South East Asian J. of Mathematics and Mathematical Sciences
Vol. 17, No. 3 (2021), pp. 261-276

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

λ_g^α-CLOSED AND λ_g^α-OPEN MAPS IN TOPOLOGICAL SPACES

S. Subhalakshmi and N. Balamani

Department of Mathematics,
Avinashilingam Institute for Home Science & Higher Education for Women,
Coimbatore - 641043, Tamil Nadu, INDIA

E-mail : subhamanu2013@gmail.com, nbalamani77@gmail.com

(**Received:** *Apr. 06, 2021* **Accepted:** *Oct. 29, 2021* **Published:** *Dec. 30, 2021)*

Abstract: In this paper, the aspect of λ_g^α-closed maps and λ_g^α-open maps are explored using the recently introduced λ_g^α-closed sets and λ_g^α-open sets in topological spaces. Initially, the standard properties of λ_g^α-closure and λ_g^α-interior with appropriate examples are studied. Further, characterizations of λ_g^α-closed maps and λ_g^α-open maps are also investigated.

Keywords and Phrases: λ-closed set, α-closed set, λ_g^α-closed set, λ_g^α-closed map and λ_g^α-open map.

2020 Mathematics Subject Classification: 54.

1. Introduction

Levine [7] introduced the notion of generalized closed sets in topological spaces. Following this, many researchers introduced several variation of generalized closed sets and investigated some stronger and weaker forms of them. Maki [10] continued the work of Levine and Dunham on generalized closed sets and closure operators by introducing the notion of Λ-sets in topological spaces.

A Λ-set is a set A which is equal to its kernel(= saturated set), i.e. to the intersection of all open supersets of A. Caldas et.al. [2] introduced the notion of λ-closure of a set by utilizing the notion of λ-closed sets defined by Francisco G Arenas et.al. [5]. They also studied the concept of λ-closed maps and studied various properties. Malghan [13] introduced the concept of generalized closed maps in topological spaces. Following this many researchers discussed various forms of closed maps and open maps. wg-closed maps and rwg-closed maps were introduced and studied by Nagavani [15]. Regular closed maps, gpr-closed maps, rg-closed maps, $rg\alpha$-closed and $\psi\widehat{g}$-closed maps have been introduced and studied by Long and Herington [8], Gnanambal [6], Arockiarani [1], Vadivel and Vairamanickam [20] and Ramya and Parvathi [17] respectively.

Moreover, the generalizations of various closed and open set concepts in general topology have been extended to ideal, digital, nano and micro topologies. Quite recently, Wadei Al-Omari and Noiri [23] presented the concept of A$\mathcal{G}_{\Im^*}$-sets , B$\mathcal{G}_{\Im^*}$-sets and $\delta\beta_{\Im}$-open sets in ideal topological spaces. Later, Wadei Al-Omari and Abu salem [22] studied some characterizations and basic properties of $\Im_g^*$-closed sets in ideal topological spaces. Recently, Subhalakshmi and Balamani [19] introduced λ_g^α-closed sets and λ_g^α-open sets in topological spaces and studied their properties. In this paper, we put forth concept of λ_g^α-closure and λ_g^α-interior of a subset A of a topological space and analyse their properties. Further we introduce λ_g^α-closed map and λ_g^α-open map and derive their fundamental properties and characterizations.

2. Preliminaries

Throughout this paper (X,τ) represents a topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X,τ), $cl(A)$ and $int(A)$ denote the closure and interior of A respectively.

Definition 2.1. [16] *Let (X,τ) be a topological space. A subset A of (X,τ) is called an α-open set if $A\subseteq int(cl(int(A)))$. The complement of an α-open set is called α-closed. The intersection of all α-closed sets containing A is called α-closure of A and is denoted by $cl_\alpha(A)$.*

Definition 2.2. [5] *Let (X,τ) be a topological space. A subset A of (X,τ) is called λ-closed if $A= L\cap D$, where L is a λ-set and D is a closed set. The complement of a λ-closed set is called λ-open set.*

Definition 2.3. [2] *The λ-closure of a subset A of a topological space (X,τ) is the intersection of all λ-closed sets containing A and is denoted by $cl_\lambda(A)$.*

Definition 2.4. *A subset A of a topological space (X,τ) is called*

(i) generalized closed (briefly g-closed) [7] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ).

(ii) generalized α-closed (briefly $g\alpha$-closed) [11] if $cl_\alpha(A)\subseteq U$ whenever $A\subseteq U$ and U is α-open in (X,τ).

(iii) α-generalized closed (briefly αg-closed) [12] if $cl_\alpha(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ).

(iv) *$g\Lambda$-closed [3] if $cl_\lambda(A)\subseteq U$ whenever $A\subseteq U$ and U is open in (X,τ).*

(v) *g^*-closed [21] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is g-open in (X,τ).*

(vi) *λ_g^α-closed [19] if $cl_\lambda(A)\subseteq U$ whenever $A\subseteq U$ and U is α-open in (X,τ).*

The complements of the above-mentioned sets are called their respective open sets.

Definition 2.5. *A map $f:(X,\tau)\to(Y,\sigma)$ is called*

(i) *closed if $f(V)$ is closed in (Y,σ) for every closed set V of (X,τ).*

(ii) *g-closed [13] if $f(V)$ is g-closed in (Y,σ) for every closed set V of (X,τ).*

(iii) *α-closed [14] if $f(V)$ is α-closed in (Y,σ) for every closed set V of (X,τ).*

(iv) *λ-closed [3] if $f(V)$ is λ-closed in (Y,σ) for every λ-closed set V of (X,τ).*

(v) *g^*-closed [18] if $f(V)$ is g^*-closed in (Y,σ) for every closed set V of (X,τ).*

(vi) *$g\alpha$-closed [4] if $f(V)$ is $g\alpha$-closed in (Y,σ) for every closed set V of (X,τ).*

(vii) *αg-closed [4] if $f(V)$ is αg-closed in (Y,σ) for every closed set V of (X,τ).*

(viii) *α-irresolute [9] if $f^{-1}(V)$ is α-closed in (X,τ) for every α-closed set V of (Y,σ).*

Remark 2.6.

(i) *In α-space, every α-closed subset of (X,τ) is closed in (X,τ).[16]*

(ii) *In $T_{1/2}$space, every g-closed subset of (X,τ) is closed in (X,τ).[7]*

(iii) *In ${}_\alpha T_b$-space, every αg-closed subset of (X,τ) is closed in (X,τ).[4]*

(iv) *In $T^*_{1/2}$-space, every g^*-closed subset of (X,τ) is closed in (X,τ).[21]*

Lemma 2.7. [19] *In a topological space (X,τ), the following properties hold:*

(i) *Every λ-closed set in (X,τ) is λ_g^α-closed.*

(ii) *Every closed set in (X,τ) is λ_g^α-closed.*

(iii) *Every open set in (X,τ) is λ_g^α-closed.*

(iv) *Every λ_g^α-closed set in (X,τ) is $g\Lambda$-closed.*

(v) *In an α-space, every α-closed set is λ_g^α-closed.*

(vi) *In a $T_{1/2}$-space, every g-closed set is λ_g^α-closed.*

(vii) *In a partition space, every λ_g^α-closed set is g-closed.*

(viii) *In a $T^*_{1/2}$-space, every g^*-closed set is λ_g^α-closed.*

Theorem 2.8. [19] *Let A be α-open and λ_g^α-closed in a topological space (X,τ). If F is λ-closed then A$\cap$F is λ_g^α-closed.*

3. Properties of λ_g^α-Closure and λ_g^α-Interior Operators

Definition 3.1. *For a subset A of a topological space (X,τ), the λ_g^α-closure of A (briefly $\lambda_g^\alpha cl(A)$) is defined to be the intersection of all λ_g^α-closed sets containing A. i.e.$\lambda_g^\alpha cl(A)$)= $\cap\{F\subseteq X | A \subseteq F$ and F is λ_g^α-closed in (X,τ)$\}$*

Proposition 3.2. *Let A and B be any two subsets of a topological space (X,τ). Then the following properties hold:*

(i) $\lambda_g^\alpha cl(\phi)=\phi$ *and* $\lambda_g^\alpha cl(X)=X$.

(ii) *If* $A\subseteq B$, *then* $\lambda_g^\alpha cl(A)\subseteq \lambda_g^\alpha cl(B)$.

(iii) $A\subseteq \lambda_g^\alpha cl(A)$.

(iv) $\lambda_g^\alpha cl(\lambda_g^\alpha cl(A))=\lambda_g^\alpha cl(A)$.

(v) *For* $A\subseteq X$, $\lambda_g^\alpha cl(A)\subseteq cl_\lambda(A)$.

Proof. (i), (ii), (iii) and (iv) follow from Definition 3.1 and (v) follows from Lemma 2.7.

Remark 3.3. *For a subset $A\subseteq X$, $\lambda_g^\alpha cl(A)$ need not be the smallest λ_g^α-closed set containing A as observed from the following example.*

Example 3.4. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d, e\}, X\}$. Then λ_g^α-closed sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{a, e\}$, $\{b, c\}$, $\{d, e\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, b, e\}$, $\{a, c, d\}$, $\{a, c, e\}$, $\{a, d, e\}$, $\{b, d, e\}$, $\{c, d, e\}$, $\{a, b, c, d\}$, {a,b,d,e}, {a,b,c,e}, {a,c,d,e}, {b,c,d,e}, X. Let A={d}. Then λ_g^αcl(A) = {a,d} $\cap$ {d,e} $\cap$ {a,b,d} $\cap$ {a,c,d} $\cap$ {a,d,e} $\cap$ {b,d,e} $\cap$ {c,d,e} $\cap$ {a,b,c,d} $\cap$ {a,b,d,e} $\cap$ {a,c,d,e} $\cap$ {b,c,d,e}$\cap$X={d} which is not the smallest λ_g^α-closed set containing A.

Proposition 3.5. *If a subset A of (X,τ) is λ_g^α-closed then $\lambda_g^\alpha cl(A)$=A, but not conversely.*

Proof. Let A be λ_g^α-closed in *(X,τ)*. By definition, $\lambda_g^\alpha cl(A)$=∩{$F \subseteq X|A \subseteq F$ and F is λ_g^α-closed in *(X,τ)*. Since A is a λ_g^α-closed set, F in the above intersection is A and hence $\lambda_g^\alpha cl(A) = A$.

Example 3.6. Consider (X,τ) as in Example 3.4. Let A={e} then λ_g^αcl(A)=A but A={e} is not a λ_g^α-closed set.

Remark 3.7. *$\lambda_g^\alpha cl(A)$ need not be a λ_g^α-closed set as observed from the Examples 3.4 and 3.6.*

Proposition 3.8. *For the subsets A and B of a topological space (X,τ), λ_g^αcl(A∩B)⊆ λ_g^αcl(A)∩λ_g^αcl(B).*

Proof. Since *A*∩*B*⊆*A* and *A*∩*B*⊆*B*, by Proposition 3.2 (ii) we have λ_g^α*cl(A*∩*B)*⊆ λ_g^α*cl(A)* and λ_g^α*cl(A*∩*B)*⊆ λ_g^α*cl(B)*. Hence λ_g^α*cl(A*∩*B)*⊆ λ_g^α*cl(A)*∩λ_g^α*cl(B)*.

Remark 3.9. *The reverse inclusion of Proposition 3.8 may not be true as observed from the following example.*

Example 3.10. Let $X = \{a,b,c,d,e\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, \{a,b,c,d\}, \{b,c,d,e\}, X\}$. Then λ_g^α-closed sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{e\}$, $\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{a,e\}$, $\{b,c\}$, $\{c,d\}$, $\{d,e\}$, $\{a,b,c\}$, $\{a,c,d\}$, $\{a,d,e\}$, $\{b,c,d\}$, $\{c,d,e\}$, $\{a,b,c,d\}$, $\{a,c,d,e\}$, $\{b,c,d,e\}$, X. Let A={b,d} and B={b,e}. Then $A \cap B = \{b\}$, $\lambda_g^\alpha cl(A \cap B) = \{b\}$, $\lambda_g^\alpha cl(A) = \{b,c,d\}$ and $\lambda_g^\alpha cl(B) = \{b,c,d,e\}$. Therefore $\lambda_g^\alpha cl(A) \cap \lambda_g^\alpha cl(B) = \{b,c,d\}$ but $\lambda_g^\alpha cl(A \cap B) = \{b\}$. Hence $\lambda_g^\alpha cl(A) \cap \lambda_g^\alpha cl(B) \nsubseteq \lambda_g^\alpha cl(A \cap B)$.

Proposition 3.11. *For the subsets A and B of topological space (X,τ), $\lambda_g^\alpha cl(A) \cup \lambda_g^\alpha\ cl(B) \subseteq \lambda_g^\alpha\ cl(A \cup B)$.*

Proof. Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, by Proposition 3.2 (ii), $\lambda_g^\alpha cl(A) \subseteq \lambda_g^\alpha cl(A \cup B)$ and $\lambda_g^\alpha cl(B) \subseteq \lambda_g^\alpha cl(A \cup B)$. Hence $\lambda_g^\alpha cl(A) \cup \lambda_g^\alpha cl(B) \subseteq \lambda_g^\alpha cl(A \cup B)$.

Remark 3.12. *The reverse inclusion of Proposition 3.11 may not be true as observed from the following example.*

Example 3.13. Consider (X,τ) as in Example 3.10. Let A={b} and $B = \{d\}$. Then A∪B={b,d}, λ_g^αcl(A)={b}, λ_g^αcl(B)={d} and λ_g^αcl(A∪B)={b,c,d}. Therefore $\lambda_g^\alpha\ cl(A) \cup \lambda_g^\alpha\ cl(B) = \{b,d\}$ but λ_g^αcl(A∪B)={b,c,d}. Hence $\lambda_g^\alpha\ cl(A \cup B) \nsubseteq \lambda_g^\alpha\ cl(A) \cup \lambda_g^\alpha cl(B)$.

Remark 3.14. *λ_g^α-closure operator is not a Kuratowski closure operator as it does not satisfy*
λ_g^αcl(A∪B)=λ_g^αcl(A)∪λ_g^αcl(B).

Definition 3.15. *For a subset A of topological space (X,τ), the λ_g^α-interior of A* *(briefly $\lambda_g^\alpha int(A)$) is defined to be the union of all λ_g^α-open sets contained in A. i.e. $\lambda_g^\alpha int(A) = \cup\{F \subseteq X | F \subseteq A$ and F is λ_g^α-open in $(X,\tau)\}$*

Proposition 3.16. *Let A and B any two subsets of a topological space (X,τ). Then the following properties hold:*

(i) $\lambda_g^\alpha int(\phi) = \phi$ and $\lambda_g^\alpha int(X) = X$.

(ii) If $A \subseteq B$, then $\lambda_g^\alpha int(A) \subseteq \lambda_g^\alpha int(B)$.

(iii) $\lambda_g^\alpha int(A) \subseteq (A)$.

(iv) $\lambda_g^\alpha int(\lambda_g^\alpha int(A)) = \lambda_g^\alpha int(A)$.

(v) For $A \subseteq X$, $int_\lambda(A) \subseteq \lambda_g^\alpha int(A)$.

Proof. Obvious.

Remark 3.17. *For a subset $A \subseteq X$, $\lambda_g^\alpha int(A)$ need not be the largest λ_g^α-open set contained in A as observed from the following example.*

Example 3.18. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d, e\}, X\}$. Then λ_g^α-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{e\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{b, d\}$, $\{b, e\}$, $\{c, d\}$, $\{c, e\}$, $\{d, e\}$, $\{a, b, c\}$, $\{a, d, e\}$, $\{b, c, d\}$, $\{b, c, e\}$, $\{b, d, e\}$, $\{c, d, e\}$, $\{a, b, d, e\}$, $\{a, c, d, e\}$, $\{b, c, d, e\}$, X. Let $A = \{a, b, d\}$. Then λ_g^αint(A) = {a}$\cup${b}$\cup${d}$\cup${a,b}$\cup${b,d} = {a,b,d}, which is not the largest λ_g^α-open set contained in A.

Proposition 3.19. *If a subset A of (X, τ) is λ_g^α-open then $\lambda_g^\alpha int(A) = A$, but not conversely.*
Proof. Obvious.

Example 3.20. Consider (X, τ) as in Example 3.18. Let $A = \{a, e\}$. Then $\lambda_g^\alpha int(A) = A$ but $A = \{a, e\}$ is not a λ_g^α-open set.

Remark 3.21. *$\lambda_g^\alpha int(A)$ need not be a λ_g^α-open set as observed from the Examples 3.18 and 3.20.*

Proposition 3.22. *For the subsets A and B of topological space (X, τ), $\lambda_g^\alpha int(A \cup B) \supseteq \lambda_g^\alpha int(A) \cup \lambda_g^\alpha int(B)$.*
Proof. Obvious.

Remark 3.23. *The reverse inclusion of Proposition 3.20 may not be true as observed from the following example.*

Example 3.24. Consider (X, τ) as in Example 3.10. Here λ_g^α-open sets are ϕ, $\{a\}$, $\{b\}$, $\{e\}$, $\{a, b\}$, $\{a, e\}$, $\{b, c\}$, $\{b, e\}$, $\{d, e\}$, $\{a, b, c\}$, $\{a, b, e\}$, $\{a, d, e\}$,

$\{b, c, d\}$, $\{b, c, e\}$, $\{b, d, e\}$, $\{c, d, e\}$, $\{a, b, c, d\}$, $\{a, b, c, e\}$, $\{a, b, d, e\}$, $\{a, c, d, e\}$, $\{b, c, d, e\}$, $X\}$. Let A={c} and B={d,e}. Then $A \cup B$ = {c,d,e}, $\lambda_g^\alpha int(A \cup B) = \{c, d, e\}, \lambda_g^\alpha int(A) = \phi$ and $\lambda_g^\alpha int(B) = \{d, e\}$. Therefore $\lambda_g^\alpha int(A) \cup \lambda_g^\alpha int(B) = \{d, e\}$ but $\lambda_g^\alpha int(A \cup B) = \{c, d, e\}$. Hence $\lambda_g^\alpha int(A \cup B) \nsubseteq \lambda_g^\alpha int(A) \cup \lambda_g^\alpha int(B)$.

Proposition 3.25. *For the subsets A and B of a topological space* (X, τ), $\lambda_g^\alpha int(A \cap B) \subseteq \lambda_g^\alpha int(A) \cap \lambda_g^\alpha int(B)$.
Proof. Obvious.

Remark 3.26. *The reverse inclusion of Proposition 3.25 may not be true as observed from the following example.*

Example 3.27. Consider (X, τ) as in Example 3.10. Let A={b,c,d} and B={c,d,e}. Then $A \cap B = \{c, d\}$, $\lambda_g^\alpha int(A \cap B) = \{\phi\}, \lambda_g^\alpha int(A) = \{b, c, d\}$ and $\lambda_g^\alpha int(B) = \{c, d, e\}$. Therefore $\lambda_g^\alpha int(A) \cap \lambda_g^\alpha int(B) = \{c, d\}$ but $\lambda_g^\alpha int(A \cap B) = \{\phi\}$. Hence $\lambda_g^\alpha int(A) \cap \lambda_g^\alpha int(B) \nsubseteq \lambda_g^\alpha int(A \cap B)$.

Lemma 3.28. *For a subset A of (X,τ), the following properties hold:*

(i) $\lambda_g^\alpha cl(A^c) = (\lambda_g^\alpha int(A))^c$.

(ii) $(\lambda_g^\alpha cl(A^c))^c = \lambda_g^\alpha int(A)$.

(iii) $\lambda_g^\alpha cl(A) = (\lambda_g^\alpha int(A^c))^c$.

4. λ_g^α-Closed Maps and its Properties

Definition 4.1. *A map f:$(X, \tau) \rightarrow (Y,\sigma)$ is called a λ_g^α-closed map if the image of each closed set in (X,τ) is λ_g^α-closed in (Y,σ), i.e. if f(V) is λ_g^α-closed in (Y,σ) for every closed set V in (X,τ).*

Example 4.2. Let X=Y={a,b,c}, $\tau = \{\phi$,{a},X} and $\sigma = \{\phi$,{a,b},Y}. Let f:$(X, \tau) \rightarrow$ (Y,σ) be a map defined by f(a) = b, f(b) = c and f(c) = a. Then f is a λ_g^α-closed map as the image of every closed set in (X,τ) is λ_g^α-closed in (Y, σ).

Proposition 4.3. *Every closed map is a λ_g^α-closed map, but not conversely.*
Proof. Let f:*(X,τ)→(Y,σ)* be a closed map and V be a closed set in *(X,τ)*. Then $f(V)$ is a closed set in *(Y,σ)*. As every closed set is a λ_g^α-closed set by Lemma 2.7, $f(V)$ is λ_g^α-closed in *(Y,σ)*. Thus f is a λ_g^α-closed map.

Example 4.4. Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\phi$, $\{a\}$, $\{b\}$, $\{a, b\}$, $\{b, c\}$, $\{a, b, c\}$, $\{b, c, d\}$, $\{a, b, c, d\}$, $\{b, c, d, e\}$, $X\}$ and $\sigma = \{\phi$, {a}, {b,c,d}, {a,b,c,d},Y}. Let f:$(X, \tau) \rightarrow$(Y,σ) be a map defined by f(a) = c, f(b) = a, f(c) = b, f(d) = d and f(e) = e. Then f is a λ_g^α-closed map but not a closed map, since for the closed set {a} in (X,τ) , f({a})={c} is not a closed set in (Y,σ).

Proposition 4.5. *Every λ-closed map is a λ_g^α-closed map, but not conversely.*
Proof. Let f:*(X,τ)$\rightarrow$(Y,σ)* be a λ-closed map and V be a closed set in *(X,τ)*. As every closed set is λ-closed, V is a λ-closed set in *(X,τ)*. Since f is a λ-closed map, $f(V)$ is λ-closed in *(Y,σ)*. As every λ-closed set is λ_g^α-closed set by Lemma 2.7, $f(V)$ is λ_g^α-closed. Hence f is λ_g^α-closed map.

Example 4.6. Consider (X,τ) and (Y,σ) as in Example 4.4. Let f:(X,τ) $\rightarrow$(Y,σ) be a map defined by f(a)=a, f(b)=c, f(c)=b, f(d)=e and f(e)=d. Then f is λ_g^α-closed map but not a λ-closed map, since for the λ-closed set {a,b} in (X,τ), f({a,b})={a,c} is not a λ-closed set in (Y,σ).

Definition 4.7. *A map f:(X,τ) $\rightarrow$(Y,σ) is called a gΛ-closed map if the image of each closed set in (X,τ) is gΛ-closed in (Y,σ), i.e. if f(V) is gΛ-closed in (Y,σ) for every closed set V in (X,τ).*

Proposition 4.8. *Every λ_g^α-closed map is a gΛ-closed map but not conversely.*
Proof. Let f:*(X,τ)$\rightarrow$(Y,σ)* be a λ_g^α-closed map and V be a closed set in *(X,τ)*. Then $f(V)$ is λ_g^α-closed in *(Y,σ)*. Since every λ_g^α-closed set is a gΛ-closed set by Lemma 2.7, $f(V)$ is gΛ-closed in *(Y,σ)*. Hence f is a gΛ-closed map.

Example 4.9. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi$, $\{a\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, c, d\}$, $X\}$ and $\sigma = \{\phi$, {a}, {a,b},Y}. Let f:(X,τ) $\rightarrow$(Y,σ) be a map defined by f(a)=d, f(b)=c, f(c)=b and f(d)=a. Then f is a gΛ-closed map but not a λ_g^α-closed map, since for the closed set {b,d} in (X,τ), f({b,d})={a,c} is not a λ_g^α-closed set in (Y,σ).

Remark 4.10. *g-closed maps and λ_g^α-closed maps are independent of each other as observed from the following examples.*

Example 4.11. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi$, $\{a\}$, $\{b\}$, $\{a, b\}$, $X\}$ and $\sigma = \{\phi$, $\{a\}$, $\{a, b\}$, $Y\}$. Let f:(X,τ) $\rightarrow$(Y,σ) be a map defined by f(a)=b, f(b)=a and f(c)=c. Then f is a g-closed map but not a λ_g^α-closed map, since for the closed set {b,c} in (X,τ), f({b,c})={a,c} is not a λ_g^α-closed set in (Y,σ).

Example 4.12. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi$, $\{a\}$, $\{a, b\}$, $\{a, c\}$, $X\}$ and $\sigma = \{\phi$, $\{a\}$, $\{a, b\}$, $Y\}$. Let f:(X,τ) $\rightarrow$(Y,σ) be the identity map. Then f is a λ_g^α-closed map but not a g-closed map, since for the closed set {b} in (X,τ), f({b}) = {b} is not a g-closed set in (Y,σ).

Remark 4.13. *α-closed maps and λ_g^α-closed maps are independent of each other as observed from the following examples.*

Example 4.14. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi$, $\{a\}$, $\{b, c\}$, $X\}$ and $\sigma = \{\phi$, $\{a\}$,

$\{a, b\}$, $Y\}$. Let f:(X, τ) →(Y,σ) be the identity map. Then f is a λ_g^α-closed map but not an α-closed map, since for the closed set {a} in (X,τ), f({a})={a} is not an α-closed set in (Y,σ).

Example 4.15. Let X=Y={a,b,c,d}, $\tau = \{\phi$, {a,b,c},X} and $\sigma = \{\phi$, {a},Y}. Let f:(X, τ) →(Y,σ) be a map defined by f(a)=c, f(b)=d, f(c)=a and f(d)=b. Then f is an α-closed map but not a λ_g^α-closed map, since for the closed set {d} in (X,τ), f({d})={b} is not a λ_g^α-closed set in (Y,σ).

Remark 4.16. *αg-closed maps and λ_g^α-closed maps are independent of each other as observed from the following examples.*

Example 4.17. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi$, $\{a\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, c, d\}$, $X\}$ and $\sigma = \{\phi$, $\{a\}$, $Y\}$. Let f:(X, τ) →(Y,σ) be the identity map. Then f is an αg-closed map but not a λ_g^α-closed map, since for the closed set {b} in (X,τ), f({b})={b} is not a λ_g^α-closed set in (Y,σ).

Example 4.18. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi$, $\{a\}$, $\{b\}$, $\{a, b\}$, $\{a, b, c\}$, $\{a, b, d\}$, $X\}$ and $\sigma = \{\phi$, $\{a\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, c, d\}$, $Y\}$. Let f:(X, τ) →(Y,σ) be a map defined by f(a)=a, f(b)=d, f(c)=c and f(d)=b. Then f is a λ_g^α-closed map but not an αg-closed map, since for the closed set {a,c,d} in (X,τ), f({a,c,d})={a,b,c} is not an αg-closed set in (Y,σ).

Remark 4.19. *gα-closed maps and λ_g^α-closed maps are independent of each other as observed from the following examples.*

Example 4.20. Let X=Y={a,b,c,d}, $\tau = \{\phi$, {a,b,c},X} and $\sigma = \{\phi$, {a}, {a,b},Y}. Let f:(X, τ) →(Y,σ) be the identity map. Then f is a gα-closed map but not a λ_g^α-closed map, since for the closed set {d} in (X,τ), f({d})={d} is not a λ_g^α-closed set in (Y,σ).

Example 4.21. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi$, $\{a\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, c, d\}$, $X\}$ and $\sigma = \{\phi$, $\{a, b, c\}$, $Y\}$. Let f:(X, τ) →(Y,σ) be a map defined by f(a)=b, f(b)=c, f(c)=d and f(d)=a. Then f is a λ_g^α-closed map but not a gα-closed map, since for the closed set {b} in (X,τ), f({b})={c} is not a gα-closed set in (Y,σ).

Theorem 4.22. *If f:(X, τ) →(Y,σ) is a λ_g^α-closed map and (Y,σ) is a partition space then f is a g-closed map.*

Proof. Let V be a closed set in *(X,τ)*. Since f is a λ_g^α-closed map, $f(V)$ is a λ_g^α-closed set in *(Y,σ)*. As *(Y,σ)* is a partition space, by Lemma 2.7, $f(V)$ is g-closed in *(Y,σ)*. Therefore f is a g-closed map.

Theorem 4.23. *If f:(X,τ) →(Y,σ) is a g-closed (resp. α-closed, αg-closed, g^*-closed) map and (Y,σ) is a $T_{1/2}$-space (resp. α-space, ${}_\alpha T_b$-space, $T^*_{1/2}$-space) then f is a λ_g^α-closed map.*

Proof. Let V be a closed set in *(X,τ)*. As f is a g-closed (resp. α-closed, αg-closed, g^*-closed) map, *f(V)* is a g-closed (resp. α-closed, αg-closed, g^*-closed) set in *(Y,σ)*. Since *(Y,σ)* is a $T_{1/2}$-space (resp. α-space, ${}_\alpha T_b$-space, $T^*_{1/2}$-space) by Remark 2.6, $f(V)$ is a closed set in *(Y,σ)*. As every closed set is a λ_g^α-closed set, $f(v)$ is a λ_g^α-closed set in *(Y,σ)*. Therefore f is a λ_g^α-closed map.

Theorem 4.24. *If a map f:(X,τ) →(Y,σ) is α-irresolute and λ-closed then for every λ_g^α-closed set G of (X,τ), f(G) is a λ_g^α-closed set in (Y,σ).*

Proof. Let G be a λ_g^α-closed set in *(X,τ)*. Let U be an α-open set of *(Y,σ)* such that $f(G) \subseteq U$ then $G \subseteq f^{-1}(U)$. As f is an α-irresolute map, $f^{-1}(U)$ is α-open in*(X,τ)*. Since G is a λ_g^α-closed set and f^{-1}(U) is an α-open set by definition of λ_g^α-closed set, $cl_\lambda(G) \subseteq f^{-1}(U)$ which implies $f(cl_\lambda(G)) \subseteq$ U. As f is λ-closed, $f(cl_\lambda(G))$ is λ-closed. Now $cl_\lambda(f(G)) \subseteq cl_\lambda(f(cl_\lambda(G))) = f(cl_\lambda(G)) \subseteq U$. Therefore $f(G)$ is a λ_g^α-closed set in *(Y,σ)*.

Theorem 4.25. *A map f:(X,τ) →(Y,σ) is λ_g^α-closed if and only if for each subset S of (Y,σ) and for each open set U of (X,τ) containing $f^{-1}(S)$, there exists a λ_g^α-open set V of (Y,σ) such that S⊆V and $f^{-1}(V)$ ⊆U.*

Proof. **(Necessity)** Let f be a λ_g^α-closed map. Suppose that S is a subset of *(Y,σ)* and U is an open set of *(X,τ)* such that $f^{-1}(S) \subseteq U$. Since f is a λ_g^α-closed, $f(X\backslash U)$ is a λ_g^α-closed set in *(Y,σ)* implies $Y\backslash[f(X\backslash U)]$ is a λ_g^α-open set.
Let $V = Y\backslash[f(X\backslash U)]$. Since $f^{-1}(S) \subseteq U, [X\backslash$ U]$\subseteq X\backslash f^{-1}(S) = f^{-1}(Y\backslash S) \Rightarrow f(X\backslash U) \subseteq (Y\backslash S) \Rightarrow S \subseteq [Y\backslash(f(X\backslash U))] = V$. Now $f(X\backslash U)\subseteq (Y\backslash V) \Rightarrow (X\backslash U) \subseteq f^{-1}[(Y\backslash V)] = X\backslash f^{-1}(V) \Rightarrow f^{-1}(V) \subseteq U$.
Sufficiency: Let S be a closed set of *(X,τ)*, then $f^{-1}(Y\backslash f(S)) \subseteq X\backslash S$ and $X\backslash S$ is open. From the assumption, there exists a λ_g^α-open set V of *(Y,σ)* such that $[Y\backslash f(S)] \subseteq V$ and $f^{-1}(V) \subseteq [X\backslash S] \Rightarrow S \subseteq X\backslash f^{-1}(V)$. Hence $Y\backslash V \subseteq f(S) \subseteq f(X\backslash f^{-1}(V)) \subseteq Y\backslash V$, which implies $f(S) = Y\backslash V$. Since $Y\backslash V$ is λ_g^α-closed, $f(S)$ is λ_g^α-closed and thus f is a λ_g^α-closed map.

Theorem 4.26. *If a map f:(X,τ) →(Y,σ) is λ_g^α-closed map then λ_g^αcl(f(A))⊆ f (cl(A)) for every subset A of (X,τ).*

Proof. Let f be λ_g^α-closed map and $A \subseteq X$. As $cl(A)$ is closed in *(X,τ)*, $f(cl(A))$ is λ_g^α-closed in *(Y,σ)*. Since $f(cl(A))$ is λ_g^α-closed by Proposition 3.5, $\lambda_g^\alpha cl\,(f(cl(A))) = f(cl(A))$. From the fact that $f(A) \subseteq f(cl(A))$, we have λ_g^α $cl\,(f(A)) \subseteq \lambda_g^\alpha\ cl\,(f(cl(A))) = f(cl(A))$. Hence $\lambda_g^\alpha cl(f(A)) \subseteq f(cl(A))$.

Remark 4.27. *Converse of the Theorem 4.26 need not be true as observed from the following example.*

Example 4.28. Let X=Y={a,b,c,d}, $\tau = \{\phi, \{a,b,c\}, X\}$ and $\sigma = \{\phi, \{a,b\}, Y\}$. Let f:$(X,\tau)$ →(Y,σ) be the identity map. Then for every subset A⊆X we have λ_g^αcl(f(A))⊆f(cl(A)) but f is not a λ_g^α-closed map, since for the closed set {d} in (X,τ), f({d})={d} is not a λ_g^α-closed set in (Y,σ).

Theorem 4.29. *If f:(X,τ) →(Y,σ) is a λ_g^α-closed map and A is a closed subset of (X,τ) then the restriction $f|_A$:(A,$\tau|_A$) →(Y,σ) is a λ_g^α-closed map.*
Proof. Let $B \subseteq A$ be a closed set in $(A, \tau|_A)$, then $B = A \cap V$ for some closed set V of (X,τ). As A is closed in (X,τ), B is also closed in (X,τ). Since f is a λ_g^α-closed map, $f(B) = (f|_A)(B)$ is a λ_g^α-closed set in *(Y,σ)*. Hence $f|_A$ is a λ_g^α-closed map.

Theorem 4.30. *Let G be an α-open and λ_g^α-closed set of (Y,σ). If a bijective map $f : (X,\tau) \to (Y,\sigma)$ is λ-closed and A=$f^{-1}(G)$ then the restriction $f|_A : (A,\tau|_A) \to (Y,\sigma)$ is λ_g^α-closed.*
Proof. Let $V \subseteq A$ be a closed set in $(A,\tau|_A)$, then $V = A \cap H$ for some closed set H of (X,τ). Since H is closed in (X,τ), H is also λ-closed in *(X,τ)*. As f is a λ-closed map, $f(H)$ is λ-closed in (Y,σ). As G is α-open and $f(H)$ is λ-closed, by Theorem 2.8, $f(H) \cap G$ is λ_g^α-closed. Using the fact, $f|_A(V) = f(V) = f(A \cap H) = f(f^{-1}(G) \cap H)$=$G \cap f(H)$ is λ_g^α-closed. Hence $f|_A : (A,\tau|_A) \to (Y,\sigma)$ is λ_g^α-closed map.

5. λ_g^α-Open Maps and its Properties

Definition 5.1. *A map f:(X,τ)→(Y,σ) is called a λ_g^α-open map if the image of each open set in (X,τ) is λ_g^α-open in (Y,σ), i.e. if f(V) is λ_g^α-open in (Y,σ) for every open set V in (X,τ).*

Example 5.2. Let $X = Y = \{a,b,c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a,b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, Y\}$. Let f:$(X,\tau)$ →(Y,σ) be a map defined by f(a)=b, f(b)=c and f(c)=a. Then f is a λ_g^α-open map as the image of every open set in (X,τ) is λ_g^α-open in (Y,σ).

Proposition 5.3. *Every open map is a λ_g^α-open map.*
Proof. Similar to proof of Proposition 4.3.

Proposition 5.4. *Every λ-open map is a λ_g^α-open map.*
Proof. Similar to proof of Proposition 4.5.

Definition 5.5. *A map f:(X,τ) → (Y,σ) is called a gΛ-open map if the image of each open set in (X,τ) is gΛ-open in (Y,σ), i.e. if f(V) is gΛ-open in (Y,σ) for* *every open set V in (X,τ).*

Proposition 5.6. *Every λ_g^α-open map is a $g\Lambda$-open map.*
Proof. Similar to proof of Proposition 4.8.

Theorem 5.7. *A bijection f:(X,τ)→(Y,σ) is a λ_g^α-closed map if and only if f is a λ_g^α-open map.*
Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a λ_g^α-closed map and A be an open set in (X, τ). Then A^c is closed in (X, τ). As f is a bijection map, $f(A^c) = (f(A))^c$, which is a λ_g^α-closed set in *(Y,σ)*. Hence $f(A)$ is a λ_g^α-open set in (Y, σ). Thus f is a λ_g^α-open map.

Conversely, let $f : (X, \tau) \to (Y, \sigma)$ be a λ_g^α-open map and A be a closed set in (X, τ). Then A^c is open in (X,τ). As f is a bijection map, $f(A^c) = (f(A))^c$, which is a λ_g^α-open set in (Y,σ). Hence $f(A)$ is a λ_g^α-closed set in (Y, σ). Thus f is a λ_g^α-closed map.

Theorem 5.8. *A map f:$(X, \tau) \to (Y, \sigma)$ is a λ_g^α-open, then for every subset A of (X,τ), f(int(A))⊆λ_g^αint(f(A)).*
Proof. Let f be a λ_g^α-open map and A be any subset of (X, τ) such that $A \subseteq X$. Since $int(A)$ is open in (X, τ), $f(int(A))$ is λ_g^α-open in (Y, σ). Since $f(int(A))$ is λ_g^α-open by Proposition 3.19, $\lambda_g^\alpha int(f(int(A))) = f(int(A))$. From the fact that $f(int(A)) \subseteq f(A)$, we have $f(int(A)) = \lambda_g^\alpha int(f(int(A))) \subseteq \lambda_g^\alpha int(f(A))$. Hence $f(int(A)) \subseteq \lambda_g^\alpha int(f(A))$.

6. Composition of λ_g^α-Closed Maps and λ_g^α-Open Maps

Proposition 6.1. *If a map f:(X,τ)→(Y,σ) is a g-closed (resp. α-closed, αg-closed, g^*-closed) map, (Y,σ) is a $T_{1/2}$-space (resp. α-space, ${}_\alpha T_b$-space, $T^*_{1/2}$-space) and g: (Y,σ)→(Z, η) is a λ_g^α-closed map then g∘f:(X,τ)→(Z,η) is a λ_g^α-closed map.*
Proof. Let V be a closed set in *(X,τ)*. Since f is a g-closed (resp. α-closed, αg-closed, g*-closed) map, $f(V)$ is a g-closed (resp. α-closed, αg-closed, g*-closed) set in (Y, σ). As (Y, σ) is a $T_{1/2}$-space (resp.α-space, ${}_\alpha T_b$-space, $T^*_{1/2}$-space) by Remark 2.6, we have $f(V)$ is closed in (Y, σ).

Since $g : (Y, \sigma) \to (Z, \eta)$ is a closed map, $g(f(V)) = (g \circ f)(V)$ is a closed set in (Z, η). As every closed set is a λ_g^α-closed set, $(g \circ f)(V)$ is a λ_g^α-closed set in (Z, η). Thus $g \circ f$ is a λ_g^α-closed map.

Proposition 6.2. *If a map f:(X,τ)→(Y,σ) is g-closed (resp. α-closed, αg-closed, g^*-closed) map, (Y,σ) is a $T_{1/2}$-space (resp. α-space, ${}_\alpha T_b$-space, $T^*_{1/2}$-space) and g:(Y,σ)→(Z,η) is a λ_g^α-closed map then g∘f:(X,τ)→(Z,η) is a λ_g^α-closed map.*
Proof. Similar to the proof of Proposition 6.1.

Proposition 6.3. *Composition of closed (resp. open) maps is a* λ_g^α*-closed (resp.* λ_g^α*-open) map.*

Proof. Follows from the fact that every closed (resp. open) set is a λ_g^α-closed (resp. λ_g^α-open) set.

Remark 6.4. *Composition of* λ_g^α*-closed (resp.* λ_g^α*-open) maps is not a* λ_g^α*-closed (resp.* λ_g^α*-open) map as observed from the following example.*

Example 6.5. Let X=Y=Z={a,b,c}, $\tau = \{\phi, \{a\}, X\}, \sigma = \{\phi, \{a,b\}, Y\}$ and $\eta = \{\phi, \{a\}, \{a,b\}, Z\}$. Let f:$(X,\tau) \to (Y,\sigma)$ be a map defined by f(a)=b, f(b)=a and f(c)=c and g:(Y,σ)$\to$(Z,η) be the identity map. Then the maps f and g are both λ_g^α-closed, but their composition g$\circ$f:(X,τ)$\to$(Z,η) is not a λ_g^α-closed map, since for the closed set {b,c} in (X,τ), (g$\circ$f)($\{b,c\}$)={a,c} is not a λ_g^α-closed set in (Z,η).

Also the maps f and g are both λ_g^α-open, but their composition g$\circ$f:(X,τ)$\to$(Z,η) is not a λ_g^α-open map, since for an open set {a} in (X,τ), (g$\circ$f)($\{a\}$)={b} is not a λ_g^α-open set in (Z,η).

Remark 6.6. *If f:(X,τ)$\to$(Y,σ) is a* λ_g^α*-closed map and g:(Y,σ)$\to$(Z,η) is a closed map then their composition g$\circ$f:(X,τ)$\to$(Z,η) need not be a* λ_g^α*-closed map as observed from the following example.*

Example 6.7. Consider (X,τ), (Y,σ), (Z,η) and the maps f, g as in Example 6.5. Here f is a λ_g^α-closed map and g is a closed map, but their composition g$\circ$f:(X,τ)$\to$(Z,η) is not a λ_g^α-closed map since for a closed set ($\{b,c\}$) in (X,τ), (g$\circ$f)($\{b,c\}$)={a,c} is not a λ_g^α-closed set in (Z,η).

Proposition 6.8. *If f:(X,τ)$\to$(Y,σ) is a closed map and g:(Y,σ)$\to$(Z,η) is a* λ_g^α*-closed map then their composition g$\circ$f:(X,τ)$\to$(Z,η) is a* λ_g^α*-closed map.*

Proof. Let V be a closed set in (X,τ). As f is a closed map, $f(V)$ is closed in (Y,σ). Since $g : (Y,\sigma) \to (Z,\eta)$ is a λ_g^α-closed map, $g(f(V)) = (g \circ f)(V)$ is a λ_g^α-closed set in (Z,η). Therefore $g \circ f$ is a λ_g^α-closed map.

Proposition 6.9. *If f:(X,τ)$\to$(Y,σ) is a closed map and g:(Y,σ)$\to$(Z,η) is a λ-closed map then their composition g$\circ$ f:(X,τ)$\to$(Z,η) is a* λ_g^α*-closed map.*

Proof. Let f be a closed map and V be a closed set in (X,τ). Then $f(V)$ is a closed set in (Y,σ). As every closed set is λ-closed, $f(v)$ is a λ-closed set in (Y,σ). Since g is λ-closed, the $(g \circ f)(V) = g(f(V))$ is a λ closed set in (Z,η). From Lemma 2.7, $g(f(V))$ is a λ_g^α-closed set in (Z,η). Hence $g \circ f$ is a λ_g^α-closed map.

7. Conclusion

Initially, in this article we have defined and studied the properties related to λ_g^α-closure and λ_g^α-interior. In addition, we have given all the standard properties and theorems related to λ_g^α-closed map and λ_g^α-open map utilizing the definitions of λ_g^α-closed sets and λ_g^α-open sets in topological spaces. By the definitions of the proposed λ_g^α-closed maps and λ_g^α-open maps, the research can be extended to λ_g^α-homeomorphisms, special maps of λ_g^α-continuity, λ_g^α-quotient maps in topological spaces and their corresponding properties might have some interesting real time applications in the near future.

The proposed definitions can be applied and extended to various topologies like ideal topology, in which the topological ideals are connected with general topology for finding many application-oriented properties and also these definitions can be used as a tool in digital topology, in which the image arrays are studied by using the topological properties.

References

[1] Arockiarani I., Studies on Generalizations of Generalized Closed Sets and Maps in Topological Spaces, Ph.D Thesis, Bharathiar University, Coimbatore (1997).

[2] Caldas, M., Jafari, S. and Navalagi, T., More on λ-closed sets in topological spaces, Revista Colombina de Mathematics, 41(2) (2007), 355-369.

[3] Caldas, M., Jafari, S. and Noiri, T., On λ-generalised closed sets in topological spaces, Acta Math. Hungar, 118(4) (2008), 337-343.

[4] Devi, R. Balachandran, K. and Maki, H., Generalized α-closed maps and α-generalized closed maps, Indian J. Pure. Appl. Math., 29 (1998), 37-49.

[5] Francisco G Arenas, Julian Dontchev and Maxmillian Ganster, On λ-sets and the dual of generalized continuity, Question answers GEN. Topology, 15 (1997), 3-13.

[6] Gnanambal Y., On Generalized Pre-regular Closed Sets in Topological Spaces, Indian J. Pure Appl. Math., 28 (1997), 351-360.

[7] Levine, N., Generalized closed sets in Topology, Rend, Cir, Math. Palermo, 19 (1970), 89-96.

[8] Long P. E. and Herington L. L., Basic Properties of Regular Closed Functions, Rend. Cir. Mat. Palermo, 27 (1978), 20-28.

[9] Maheshwari, S. N. and Thakur, S. S., On α-irresolute maps, Tamkang J. Math., 11 (1980), 209-214.

[10] Maki, H., Generalized λ-sets and the associated closure operator, Special Issue in Commemoration of Prof. Kazuasada Ikeda's Retirement, (1986), 139-146.

[11] Maki, H., Devi, R. and Balachandran, K., Generalized α-closed sets in topology, Bull. Fukuoka Univ, Ed., Part III., 42 (1993), 13-21.

[12] Maki, H., Devi, R. and Balachandran, K., Associated topologies of generalized α-closed sets and (α-generalized closed sets, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 15 (1994), 51-63.

[13] Malghan, S. R., Generalized closed maps, J. Karnataka Univ. Sci., 27 (1982), 82-88.

[14] Mashhour, A. S., Hasanein, I. A. and EI-Deeb, S. N., On α-continuous and α-open mappings, Acta. Math. Hungarica, 41 (1983), 213-218.

[15] Nagaveni N., Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph.D., Thesis, Bharathiar University, Coimbatore (1999).

[16] Njastad, O., On some classes of nearly open sets, Pacific J. Math., 15 (1965), 961-970.

[17] Ramya, N. and Parvathi, A., A study on $\psi\widehat{g}$-closed sets in topological, bitopological and biminimal structure spaces, Ph.D. Thesis, Avinashilingam University, Coimbatore, India (2013).

[18] Saraf Ratnesh K. and Caldas M., Between closed maps and g-closed maps, Bulletin of the Greek Mathematical Society, 53 (2007), 135-146.

[19] Subhalakshmi, S. and Balamani, N., On λ_g^α-Closed and λ_g^α-Open Sets in Topological Spaces, Malaya Journal of Matematik, 8 (4) (2020), 2248-2252.

[20] Vadivel A. and Vairamanickam K., $rg\alpha$-closed and $rg\alpha$-open maps in Topological spaces, Int. Journal of Math. Analysis, 4 (10) (2010), 453-468.

[21] Veera Kumar, M. K. R. S., Between closed sets and g-closed sets, Mem. Fac. Sci. Koch. Univ. Ser. A, Math., 21 (2000), 1-19.

[22] Wadei Al-Omari and Abu Salem M., $\Im_g^*$- closed Sets via Ideal Topological Spaces, Missouri J. Math. Sci. 31 (2) (2019), 174-191.

[23] Wadei Al-Omari and Noiri, T., A$\mathcal{G}_{\Im^*}$-sets , B$\mathcal{G}_{\Im^*}$-sets and $\delta\beta_{\Im}$ in Ideal Topological Spaces, International Journal of Advances in Mathematics, 2018 (4) (2018), 25-33.