South East Asian J. of Mathematics and Mathematical Sciences
Vol. 17, Proceedings (2021), pp. 149-158

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

INFLUENCE OF PRESSURE, THICKNESS, AND DIAMETER ON RELIABILITY OF THIN CYLINDERS

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(**Received:** *Aug. 08, 2021* **Accepted:** *Oct. 01, 2021* **Published:** *Nov. 30, 2021)*

Special Issue
Proceedings of International Virtual Conference on
"Mathematical Modelling, Analysis and Computing IC- MMAC- 2021"

Abstract: Thin cylinders play an important role in chemical industry and engineering applications. In this paper, reliability analysis has been carried out for both ends closed and made with aluminum alloy of 2024-T6 thin cylinder by using MATLAB software. Reliability is obtained on the basis of probabilistic approach by considering material strength, pressure, thickness and inner diameter of cylinder as random variables. Stress and strength follow normal distribution. Resultant stress of the cylinder is taken in terms of circumferential stress, longitudinal stress by neglecting radial stress.

Keywords and Phrases: Thin cylinder- Reliability- Stress- Strength- Normal distribution- Pressure- Thickness- Inner diameter of cylinder- Mean- Variance.

2020 Mathematics Subject Classification: 62N05.

1. Notation and Introduction

R	Reliability
S	Stress
C	Strength
p	Pressure
d	Inner diameter of cylinder
t	Thickness of cylinder
s_1	Circumferential stress
s_2	Longitudinal stress
s_3	Radial stress
μ_C	Mean Strength of cylinder
μ_S	Mean of stress
μ_p	Mean pressure of cylinder
μ_d	Mean inner diameter of cylinder
μ_t	Mean thickness of cylinder
σ_C	Standard deviation of strength
σ_p	Standard deviation of pressure
σ_d	Standard deviation of inner diameter of cylinder
σ_t	Standard deviation of thickness of cylinder

Safety and economy play crucial role in the design of a structure. The quality, durability and applicability of the structure majorly decided by its reliability, in spite of this, finding the reliability is mandatory. Reliability is the probability of a design, should satisfy certain needs under the prescribed environment for certain period. In the designing of a structure, the aim of the designer is to design economical and safe structure. So, for a design, reliability analysis is essential.

Thin-walled pressure vessels are used in various fields such as petrochemical industry, transporting, storing of liquids, gases. Size of the cylinder plays an important role during transportation. For this purpose, it is necessary to find the reliability as a function of mean stress and strength.

In the reliability analysis of the pressure vessel, the pressure, material strength and stress are random variables, which follow normal distribution. Considering, $Y = C - S$, the element is safe if $Y > 0$ and is called steady state function.

Lot of research has been done on pressure vessel. N. K. Gupta and Venkatesh [2] studied about how the diameter and wall thickness influenced cylindrical tubes on their axial collapse. Abubakar Idris and Pius Edache [3] explained the reliability of simply supported beams using first order reliability method. Xinxu Li and Chenghong [5] obtained reliability of design of thin -walled pressure vessel.

Xinchen Wei and Chenghong [9] derived reliability formula for cylinder-elliptical head structure. Zheng Yulong et al. [10] applied Hasofer -Lind Method for finding reliability of thin -walled Cylindrical Shell.

2. Methodology

Thickness is very important in the design for the safety of the cylinder. Thin cylinders are cylinders in which the thickness is 20th part of diameter or even less [2]. The stresses act on thin cylinders are circumferential stress, longitudinal stress and radial stress. The radial stress may be negligible. According to the elastic mechanics, the stresses of cylinder can be derived as follows

$$s_1 = \frac{pd}{2t}, s_2 = \frac{pd}{4t}, s_3 = 0 \tag{1}$$

Where s_1 is circumferential stress, s_2 is longitudinal stress and these are major stresses of the cylinder. Radial stress s_3 is negligible.
The resultant stress [5]

$$S = \sqrt{s_1^2 - s_1 s_2 + s_3^2} = \frac{\sqrt{3}pd}{4t}$$

Let the probability density function for normally distributed strength C is given by

$$f_C(c) = \frac{1}{\sigma_C\sqrt{2\pi}} \exp\left(\frac{-1}{2}(\frac{c-\mu_C}{\sigma_C})^2\right), -\infty < c < \infty, \sigma_C > 0$$

Let the probability density function for normally distributed stress S is given by

$$f_S(s) = \frac{1}{\sigma_S\sqrt{2\pi}} \exp\left(\frac{-1}{2}(\frac{s-\mu_S}{\sigma_S})^2\right), -\infty < s < \infty, \sigma_S > 0$$

The reliability,

$$R = \int_0^\infty \frac{1}{\sigma_Y\sqrt{2\pi}} \exp\left(\frac{-1}{2}\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^2\right) dY \tag{2}$$

Since $Y = C - S$ and $\mu_Y = \mu_C - \mu_S$ and standard deviation of Y is $\sigma_Y^2 = \sqrt{\sigma_C^2 + \sigma_S^2}$
If $\frac{Y-\mu_Y}{\sigma_Y}$ is taken as z then $R = \int_{\frac{-(\mu_C-\mu_S)}{\sqrt{\sigma_C^2+\sigma_S^2}}}^\infty \frac{1}{\sqrt{2\pi}} \exp(\frac{-1}{2}z^2)dz$

Here z is called as standard normal variable. Then the lower limit of the integral of equation (2) is taken by

$$Z = \frac{-(\mu_C - \mu_S)}{\sqrt{\sigma_C^2 + \sigma_S^2}} \tag{3}$$

Using Taylor's formula approximation for stress S then,

$$Z = \frac{-(\mu_C - \mu_S)}{\sqrt{\sigma_C^2 + \sigma_S^2}} = -\frac{\mu_C - \frac{\sqrt{3}\mu_p\mu_d}{4\mu_t}}{\sqrt{\sigma_C^2 + (\frac{\sqrt{3}\mu_d}{4\mu_t})^2\sigma_p^2 + (\frac{\sqrt{3}\mu_p}{4\mu_t})^2\sigma_d^2 + (\frac{\sqrt{3}\mu_d\mu_p}{4\mu_t^2})^2\sigma_t^2}} \tag{4}$$

From the equations (2) and (3) Reliability,

$$R = 1 - \Phi\left(\frac{-(\mu_C - \mu_S)}{\sqrt{\sigma_C^2 + \sigma_S^2}}\right) \tag{5}$$

where Φ is standard normal cumulative distributive function.

3. Calculation of Reliability

Considering both ends are closed aluminum 2024-T6 thin cylinder with mean pressure 9 MPa, mean thickness 1.52 mm, mean diameter 142 mm, and their standard deviations 0.54 MPa, 0.09 mm, 8.52 mm respectively. Mean strength is 370 MPa and standard deviation of strength is 22.2 MPa.
For the specified reliability of cylinder 0.5517 and Z is -0.13.

3.1. Mean Thickness vs Reliability

Considering the equation (4), it is observed that if thickness increases from 1.52 mm to 2.22mm then reliability increases from 0.5517 to 0.9999. The reason for this is, with the increase of thickness, the stress is lowered at a given pressure. Therefore, reliability increases with the increase of thickness.The variation of reliability with thickness is shown below.

Table 1: Mean Thickness vs Reliability

For μ_C =370 MPa,μ_d =142 mm,μ_p=9 MPa For σ_C =22.2 MPa,, σ_p =0.54 MPa For σ_d =8.52 mm, σ_t =0.09 mm		
μ_t	-Z	R
1.52	0.13	0.5517
1.72	1.13	0.8906
1.92	2.29	0.9889
2.02	2.79	0.9973
2.22	3.73	0.9999

3.2. Mean Pressure vs Reliability

For μ_C =370 MPa,μ_d =142 mm, $\mu_t = 1.52$mm, σ_C =22.2 MPa, $\sigma_p = 0.54$ MPa, $\sigma_d = 8.52$ mm and σ_t =0.09 mm the reliability was obtained for different values of mean pressure of the cylinder, and observed that if μ_p increases from 6 MPa to 9 MPa then reliability decreases from 0.9996 to 0.5517. By the increase of pressure, the stress will be developed in the cylinder and the increment in the stress causes low reliability as shown below.

Table 2: Mean Pressure vs Reliability

μ_p	-Z	R
6	3.41	0.9996
6.5	2.80	0.9974
7	2.21	0.9864
7.5	1.65	0.9505
8	1.12	0.8686
8.5	0.61	0.7291
9	0.13	0.5517

3.3. Mean Diameter vs Reliability

If mean diameter of the design increases from 86 mm to 140 mm, then reliability decreases from 0.9999 to 0.5987. With the increase of mean diameter, the size of the cylinder increases, then the reliability decreases with increase of mean diameter. The variation of reliability with mean diameter is shown below.

Table 3: Mean Diameter vs Reliability

For μ_C =370 MPa,μ_t =1.52 mm, $\mu_p = 9MPa$ For σ_C =22.2 MPa,σ_d =8.52 mm For σ_t =0.09 mm, σ_p =0.54 MPa		
μ_d	-Z	R
86	4.12	0.9999
94	3.46	0.9997
100	2.99	0.9986
106	2.54	0.9944
112	2.10	0.9821
118	1.67	0.9525
124	1.26	0.8961
130	0.87	0.8078
136	0.49	0.6879
140	0.25	0.5987

3.4. Mean Strength vs Reliability

In this case, with the increment of mean strength, the deformation is lowered then the reliability of the cylinder enhances. The reliability increases from 0.6405 to 0.9922 with the mean strength varies from 380 MPa to 470 MPa.

Table 4: Mean Strength vs Reliability

For μ_C =370 MPa,μ_t =1.52 mm, $\mu_p = 9MPa$ For σ_C =22.2 MPa,σ_d =8.52 mm For σ_t =0.09 mm, σ_p =0.54 MPa		
μ_C	-Z	R
380	0.36	0.6405
390	0.59	0.7224
410	1.05	0.8531
430	1.50	0.9331
450	1.96	0.9750
470	2.42	0.9922

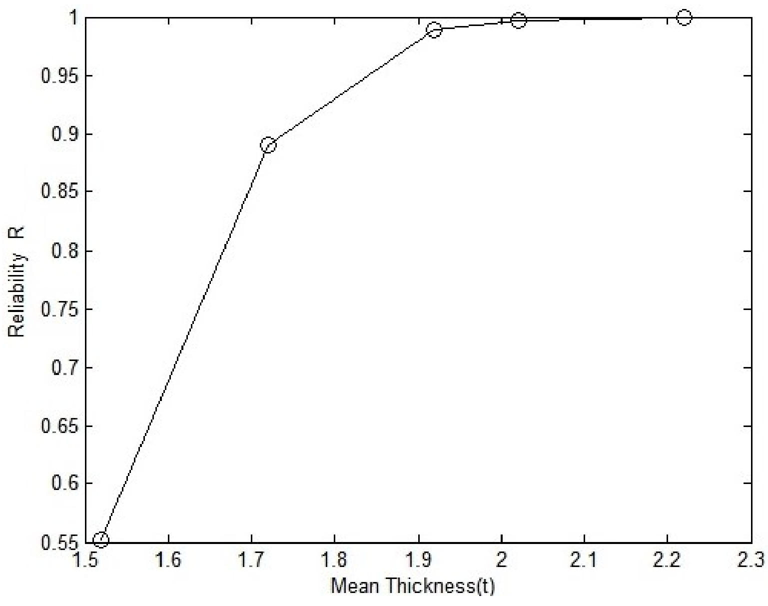

Figure 1: Variation of reliability as a function of mean thickness

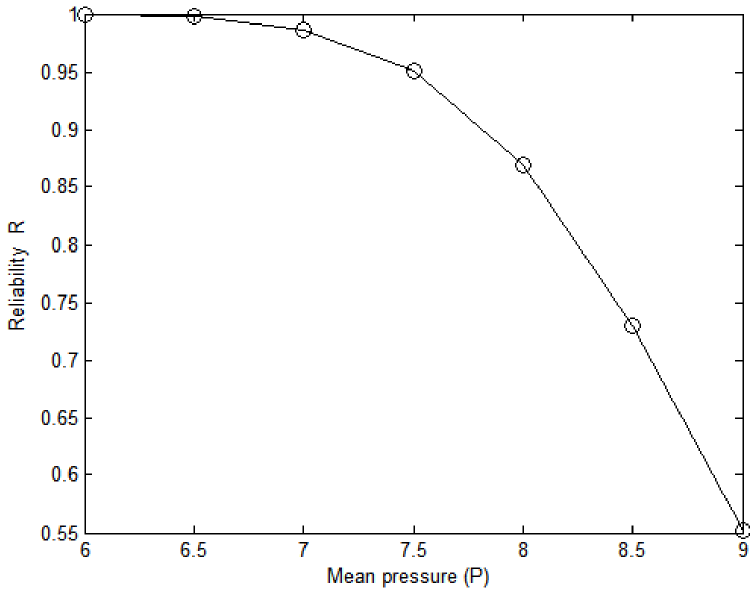

Figure 2: Variation of reliability as a function of mean pressure

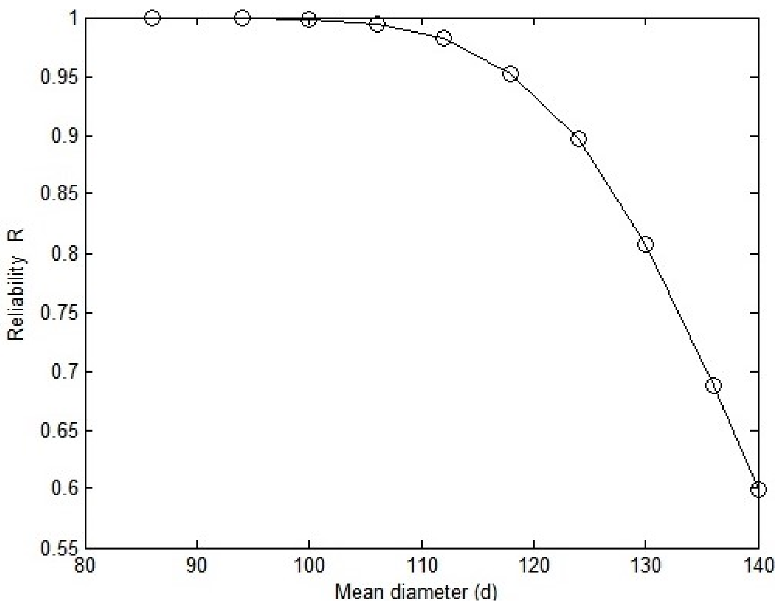

Figure 3: Variation of reliability as a function of mean diameter

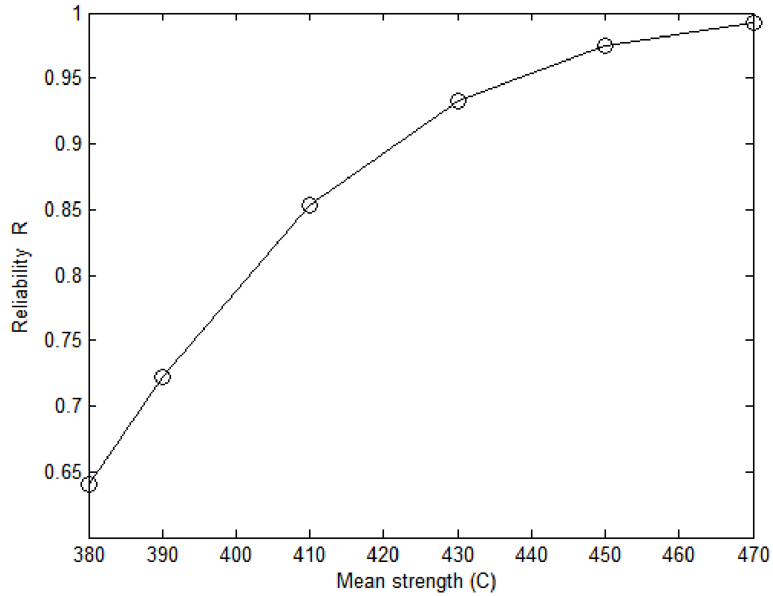

Figure 4: Variation of reliability as a function of mean strength

4. Conclusion

The reliability analysis of both ends closed thin cylindrical pressure vessel under internal pressure has been made using MATLAB. Thickness, diameter, pressure and mean strength of the cylinder will influence on reliability of the pressure vessel. The analysis shows that mean thickness of the cylinder increases from 1.52 mm to 1.72 mm then reliability increases from 0.5517 to 0.8906 drastically. If there is an increment in mean pressure of the cylinder from 6 MPa to 9 MPa then there is decrement in reliability from0.9999 to 0.5987. Change in mean diameter of the cylinder from 86 mm to 140 mm causes reliability from 0.9996 to 0.5517. If there is an increase of mean strength of the cylinder from 380 MPa to 470 MPa then reliability increases from 0.6405 to 0.9922.

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