasi solid bodies such as earth, etc.

Kirubushankumar et al. [6] have examined that a Casson fluid flow and heat transfer over an unsteady porous stretching surface. Arthur et al. [1] have analyzed the Casson fluid flow over a vertical porous surface with a chemical reaction in the presence of a magnetic field. Pramanik [9] have studied that a Casson fluid flow and heat transfer past an exponentially porous stretching surface in the presence of thermal radiation. Vijay Kumar et al. [7] have investigated that Joule heating and thermal diffusion effects on MHD radiative and convective Casson fluid flow past an oscillating semi-infinite vertical porous plate. A. Hamid et.al [2] are analyzed the Characteristics of combined heat and mass transfer on mixed convection flow of Sisko fluid model. A. Hamid and M. Khan [3] are studied the Unsteady mixed convective flow of Williamson Nano fluid with heat transfer in the presence of variable thermal conductivity and magnetic. A. Hamid et al. [4] are given the Numerical simulation for heat transfer performance in unsteady flow of Williamson fluid driven by a wedge-geometry. Hashim et al. [5] are discussed Multiple solutions for MHD transient flow of Williamson Nano fluids with convective heat transport. Muthuraj et.al [8] have examined MHD Couette flow of Powell varying fluid in an inclined porous space in the presence of temperature dependent heat source with chemical reaction. Rama Mohan et al. [10] analyzed Unsteady MHD free convection flow characteristics of a viscoelastic fluid past a vertical porous plate.

In the present investigation, the presence of radiation absorption and a transverse magnetic field is considered in an unsteady free convective flow of Casson fluid past an infinite vertical porous plate in a porous medium with time dependent oscillatory suction along with the permeability. This kind of environment is mostly present in various industries such as food processing firms, dairy industries, distilleries and beverage industries, polymer fabrication firms, glass manufacturing industries, pharmaceutical industries etc.

2. Formulation of Problem

The unsteady free convective flow of a radiative, chemically reactive, heat absorbing, Casson fluid past an infinite vertical porous plate in a porous medium with time dependent oscillatory suction as well as permeability in presence of radiation absorption and a transverse magnetic field is considered. The schematic physical model of the analyzed problem is shown in Fig. 2.1.

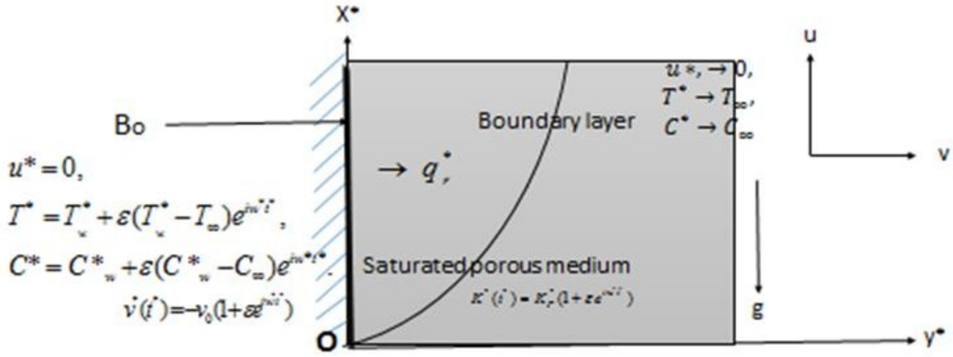

Fig. 2.1: Physical model of the problem

The following assumption are made during the formulation of the problem and those are; t* <0, v_0>0 and are positive constants, Soret and Dofour effects are neglected. Let us consider the x^1-axis be along the plate in the direction of the flow and the y^1-axis normal to it. It is also considered that the magnetic Reynolds number is much less than unity so that induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. Ii is assumed that initially, at t* <0, the plate as well as fluids are at the same temperature and also concentration of the species is very low so that the Soret and Dofour effects are neglected. When t*, the temperature of the plate is instantaneously raised to T_w^* and the concentration of the species is set to C_w^*. Let the permeability of the porous medium and the suction velocity be considered in the following forms respectively.

$$K^*(t^*) = K_p^*\left(1+\varepsilon e^{n^*t^*}\right), \nu^*(t^*) = -\nu_0\left(1+\varepsilon e^{n^*t^*}\right) \tag{1}$$

Where v_0>0 and are positive constants. Under the above assumption with usual Boussinesq's approximation, the governing equations and boundary conditions are given by [Ref 10]

From the reference [10], the fluid selected for the present problem is Casson fluids instead of viscoelastic fluid. Also considered multiple boundaries in place of uniform boundary. Therefore the governing equations modified and obtained as equations (2) and (3) is

$$\frac{\partial u^*}{\partial t^*}=\left(1+\frac{1}{\beta}\right)\nu\frac{\partial^2 u^*}{\partial y^{*2}}+g\beta\left(T^*-T^*_\infty\right)+g\beta^*\left(C^*-C^*_\infty\right)-\sigma B_0^2\frac{u^*}{\rho}-\left(1+\frac{1}{\beta}\right)\nu\frac{u^*}{k^*} \tag{2}$$

$$\frac{\partial T^*}{\partial t^*}\rho c_p=K\frac{\partial^2 T^*}{\partial y^{*2}}-\frac{\partial q^*}{\partial y^*}-Q^*\left(T^*-T^*_\infty\right)+Q^*_l\left(C^*-C^*_\infty\right) \tag{3}$$

$$\frac{\partial C^*}{\partial t^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}-K_r\left(C^*-C^*_\infty\right) \tag{4}$$

$$\begin{aligned}&u=f\left(t\right)=t,\sin t \text{ and } e^t\\&T^*=T_\infty+\varepsilon\left(T_W-T_\infty\right)e^{n^*t^*},C^*=C_\infty+\varepsilon\left(C_W-C_\infty\right)e^{n^*t^*}at\ y=0\\&u\to 0,T^*\to T_\infty,C^*\to C_\infty,\quad as\quad y\to\infty\end{aligned} \tag{5}$$

Introducing the non-dimensional quantities,

$$\begin{aligned}&y=\frac{\nu_0 t^*}{\nu},t=\frac{{\nu_0}^2 t^*}{4\nu},w=\frac{4\vartheta w^*}{{\nu_0}^2},u=\frac{u^*}{\nu_0},T=\frac{T^*-{T_\infty}^*}{T_W-T_\infty},C=\frac{C^*-{C_\infty}^*}{C_W-C_\infty},\\&S=\frac{\vartheta S^*}{{\nu_0}^2},K_p=\frac{\nu_0^2K_p^2}{\nu^2},M^2=\sigma\frac{B_0^2\nu}{\nu_0^2\rho},P_r=\frac{\nu}{K},S_c=\frac{V}{D},R_c=\frac{\nu_0^2K_0}{\nu^2\rho},\\&G_c=\frac{\nu g\beta(C_W-C_\infty)}{\nu_0^3},G_r=\frac{\nu g\beta(T_W-T_\infty)}{\nu_0^3},F=\frac{4I_1\nu}{\nu_0^2\rho C_p},s=\frac{Q\nu}{\nu_0^2\rho C_p},\\&R=\frac{Q_1\nu(C_W^*-C_\infty)}{\nu_0^2\rho(T_W^*-T_\infty)},K_c=\frac{k_r\nu}{\nu_0^2},H=F+S\end{aligned} \tag{6}$$

The equations, (2)-(5) reduce to following non-dimensional form

$$\frac{1}{4}\frac{\partial u}{\partial t}=\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}+G_rT+G_cC-\left(M^2+\frac{1}{K_p}\right)u \tag{7}$$

$$\frac{1}{4}\frac{\partial T}{\partial t}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}-HT+RC \tag{8}$$

$$\frac{1}{4}\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}-K_cC \tag{9}$$

$$\begin{aligned}&u=f\left(t\right)=t,\sin t\ \text{ and e}^t,T=1+\varepsilon e^{nt},C=1+\varepsilon e^{nt},\text{ at y}=0\\&u\to 0,\text{T}\to 0,\text{C}\to 0,\ \text{ as y}\to\infty\end{aligned} \tag{10}$$

3. Solution of the Problem

In view of periodic suction, temperature and concentration at the plate let us assume the velocity, temperature, concentration the neighborhood of the plate be

$$u\left(y,t\right)=u_0\left(y\right)+\varepsilon e^{nt}u_1\left(y\right),T\left(y,t\right)=T_0\left(y\right)+\varepsilon e^{nt}T_1\left(y\right), \quad \textit{and}\ C\left(y,t\right)=C_0\left(y\right)+\varepsilon e^{nt}C_1\left(y\right) \tag{11}$$

Substituting equations (11) into (7)-(9) and comparing the no harmonic and harmonic terms we get

$$\left(1+\frac{1}{\beta}\right)u_0^{11}-\left(M^2+\frac{1}{K_p}\right)u_0=-G_rT_0-G_cC_0 \tag{12}$$

$$\left(1+\frac{1}{\beta}\right)u_1^{11}-\left(M^2+\frac{1}{K_p}+\frac{n}{4}\right)u_1=-G_cC_1-G_rT_1 \tag{13}$$

$$T_0^{11}-P_rHT_0=-RP_rC_0 \tag{14}$$

$$T_1^{11}-\left(H+\frac{n}{4}\right)P_rT_1=-RP_rC_1 \tag{15}$$

$$C_0^{11}-K_cS_cC_0=0 \tag{16}$$

$$C_1^{11}-\left(K_c+\frac{n}{4}\right)S_cC_1=0 \tag{17}$$

The boundary conditions now reduce to

$$u_0=1,u_1=0,T_0=T_1=1,C_0=C_1=1,\quad at\quad y=0$$
$$u_0=u_1\to 0,T_0=T_1\to 0,C_0=C_1\to 0,\quad as\quad y\to\infty \tag{18}$$

Solving these differential equations (12-18) with the help of boundary conditions we get,

$$u_1(y,t)=(1-b_3-b_4)e^{-\sqrt{a_5}y}+b_3e^{-\sqrt{a_3}y}+b_4e^{-\sqrt{a_1}y} + \varepsilon e^{\mathrm{nt}}\left\{(-b_5-b_6)e^{-\sqrt{a_8}y}+b_5e^{-\sqrt{a_4}y}+b_6e^{-\sqrt{a_2}y}\right\} \tag{19}$$

$$u_2(y,t)=(1-b_3-b_4)e^{-\sqrt{a_5}y}+b_3e^{-\sqrt{a_3}y}+b_4e^{-\sqrt{a_1}y} + \varepsilon e^{\mathrm{nt}}\left\{(-b_5-b_6)e^{-\sqrt{a_8}y}+b_5e^{-\sqrt{a_4}y}+b_6e^{-\sqrt{a_2}y}\right\} \tag{20}$$

$$u_3(y,t)=(1-b_3-b_4)e^{-\sqrt{a_5}y}+b_3e^{-\sqrt{a_3}y}+b_4e^{-\sqrt{a_1}y} + \varepsilon e^{\mathrm{nt}}\left\{(-b_5-b_6)e^{-\sqrt{a_8}y}+b_5e^{-\sqrt{a_4}y}+b_6e^{-\sqrt{a_2}y}\right\} \tag{21}$$

$$T(y,t)=(1-b_1)e^{-\sqrt{a_3}y}+b_1e^{-\sqrt{a_1}y}+\varepsilon e^{\mathrm{nt}}\left\{(1-b_2)e^{-\sqrt{a_4}y}+b_2e^{-\sqrt{a_2}y}\right\} \tag{22}$$

$$C(y,t)=e^{-\sqrt{a_1}y}+\varepsilon e^{\mathrm{nt}}\left\{e^{-\sqrt{a_2}y}\right\} \tag{23}$$

The skin friction at the plate in terms of amplitude and phase angle is given by

$$\tau = \frac{\partial u_0}{\partial y} + \varepsilon e^{nt} \frac{\partial u_0}{\partial y}, aty = 0$$
$$\tau = [-(1 - b_3 - b_4)\sqrt{a_5} - b_3\sqrt{a_3} - b_4\sqrt{a_1}] + \varepsilon e^{nt} [(b_5 - b_6)\sqrt{a_8} - b_5\sqrt{a_4} - b_6\sqrt{a_2}] \tag{24}$$

The rate of heat transfer, i.e. heat flux at the (N_u) in terms of amplitude and phase is given by,

$$N_u = -\left[\frac{\partial T_0}{\partial y} + \varepsilon e^{nt} \frac{\partial T_1}{\partial y}\right] \ aty = 0 \ is$$
$$N_u = [(1 - b_3)\sqrt{a_3} + b_1\sqrt{a_1}] + \varepsilon e^{nt} [(1 - b_2)\sqrt{a_4} + b_2\sqrt{a_2}] \tag{25}$$

The mass transfer coefficient, i.e., the Sherwood number (S_h) at the plate in terms of amplitude and phase is given by

$$S_h = -\left[\frac{\partial C_0}{\partial y} + \varepsilon e^{nt} \frac{\partial C_1}{\partial y}\right] aty = 0 \ is \ S_h = [\sqrt{a_1}] + \varepsilon e^{nt} [\sqrt{a_2}] \tag{26}$$

4. Results and Discussions

In order to assess the effects of the dimensionless thermo physical parameters on the regime calculations have been carried out on velocity field, temperature field, and concentration field for various physical parameters like magnetic parameter, Prandtl parameter, Grashof number, modified Grashof number, chemical reaction parameter etc.

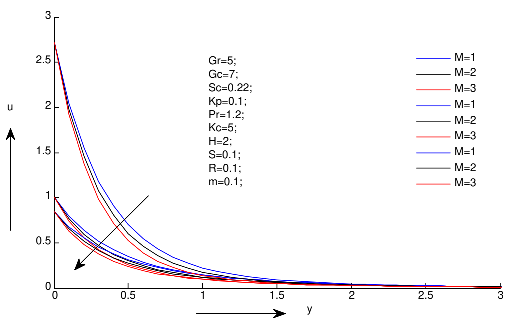

Fig.1. Effect of M on Velocity

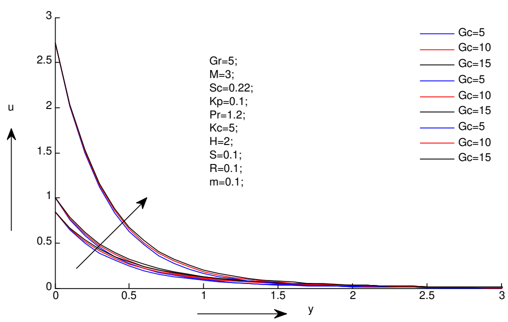

Fig.2. Effect of Gc of number on Velocity

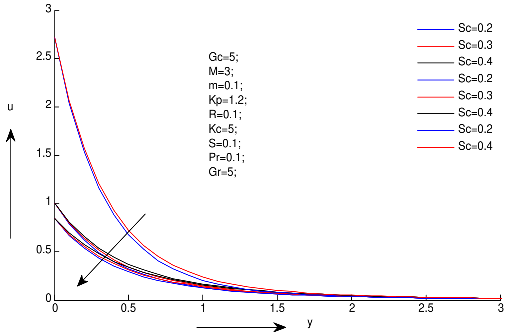

Fig.3. Effect of Sc of number on Velocity

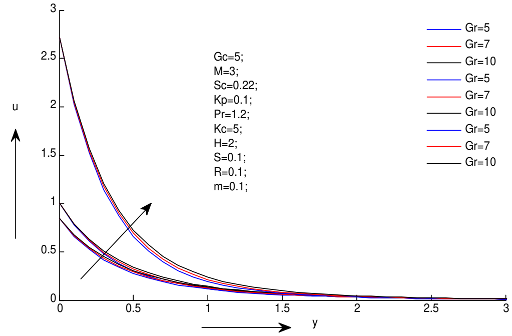

Fig.4. Effect of Grashof number on Velocity

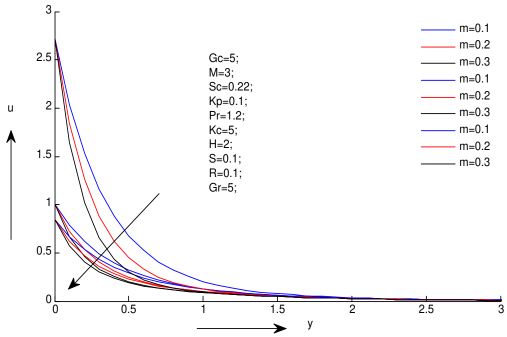

Fig.5. Effect of m=β Casson on Velocity

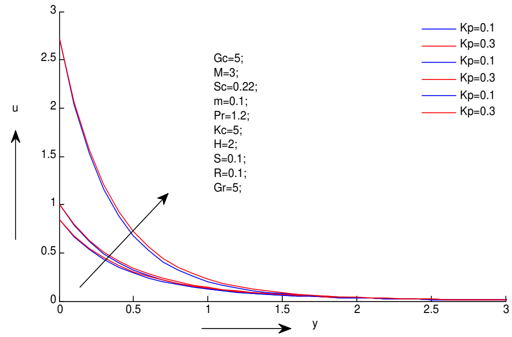

Fig.6. Effect of Kp on Velocity

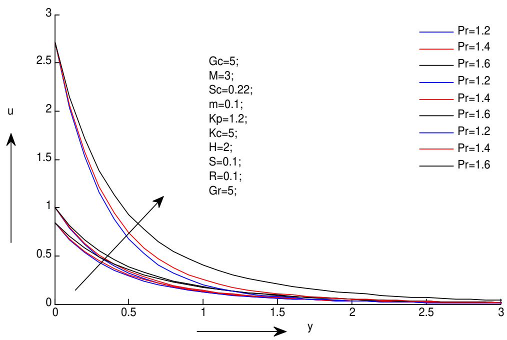

Fig.7. Effect of Pr on Velocity

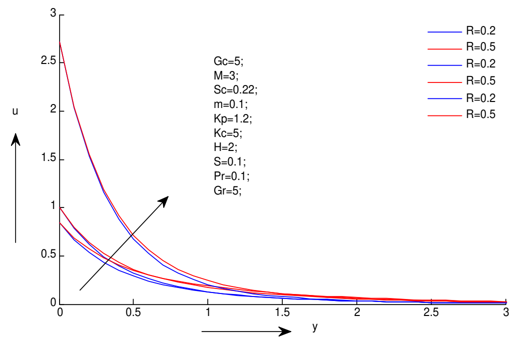

Fig.8. Effect of Ron Velocity

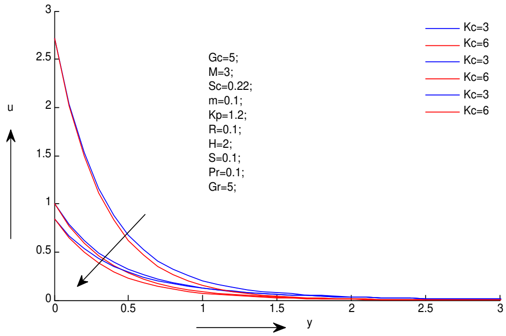

Fig.9. Effect of Kc on Velocity

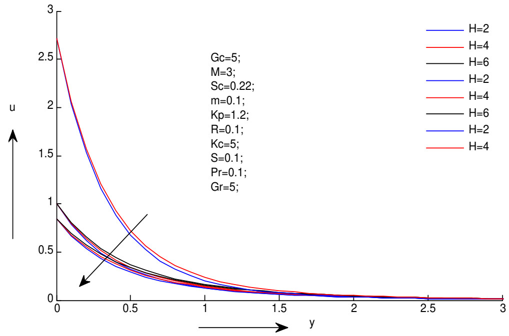

Fig.10. Effect of H on Velocity

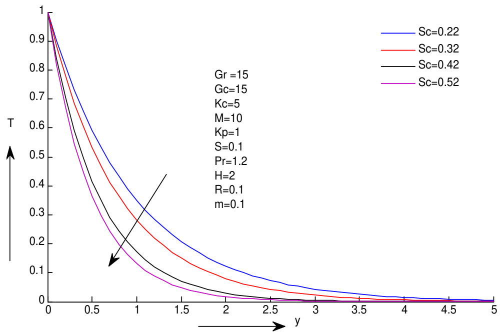

Fig.11. Effect of Sc on Temperature

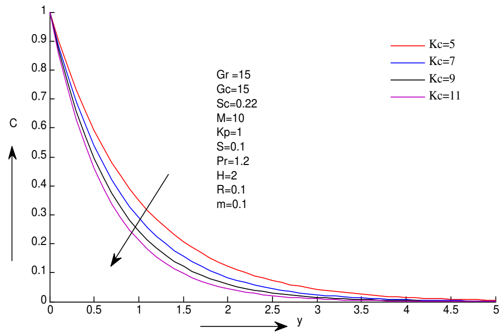

Fig.12. Effect of (Kc) on Concentration

The results are represented through graphs in figures 1 to 14. Figure 1, displays the velocity profiles for various values of magnetic parameter M. It is observed that the velocity decreases with an increase in M. This is due to fact that the applied magnetic field which acts as retarding force that condenses the momentum boundary layer. From figure 2, it displays that the velocity increases with an increase in Gc number. A similar effect is noticed from figure 3, in the presence of Schmidt number where velocity decreases. Figure 4, depicts the effects of Grashof number on velocity, from this figure it is noticed that the velocity increases with an increase in Gr. Influence of the m=β on velocity is presented in figure 5, from this figure it is noticed that the velocity decreases with an increase in m. From figure 6, it is seen that the velocity increases with an increase in Kp. From figure 7 shows velocity increases for the increasing values of Prandtl number. Effect of radiation absorption is presented in figure 8, from this figure, it is noticed that the velocity decreases with an increase in R. Form figure 9 noticed velocity decreases for increases in chemical reaction parameter. This figure 10 witnesses that velocity increases with an increase in H. Effect of Schmidt number on temperature is shown in figure 11, which concludes that temperature decreases as the values of Sc increase. From figure 12, it is concluded that the concentration decreases as Kc increases.

Table1: Effec ts of Gr, Gc, M and Kp on skin friction coefficient

Gr	Gc	M	Kp	T
11	4	0.9	1.5	10.181
12	4	0.9	1.5	11.162
13	4	0.9	1.5	12.173
9	5	0.9	1.5	12.354
9	6	0.9	1.5	11.445
9	7	0.9	1.5	10.533
9	4	1.5	1.5	12.461
9	4	2.0	1.5	12.347
9	4	2.5	1.5	11.217
9	4	0.9	0.3	8.674
9	4	0.9	0.6	9.489
9	4	0.9	0.8	9.8940

Table 2: Effec t of R and H on skin friction coefficient and Nusselt number

R	Pr	H	T	Nu
1	0.71	1	98.564	0.678
2	0.71	1	127.986	0.453
3	0.71	1	180.567	0.328
4	0.71	1	230.852	0.234
1.5	1.71	1	14.895	2.472
1.5	2.71	1	21.853	2.534
1.5	3.71	1	32.984	2.628
1.5	4.71	1	37.628	2.784
1.5	0.71	2	7.638	1.678
1.5	0.71	3	8.835	1.874
1.5	0.71	4	9.764	1.956
1.5	0.71	5	10.732	2.674

Table 3: Effec t of Sc and Kc on skin friction and Sherwood number

Sc	Kc	T	Sh
1.33	2	11.675	3.675
2.33	2	12.864	4.673
3.33	2	13.974	5.634
4.33	2	14.934	6.546
0.33	5	73.845	1.345
0.33	6	54.854	1.467
0.33	7	43.864	1.643
0.33	8	23.895	1.845

Effects of various parameters on skin friction, the rate of heat transfer and also the rate of mass transfer are presented in tables 1-3. From table 1 it is noted that skin friction increases due to an increase in Grashof number Gr. But modified Grashof number has a different effect on skin friction. Skin friction decreases due to an increase in M. From this table it is also observed that the skin friction increases due to an increase in porosity parameter. From table 2 it is observed that skin friction increases for increasing values of R whereas Nusselt number decreases with the increasing values of R. Of course, skin friction, as well as Nusselt number increase for increasing values of Pr and also heat source parameter H. From table 3, it is found that skin friction and Sherwood number both increase for increasing values of Sc. whereas skin friction decreases with increase values of Kc, but a reverse effect is noticed in the case of Sherwood number. Schmidt number and chemical reaction parameters are compared with the available results and the outcomes are shown in Table 4. The results are found in excellent agreement with the earlier work of Rama Mohan Reddy et al. [10].

Table. 4. Comparison of skin-friction coefficient and Sherwood number for various values of Sc and Kc

		Rama Mohan Reddy et al. [10]		Present	
Sc	Kc	τ	Sh	τ	Sh
1.22	1	9.1911	2.4708	9.1913	2.471
2.22	1	10.4171	3.3327	10.418	3.333
3.22	1	11.4397	4.0135	11.434	4.014
0.22	6	88.7471	1.1499	88.748	1.151
0.22	7	73.9932	1.2420	73.994	1.243
0.22	8	16.9541	1.3276	16.955	1.328

5. Conclusions

The following inferences are observed from the present investigation

- The heavier species with low conductivity reduces the flow within the boundary layer.
- An increase in elasticity of the fluid leads to decrease the velocity which is an established result.
- Impact of Casson parameter leads to decrease the fluid velocity.

Nomenclature

C^* Species concentration
D Molecular diffusivity
Gr Grashof number for heat transfer
K_p^1 Permeability of the medium
K Thermal diffusivity
Rc Elastic parameter
H Heat source parameter
Sh Sherwood number
Nu Nusselt number
T non-dimensional temperature
T non-dimensional time
U non-dimensional velocity
$v0$ Constant suction velocity
σ Electrical conductivity
ω non-dimensional frequency of oscillation
C non-dimensional species concentration
Gc Grashof number for mass transfer
g Acceleration due to gravity
Kp Porosity parameter
M Magnetic parameter
Bo Magnetic field of uniform strength
Pr Prandtl number
Sc Schmidt number
T^* Temperature of the field
t^* Time
u^* Velocity component long x-axis

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