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CORDIAL LABELING FOR NEW CLASS OF GRAPHS

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Abstract: Graph labeling is an assignment of integers to vertices or edges of a graph or both subject to a certain condition. The concept of cordial labeling was introduced by Cahit [3] in 1987. Let f be a function from the vertices of G to $(0, 1)$ and for each edge xy assigns the label $|f(x) - f(y)|$. We call f a cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ at most by 1, and the number of edges labeled 0 and the number of edges labeled 1 differ at most by 1. A graph which admits cordial labeling is called a cordial graph. In this paper, we prove the cordial labeling of a new class of graphs.

Keywords and Phrases: Cordial labeling, tadpole graph, k -polygonal snake graph.

2020 Mathematics Subject Classification: 05C78.

1. Introduction

Graph Theory is a dynamic mathematical discipline that has many applications in wide variety of subjects such as Physics, Chemistry, Operations Research and so on. The first paper in graph theory was written by Euler in 1736 when he solved the Konigsberg Bridge problem. Bondy and Murthy have proved various applications of graph theory [2]. Alex Rosa [1] introduced Graph labeling in 1967. Graph labeling has various applications like coding theory, radar, astronomy, circuit design, database management.

The concept of cordial labeling was introduced by Cahit as a weaker version of graceful and harmonious labeling. Ghodasara and Rokad [5] prove that the path union of $k_{m,n}$ is cordial. Vaidya and Shah [6] have discussed cordial labeling for some bistar related graphs. For extension survey on cordial labeling refer to Gallian Survey [4]. In this section we have proved that the tadpole graph $T_{a,b}$ attached to k- polygonal snakes by an edge is cordial,the tadpole graph $T_{a,b}$ attached to k-double polygonal snakes by an edge is cordial and the tadpole graph $T_{a,b}$ attached to alternate k polygonal snakes by an edge is cordial.

2. Preliminaries

Definition 2.1. *Let f be a function from the vertices of G to $(0, 1)$ and for each edge xy assigns the label $|f(x) - f(y)|$. We call f a cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ at most by 1, and the number of edges labeled 0 and the number of edges labeled 1 differ at most by 1.*

Definition 2.2. *The (m, n) - tadpole graph is a special type of graph consisting of a cycle graph on m vertices (atleast 3)and a path graph on n vertices connected with a bridge and it is denoted by $T_{m,n}$.*

Definition 2.3. *A k- polygonal snake is obtained by replacing every edge of a path P_n : u_1,u_2,...,u_n by k- cycle C_k for $k \geq 3$ such that each pair u_i,u_{i+1} remains adjacent for all $i = 1, 2, ..., n - 1$. It is denoted by $S_n(C_k)$. Note that $S_n(C_3)$ is the triangular snake graph and $S_n(C_4)$ is the quadrilateral snake graph.*

Definition 2.4. *A double k- polygonal snake is obtained from two k- polygonal snakes that have a common path. It is denoted by $DS_n(C_k)$.*

Definition 2.5. *An alternate k- polygonal snake is obtained by replacing every alternating edge of a path P_n : u_1,u_2,...,u_n by k- cycle C_k for $k \geq 3$ such that each pair u_i,u_{i+1} remains adjacent for all $i = 1, 2, ..., n - 1$. It is denoted by $AS_n(C_k)$.*

3. Main Result

Theorem 3.1. *The tadpole graph $T_{a,b}$ attached to k-polygonal snakes by an edge admits cordial.*

Proof. Let $T_{a,b}$ be the tadpole graph where a denotes the no. of vertices of cycle and b denotes the no. of vertices of the path attached to the cycle by an edge. Let us denote the vertices in the Tadpole graph $T_{a,b}$ as $c_1, c_2, ..., c_{a+b}$. Let $S_n(C_k)$ be the k-polygonal snake graph and we denote the vertices of the polygonal attached at the edge a_1, a_2 as $b^1_1, b^1_2, ..., b^1_k$ such that b^1_1 coincides as a_1 and b^1_k coincides as a_2. Similarly the vertices of the polygon attached at the edge a_2, a_3 are renamed as $b^2_1, b^2_2, ..., b^2_k$ such that b^2_1 coincides as a_2 and b^2_k coincides as a_3 and so on. The graphs $T_{a,b}$ and $S_n(C_k)$ are connected by an edge and is denoted as $T_{a,b}S_n(C_k)$. we have proved this result for $a \equiv 1(mod\ \ 4)$ and $b \equiv 1(mod\ \ 2)$

The total number of vertices and edges in the graph $T_{a,b}S_n(C_k)$ are denoted as p and q where $p = a + b + n(k-1) + 1$ and $q = a + b + nk + 1$.

The vertices of $T_{a,b}S_n(C_k)$ are labeled as follows

$$f(c_l) = \begin{cases} 1, & 1 \leq l \leq (a+b) \quad l \equiv 2, 3(mod\ 4) \\ 0, & 1 \leq l \leq (a+b) \quad l \equiv 0, 1(mod\ 4) \end{cases}$$

$$f(a_i) = \begin{cases} 0, & 1 \leq i \leq n \quad i \equiv 1, 2(mod\ 4) \\ 1, & 1 \leq i \leq n \quad i \equiv 0, 3(mod\ 4) \end{cases}$$

Case 1 When $k \equiv 1(mod\ \ 2)$ **and** $n \equiv 0(mod\ \ 4)$**:**

For $1 \leq i \leq n-1$ where $i \equiv 1(mod\ \ 2)$

$$f(b^i_j) = \begin{cases} 0, & 2 \leq j \leq k-1 \quad j \equiv 1, 2(mod\ 4) \\ 1, & 2 \leq j \leq k-1 \quad j \equiv 0, 3(mod\ 4) \end{cases}$$

For $1 \leq i \leq n-1$ where $i \equiv 0(mod\ \ 2)$

$$f(b^i_j) = \begin{cases} 1, & 2 \leq j \leq k-1 \quad j \equiv 1, 2(mod\ 4) \\ 0, & 2 \leq j \leq k-1 \quad j \equiv 0, 3(mod\ 4) \end{cases}$$

Case 2 When $k \equiv 0(mod\ \ 2)$ **and** $n \equiv 0(mod\ \ 2)$**:**

For $1 \leq i \leq n-1$ where $i \equiv 1, 2(mod\ \ 4)$

$$f(b^i_j) = \begin{cases} 0, & 2 \leq j \leq k-1 \quad j \equiv 1, 2(mod\ 4) \\ 1, & 2 \leq j \leq k-1 \quad j \equiv 0, 3(mod\ 4) \end{cases}$$

For $1 \leq i \leq n-1$ where $i \equiv 0, 3(mod\ \ 4)$

$$f(b^i_j) = \begin{cases} 1, & 2 \leq j \leq k-1 \quad j \equiv 1, 2(mod\ 4) \\ 0, & 2 \leq j \leq k-1 \quad j \equiv 0, 3(mod\ 4) \end{cases}$$

From the above definition it is clear that $|v_f(0) - v_f(1)| \leq 1$

The edges of $T_{a,b}S_n(C_k)$ are computed as follows

$$|f(a_i) - f(a_{i+1})| = \begin{cases} 0, & 1 \leq i \leq n \quad i \equiv 1(mod\ 2) \\ 1, & 1 \leq i \leq n \quad i \equiv 0(mod\ 2) \end{cases}$$

$$|f(b_j^i)-f(b_{j+1}^i)| = \begin{cases} 0, & 2 \le j < k-1 \quad j \equiv 1(mod\ 2) \\ 1, & 2 \le j < k-1 \quad j \equiv 0(mod\ 2) \end{cases}$$

$$|f(a_i)-f(b_2^i)| = \begin{cases} 0, & 1 \le i \le n \quad i \equiv 0,1(mod\ 4) \\ 1, & 1 \le i \le n \quad i \equiv 2,3(mod\ 4) \end{cases}$$

For $k \equiv 1(mod\ 2)$

$$|f(b_{k-1}^i)-f(a_{i+1})| = \begin{cases} 1, & 1 \le i \le n \quad i \equiv 0,1(mod\ 4) \\ 0, & 1 \le i \le n \quad i \equiv 2,3(mod\ 4) \end{cases}$$

For $k \equiv 0(mod\ 4)$

$$|f(b_{k-1}^i)-f(a_{i+1})| = \begin{cases} 1, & 1 \le i \le n \quad i \equiv 1(mod\ 2) \\ 0, & 1 \le i \le n \quad i \equiv 0(mod\ 2) \end{cases}$$

For $k \equiv 2(mod\ 4)$

$$|f(b_{k-1}^i)-f(a_{i+1})| = \begin{cases} 0, & 1 \le i \le n \quad i \equiv 1(mod\ 2) \\ 1, & 1 \le i \le n \quad i \equiv 0(mod\ 2) \end{cases}$$

$$|f(c_l)-f(c_{l+1})| = \begin{cases} 0, & 1 \le l \le a+b \quad l \equiv 0(mod\ 2) \\ 1, & 1 \le l \le a+b \quad l \equiv 1(mod\ 2) \end{cases}$$

$$|f(c_l)-f(c_a)| = 0$$

$$|f(c_{a+b})-f(a_1)| = \begin{cases} 0, & b \equiv 3(mod\ 4) \\ 1, & b \equiv 1(mod\ 4) \end{cases}$$

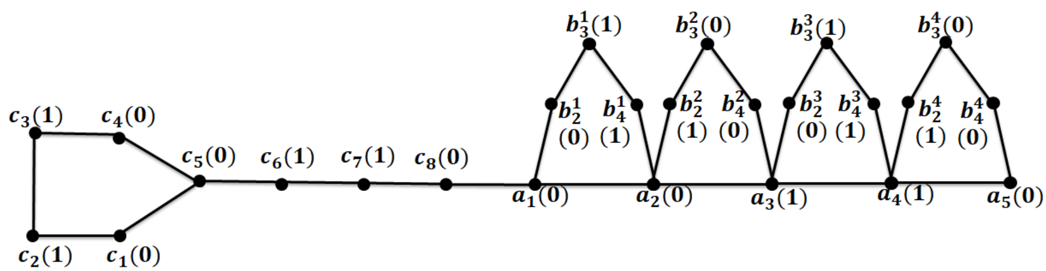

Figure 1: Tadpole graph $T_{5,3}$ attached to $S_5(C_5)$ is cordial

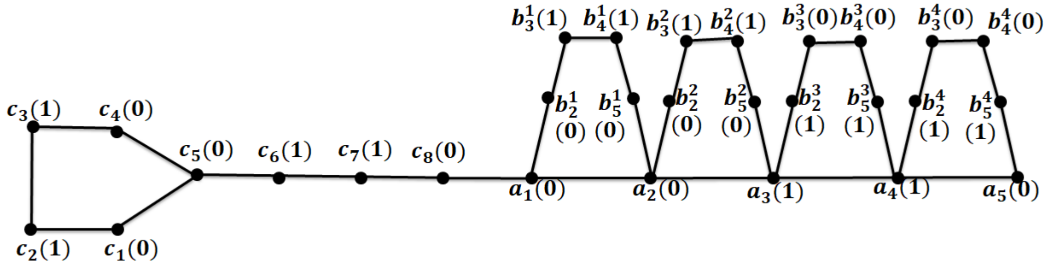

Figure 2: Tadpole graph $T_{5,3}$ attached to $S_5(C_6)$ is cordial

From the above calculations it is clear that $|e_f(0) - e_f(1)| \leq 1$
Thus the graph $T_{a,b}S_n(C_k)$ is cordial which is illustrated below in Figure 1 and figure 2.

Theorem 3.2. *The tadpole graph $T_{a,b}$ attached to double k-polygonal snakes by an edge admits cordial.*

Proof. Let $T_{a,b}$ be the tadpole graph as described above in theorem 3.1. Consider a k-polygonal snake as described above and attach that to the edges of the path upward and downward such the vertex of the polygon b_1^i coincides with the vertex a_i and the vertex b_k^i with the vertex a_{i+1} and vertex of the polygon c_1^i coincides with the vertex a_i and the vertex c_k^i with the vertex a_{i+1} for $1 \leq i < n$
Here $k \equiv 0(mod\ 2)$

$$f(c_l) = \begin{cases} 1, & 1 \leq l \leq (a+b) \quad l \equiv 2,3(mod\ 4) \\ 0, & 1 \leq l \leq (a+b) \quad l \equiv 0,1(mod\ 4) \end{cases}$$

$$f(a_i) = \begin{cases} 0, & 1 \leq i \leq n \quad i \equiv 1,2(mod\ 4) \\ 1, & 1 \leq i \leq n \quad i \equiv 0,3(mod\ 4) \end{cases}$$

For $1 \leq i \leq n-1$ where $i \equiv 1(mod\ \ 2)$

$$f(b_j^i) = \begin{cases} 1, & 2 \leq j \leq k-1 \quad j \equiv 2,3(mod\ 4) \\ 0, & 2 \leq j \leq k-1 \quad j \equiv 0,1(mod\ 4) \end{cases}$$

$$f(c_j^i) = \begin{cases} 1, & 2 \leq j \leq k-1 \quad j \equiv 1,2(mod\ 4) \\ 0, & 2 \leq j \leq k-1 \quad j \equiv 0,3(mod\ 4) \end{cases}$$

For $1 \leq i \leq n-1$ where $i \equiv 1(mod\ \ 2)$

$$f(b_j^i) = \begin{cases} 1, & 2 \leq j \leq k-1 \quad j \equiv 0,1(mod\ 4) \\ 0, & 2 \leq j \leq k-1 \quad j \equiv 2,3(mod\ 4) \end{cases}$$

$$f(c_j^i) = \begin{cases} 1, & 2 \leq j \leq k-1 \quad j \equiv 0, 3(mod\ 4) \\ 0, & 2 \leq j \leq k-1 \quad j \equiv 1, 2(mod\ 4) \end{cases}$$

From the above observations it is clear that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Thus the graph $T_{a,b}DS_n(C_k)$ is cordial which is illustrated below in Figure 3.

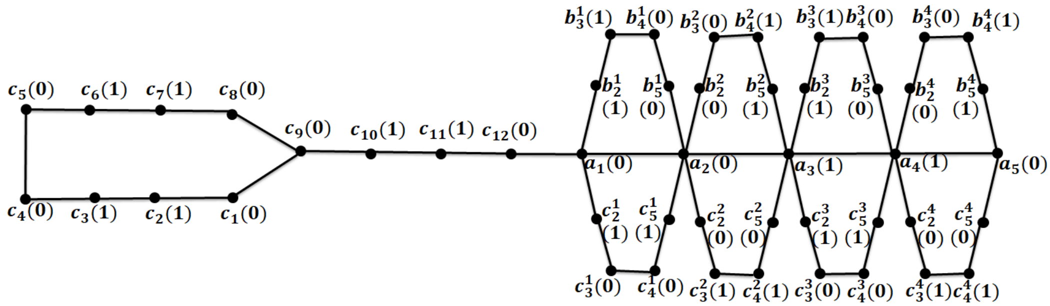

Figure 3: Tadpole graph $T_{9,3}$ attached to $DS_5(C_6)$ is cordial

Theorem 3.3. *The tadpole graph $T_{a,b}$ attached to Alternate k-polygonal snakes by an edge admits cordial.*

Proof. Let $T_{a,b}$ be the tadpole graph as described above in theorem 3.1. Consider a k-polygonal snake as described above and attach that to the edges of the path alternately such the vertex of the polygon b_1^1 coincides with the vertex a_1 and the vertex b_k^1 with the vertex a_2. Next the vertex of the polygon b_1^2 coincides with the vertex a_3 and the vertex b_k^2 with the vertex a_4 and continually attach the polygon snakes alternately for $1 \leq i \leq n$

$$f(c_l) = \begin{cases} 1, & 1 \leq l \leq (a+b) \quad l \equiv 2, 3(mod\ 4) \\ 0, & 1 \leq l \leq (a+b) \quad l \equiv 0, 1(mod\ 4) \end{cases}$$

Case 1 When $k \equiv 0(mod\ \ 4)$ **and** $n \equiv 0(mod\ \ 2)$**:**
For $1 \leq i \leq \frac{n}{2}$ where $i \equiv 1, 2(mod\ \ 4)$

$$f(b_j^i) = \begin{cases} 0, & 1 \leq j \leq k \quad j \equiv 1, 2(mod\ 4) \\ 1, & 1 \leq j \leq k \quad j \equiv 3, 0(mod\ 4) \end{cases}$$

For $1 \leq i \leq \frac{n}{2}$ where $i \equiv 3, 0(mod\ \ 4)$

$$f(b_j^i) = \begin{cases} 1, & 1 \leq j \leq k \quad j \equiv 1, 2(mod\ 4) \\ 0, & 1 \leq j \leq k \quad j \equiv 3, 0(mod\ 4) \end{cases}$$

Case 2 When $k \equiv 1(mod\ \ 2)$ **and** $n \equiv 0(mod\ \ 2)$**:**
For $1 \leq i \leq \frac{n}{2}$

$$f(b_j^i) = \begin{cases} 0, & 1 \leq j \leq k \quad j \equiv 1, 2(mod\ 4) \\ 1, & 1 \leq j \leq k \quad j \equiv 3, 0(mod\ 4) \end{cases}$$

From the above observations it is clear that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$

Thus the graph $T_{a,b}AS_n(C_k)$ is cordial which is illustrated below in Figure 4.

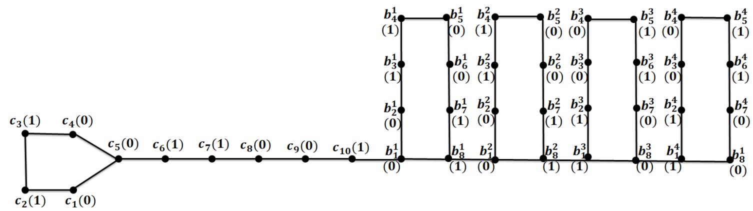

Figure 4: Tadpole graph $T_{5,5}$ attached to $AS_8(C_8)$ is cordial

4. Conclusion

In this paper we have proved that the tadpole graph $T_{a,b}$ attached to k - polygonal snakes, double polygonal snakes and alternate k - polygonal snakes by an edge is cordial. Further we intend to prove some other connected graphs is cordial.

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