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EDGE-ODD GRACEFUL LABELING OF JAHANGIR GRAPH

J. Jeba Jesintha and C. Yogalakshmi

PG Department of Mathematics,
Women's Christian College, University of Madras,
Chennai - 600006, Tamil Nadu, INDIA

E-mail : jjesintha_75@yahoo.com, yogalakshmi773@gmail.com

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Abstract: In 2008, Solairaju and Chithra [10] introduced edge-odd graceful labeling. A graph G with p vertices and q edges is called an edge odd graceful graph if there is a bijection f from the edge set of the graph to the set $\{1, 3, \ldots, (2q-1)\}$ such that, when each vertex is assigned to the sum of all edges incident to it modulo $2q$, the resulting vertex labels are distinct. In this paper, we have proved the Jahangir graph $J_{m,n}$ for $m = 3$ and $n \geq 3$ is edge odd graceful and in general, the Jahangir graph $J_{m,n}$ for m, n is odd, $m \geq 5$ and $n \geq 3$ is also edge odd graceful.

Keywords and Phrases: Edge-Odd graceful labeling, Jahangir graph.

2020 Mathematics Subject Classification: 05C78.

1. Introduction

A Graph labeling is one of the important areas in graph theory. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The first graph labeling method was graceful labeling introduced by Rosa [6] in 1967. The graceful labeling of a graph G with q edges is an injection from the vertices of G to the set $\{0, 1, 2, \ldots, q\}$ such that when each edge xy is assigned the label $|f(x)–f(y)|$, the resulting edge labels are distinct.

In 1985, Lo [4] introduced a labeling for the graph G called edge graceful labeling, which is a bijection f from the set of edges $E(G)$ to the set $\{1, 2, \ldots, q\}$ such that the induced map f^* from the set of vertices $V(G)$ to $\{0, 1, 2, \ldots, (p-1)\}$

given by $f^*(u) = \sum_{uv \in E(G)} f(uv)$ $(mod\ p)$ is a bijection. A graph which admits edge graceful labeling is called an edge graceful graph.

In 1991, Gnanojothi [2] introduced odd graceful labeling as an injection f from $V(G)$ to the set $\{0, 1, 2, \ldots, (2q-1)\}$ such that when each edge xy is assigned the label $|f(x)–f(y)|$, the resulting edge labels are in the set $\{1, 3, 5, \ldots, (2q-1)\}$.

In 2008, Solairaju and Chithra [10] defined edge odd graceful labeling as a bijection f from $E(G)$ to the set $\{1, 3, 5, \ldots, (2q-1)\}$ such that the induced mapping f from $V(G)$ to the set $\{0, 1, 2, \ldots, (2q-1)\}$ is given by $f(u) = \sum f(uv)$ $(mod\ 2q)$, where the vertex u is adjacent to the vertex v, the edge labels and vertex labels being distinct. A graph that admits edge odd graceful labeling is called an edge odd graceful graph.

Solairaju and Chithra [10] have proved the following graphs are edge odd graceful: paths with atleast 3 vertices, odd cycles, ladders $P_n \times P_2$ $(n \geq 3)$, stars with an even number of edges, and crowns $C_n \odot K_1$. In [9], Solairaju and Chithra have proved the following graphs to be edge odd graceful: P_n $(n > 1)$ with a pendant edges attached to each vertex (combs), the graph obtained by appending $2n+1$ pendant edges to each endpoints of P_2 or P_3, and the graph obtained by subdividing each edge of the star $K_{1,2n}$. Jeba Jesintha and Ezhilarasi Hilda [3] proved that the shell butterfly graphs have edge odd graceful labelings. Singhun [8] proved the following graphs have edge odd graceful labeling $W_{2n}, W_n \odot K_1$ and $W_n \odot K_m$, where n is odd, m is even and n divides m. Seoud and Salim [7] have shown the edge odd graceful labeling for the following families of graph W_n for $n \equiv$ 1,2,3 (mod 4), $C_n \odot K_{2m-1}$, even helms, $P_n \odot K_{2m}$ and $K_{2,s}$. Joseph Gallian [1] has given a broad and a dynamic survey on various graph labeling methods including edge odd graceful labeling.

2. Main Results

In this section we prove the Jahangir graph $J_{m,n}$ when $m = 3$ and $n \geq 3$, admits edge odd graceful labeling and in general, Jahangir graph $J_{m,n}$ for m, n is odd, $m \geq 5$ and $n \geq 3$ is also edge odd graceful.
The concept of Jahangir graph was introduced by Tomescu and Javaid [5].

Definition

The Generalized Jahangir graph $J_{m,n}$ for $n \geq 3$ is a graph on $mn+1$ vertices, consisting of a cycle C_{mn} with one additional vertex as centre that is adjacent to n vertices of C_{mn} at distance m to each other on C_{mn}.

Theorem 1. *The Jahangir graph $J_{m,n}$ for $m = 3$, $n \geq 3$ is an edge odd graceful graph.*

Proof. Let G denote the Jahangir graph $J_{m,n}$ for $m = 3$ and $n \geq 3$. Let $|V(G)|$ and $|E(G)|$ denote the number of vertices and edges respectively. Let v_0 be the apex vertex. The vertices adjacent to the apex vertex v_0 are labeled as $v_1, v_2, v_3, ..., v_n$ in a clockwise direction. The vertices which are not adjacent to the apex vertex v_0 are labeled as $u_1, u_2, u_3, ..., u_{2n}$ in a clockwise direction.

Case 1. When n $\equiv$ 3(mod 8)
Let $p = |V(G)| = mn + 1$, $q = |E(G)| = (m + 1)n$.
Let $f : E \rightarrow \{1, 3, ..., 8n - 3, 8n - 1\}$ be defined by

$$f(v_0v_1)=1,\ f(v_0v_n)= 3,\ f(v_0v_{n-1})= 5\ ,...,\ f(v_0v_3)= 2n\text{- }3,\ f(v_0v_2)= 2n\text{–}1$$

$$\begin{aligned} f(v_ru_{2r-1}) &= 2n + 2r - 1, && for\ 1 \leq r \leq n \\ f(u_{2i}v_{i+1}) &= 4n + 2i\text{–}1, && for\ 1 \leq i < n \\ f(u_{2r-1}u_{2r}) &= 6n + 2r - 1, && for\ 1 \leq r \leq n \\ f(u_{2n}v_1) &= 6n\text{–}1 \end{aligned}$$

The induced mapping are the vertex labels given below

$$\begin{aligned} f^*(v_r) &= 2r\text{–}1, && for\ 1 \leq r \leq n \\ f^*(u_{2r}) &= 10n + 4r\text{–}2(mod 8n), && for\ 1 \leq r \leq n \\ f^*(u_j) &\equiv 2j(mod 8n), && for\ j = 1, 3, 5, ..., 2n - 1 \end{aligned}$$

The label assigned to the central vertex v_0 is given by

$$f^*(v_0) \equiv 3n(mod 8n)$$

Case 2. When n $\not\equiv$ 3(mod 8)
Let $p = |V(G)| = mn + 1$, $q = |E(G)| = (m + 1)n$.
Let $f : E \rightarrow \{1, 3, ..., 8n - 3, 8n - 1\}$ be defined by

$$f(v_0v_1) = 1,\ f(v_0v_n) = 3,\ f(v_0v_{n-1}) = 5,...,\ f(v_0v_3) = 2n\text{ - }3,\ f(v_0v_2) = 2n - 1$$

$$\begin{aligned} f(v_ru_{2r-1}) &= 2n + 2r - 1, && for\ 1 \leq r \leq n \\ f(u_{2r-1}u_{2r}) &= 4n + 2r - 1, && for\ 1 \leq r \leq n \\ f(u_{2r}v_{r+1}) &= 6n + 2r - 1, && for\ 1 \leq r < n \\ f(u_{2n}v_1) &= 8n\text{–}1 \end{aligned}$$

The induced mapping are the vertex labels given below

$$
\begin{array}{llll}
f^*(v_r) & \equiv & 2n + 2r{-}1, & for\ 1 \le r \le n \\
f^*(u_{2r}) & \equiv & 10n + 4r{-}2(mod 8n), & for\ 1 \le r \le n \\
f^*(u_r) & \equiv & 6n + 4i - 2 & for\ r = 1, 3, ..., 2n - 1 \\
 & & & for\ 1 \le i \le n
\end{array}
$$

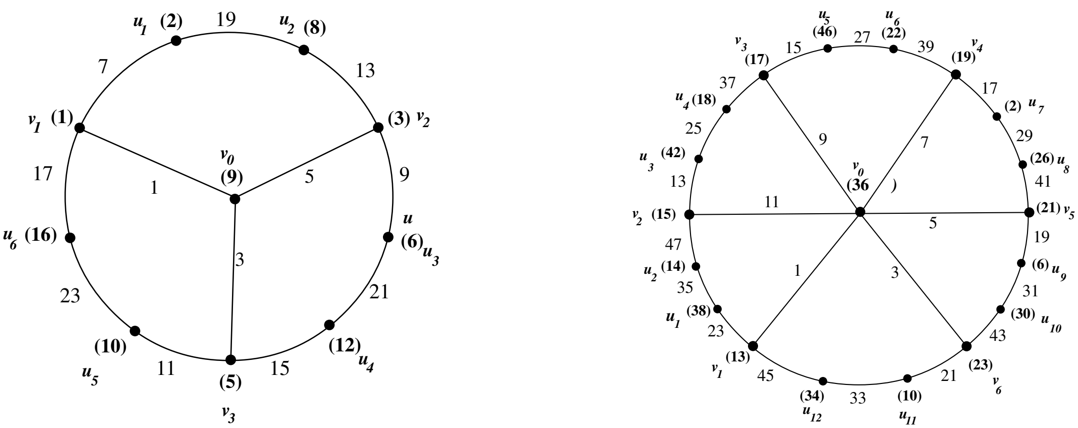
The label assigned to the central vertex v_0 is given by

$$
\begin{array}{c}
\text{When } n \equiv 0(\text{mod } 8),\ f^*(v_0) \equiv 0 \ (\text{mod } 8n) \\
\text{When } n \equiv 1(\text{mod } 8),\ f^*(v_0) \equiv n \ (\text{mod } 8n) \\
\text{When } n \equiv 2(\text{mod } 8),\ f^*(v_0) \equiv 2n(\text{mod } 8n) \\
\text{When } n \equiv 4(\text{mod } 8),\ f^*(v_0) \equiv 4n(\text{mod } 8n) \\
\text{When } n \equiv 5(\text{mod } 8),\ f^*(v_0) \equiv 5n(\text{mod } 8n) \\
\text{When } n \equiv 6(\text{mod } 8),\ f^*(v_0) \equiv 6n(\text{mod } 8n) \\
\text{When } n \equiv 7(\text{mod } 8),\ f^*(v_0) \equiv 7n(\text{mod } 8n)
\end{array}
$$

In both the Case 1 and 2, it is observed that the vertex labels are distinct.
Thus we have proved the Jahangir graph $J_{m,n}$ for $m = 3$ is edge odd graceful. The above theorem is illustrated in Figure 1

Illustration

Case 1: When m = 3 and n = 3 **Case 2: When m = 3 and n = 6**



(a) Jahangir Graph $J_{3,3}$ (b) Jahangir graph $J_{3,6}$

Figure 1

Theorem 2. *The Jahangir graph $J_{m,n}$ for m, n is odd, $m \geq 5$ and $n \geq 3$ is an edge odd graceful graph.*

Proof. Let G denote the Jahangir graph $J_{m,n}$ for m, n is odd, $m \geq 5$ and $n \geq 3$. Let $|V(G)|$ and $|E(G)|$ denote the number of vertices and edges respectively. Let v_0 be the apex vertex. The vertices adjacent to the apex vertex v_0 are labeled as $v_1, v_2, v_3, \ldots, v_n$ in a clockwise direction. The vertices which are not adjacent to the apex vertex v_0 are labeled as $u_1, u_2, u_3, \ldots, u_{(m-1)n}$ in a clockwise direction as in Figure 2.

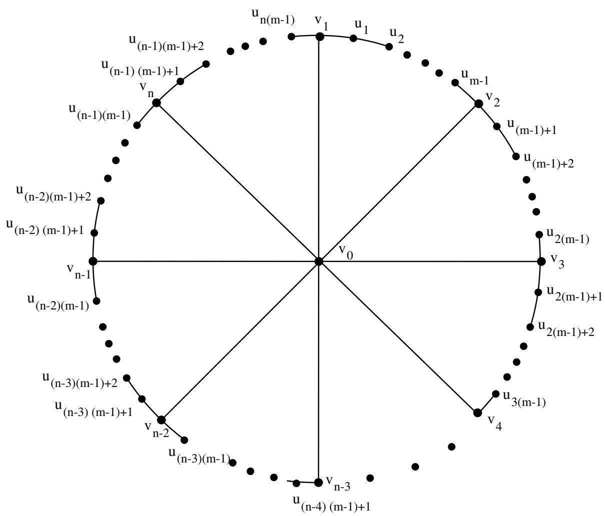


Figure 2: Jahangir Graph $J_{m,n}$

Case 1. When $\mathbf{m \geq 5, n \geq 3}$ except $\mathbf{m = 9}$ and $\mathbf{n = 5}$
Let $p = |V(G)| = mn + 1$, $q = |E(G)| = (m + 1)n$.
Let $f : E \to \{1, 3, \ldots, 2n(m + 1)\}$ be defined by

$$f(v_0v_1)=1,\ f(v_0v_n)=3,\ f(v_0v_{n-1})= 5,\ldots,\ f\ (v_0v_3)=2n\text{-}3,\ f\ (v_0v_2)=2n\text{–}1$$

$$\begin{aligned}
f(v_1u_1) &= 2n + 1 \\
f(v_2u_{(m-1)+1}) &= 2n + 1 + m(n + 1) \\
f(v_ru_{(r-1)(m-1)+1}) &= f(v_{(r-2)}u_{(r-3)(m-1)+1}) + 2m, && for\ 3 \leq r \leq n \\
f(u_{m-1}v_2) &= m + 2n \\
f(u_{n(m-1)}v_1) &= n(m + 2) \\
f(v_3u_{2(m-1)}) &= f(u_{n(m-1)}v_1) + 2m \\
f(u_{(r-1)(m-1)}v_r) &= f(u_{(r-3)(m-1)}v_{r-2}) + 2m, && for\ 4 \leq r \leq n
\end{aligned}$$

$$f(u_ju_{j+1}) = \begin{cases} 2n+j+1, & for\ j\ is\ even \\ q+n+j+1, & for\ j\ is\ odd \end{cases}$$

$$for\ 1 \leq j \leq m-2$$

$$f(u_{(m-1)+j}u_{(m-1)+(j+1)}) = \begin{cases} m+2n+j+1, & for\ j\ is\ odd \\ m+q+n+j+1, & for\ j\ is\ even. \end{cases}$$

$$for 1 \leq j \leq m-2$$

$$f(u_{i(m-1)+j}u_{i(m-1)+(j+1)}) = f(u_{(i-2)(m-1)+j}u_{(i-2)(m-1)+(j+1)}) + 2m,$$

$$for\ 2 \leq i \leq n-1, 1 \leq j \leq m-2$$

The induced mapping are the vertex labels given below

$$f^*(v_1) = n(m+4)+2$$

$$f^*(u_{k(m-1)+i}) \equiv n(m+4)+2km+2i+1\ (mod\ 2n(m+1))$$

$$for\ 0 \leq k \leq n-1, 1 \leq i \leq m-1$$

$$f^*(v_r) \equiv 6n+m(n+2)+2(r-2)(m-1)\ (mod\ 2n(m+1))$$

$$for\ 2 \leq r \leq n$$

The label assigned to the central vertex v_0 is given by

$$f^*(v_0) \equiv n^2 (mod\ 2n(m+1))$$

Case 2. When m = 9, n = 5
Let $p = |V(G)| = mn+1$, $q = |E(G)| = (m+1)n$.
Let $f : E \to \{1, 3, \ldots, 2n(m+1)\}$ be defined by

$f\ (v_0v_1)$= 1, $f(v_0v_2)$=3, $f(v_0v_3)$=5,..., $f(v_0v_{n-1})$=2n-3, $f(v_0v_n)$= 2n–1

The edge labels for the edges v_1u_1, $v_2u_{(m-1)+1}$, $v_ru_{(r-1)(m-1)+1}$ for $3 \leq r \leq n$, $u_{m-1}v_2$, $u_{n(m-1)}v_1$, $v_3u_{2(m-1)}$, $u_{(r-1)(m-1)}v_r$ for $4 \leq r \leq n$, u_ju_{j+1} for $1 \leq j \leq m-2$, $u_{(m-1)+j}u_{(m-1)+(j+1)}$ for $1 \leq j \leq m-2$, $u_{i(m-1)+j}u_{i(m-1)+(j+1)}$ for $1 \leq j \leq m-2$, $2 \leq i \leq n-1$ follows the same as in previous case.

The induced mapping are the vertex labels given below

$$f^*(v_r) = n(m+4)+20r-18, \qquad for\ 1 \leq r \leq n$$

The vertex labels for the vertices $v_1, v_0, u_{k(m-1)+i}$ for $0 \leq k \leq n-1, 1 \leq i \leq m-1$ follows the same as in previous case. In both the Case 1 and 2, it is observed that the vertex labels are distinct.
Thus we have proved the Jahangir graph $J_{m,n}$ for m, n is odd, $m \geq 5$ and $n \geq 3$ is edge odd graceful. The illustration for Case 1 is given in Figure 3.

Illustration

Case 1: When m = 5 and n = 5

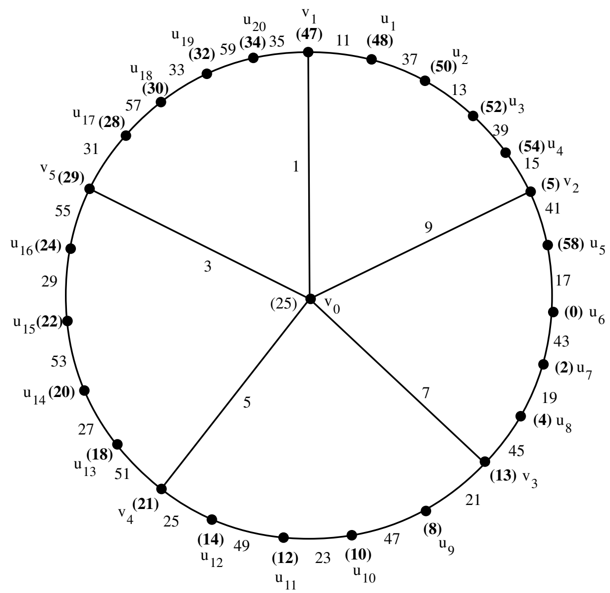


Figure 3: Jahangir Graph $J_{5,5}$

3. Conclusion

In this paper, we have proved the edge odd gracefulness on Jahangir graph $J_{m,n}$ for $m = 3$ and $n \geq 3$. In general, the Jahangir graph $J_{m,n}$ for m, n as odd and $m \geq 5, n \geq 3$ is also proved to be edge odd graceful.

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