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MULTI-CRITERIA DECISION MAKING USING COMPLEX CUBIC PYTHAGOREAN FUZZY SET

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Abstract: In this article, we introduce the notion of a complex cubic Pythagorean fuzzy set (CCPyFS). We discuss some of its properties. Also, we present the algebraic and aggregation operators on CCPyFS. Finally, we analyze two case studies using aggregation operation to select the best cotton variety from vendors.

Keywords and Phrases: Complex Pythagorean fuzzy set, complex interval-valued Pythagorean fuzzy set, complex cubic Pythagorean fuzzy set.

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1. Introduction

Zadeh [13] introduced the concept of a fuzzy set (FS). Also, he discussed the concept of interval-valued FS (IVFS) [14]. The concept of intuitionistic FS (IFS), a generalization of FS was introduced by Attanassov [2]. Later, Atanassov and Gargov [3] presented the notion of interval-valued intuitionistic FS (IVIFS) to deal with uncertainty in a broader perspective than FS. Yager [12] introduced the concept of Pythagorean FS with a condition that the square sum of its membership value and non-membership value is less than or equal to one. The concept of interval-valued PFS, a generalization of PFS and IVIFS was presented by Peng and Yang [9]. The notion of complex FS (CFS), a tool for providing membership value in terms of complex numbers was introduced by Ramot et al. [10]. Greenfield et al. [7] extended the notion of CFS to interval-valued CFS (IVIFS). Alkouri and Salleh [1] introduced the notion of complex IFS (CIFS) by adding the degree of non-membership and discussed some of its properties. Garg and Dimple [8] coined the definition of complex IVIFS (CIVIFS) to deal with time-periodic problems. Ullah et al. [11] presented the concept of complex PFS (CPFS) and its properties. Chinnadurai et al. [6] discussed the notion of a complex cubic set (CSS), complex cubic IFS (CCIFS) [4] and complex interval-valued Pythagorean FS (CIVPyFS)[5].

In this manuscript, we present the notion of complex cubic Pythagorean FS (CCPFS), a combination of complex interval-valued Pythagorean FS (CIVPyFS) and complex Pythagorean FS (CPyFS).

2. Preliminaries

In this section, we present the basic concepts of CFS, CPFS and CIVPFS. Through out the discussion U represents universal set.

Definition 2.1. [10] *A CFS* $\mathcal{A}_t$ *represented as* $\mathcal{A}_t = \{(x, P_{\mathcal{A}_t}(x))|x \in U\}$ *where* $P_{\mathcal{A}_t}(x) : U \to \{\grave{a} : \grave{a} \in C : |\grave{a}| \leq 1\}$ *is a membership function which assigns a grade of membership. The degree of membership value* $P_{\mathcal{A}_t}(x)$ *is deceive unit circle in the complex plane and given as* $\gamma_t(x).e^{i\theta_{\gamma_t}(x)}$ *where* $i = \sqrt{-1}, \gamma_t(x) \in [0,1]$ *and* $\theta_{\gamma_t}(x) \in [0, 2\pi]$.

Definition 2.2. [11] *A CPFS* $\mathcal{F}_t$ *represented as* $\mathcal{F}_t = \{(x, P_{\mathcal{F}_t}(x), Q_{\mathcal{F}_t}(x))|x \in U\}$, *where* $P_{\mathcal{F}_t} : U \to \{\grave{z}_1 : \grave{z}_1 \in C : |\grave{z}_1| \leq 1\}$, $Q_{\mathcal{F}_t} : U \to \{\grave{z}_2 : \grave{z}_2 \in C : |\grave{z}_2| \leq 1\}$ *provided that* $0 \leq |\grave{z}_1|^2 + |\grave{z}_2|^2 \leq 1$ *or* $P_{\mathcal{F}_t}(x) = \gamma_t(x).e^{i2\pi\theta_{\gamma_t}(x)}$ *and* $Q_{\mathcal{F}_t}(x) = \kappa_t(x).e^{i2\pi\theta_{\kappa_t}(x)}$, *satisfying the condition* $0 \leq \gamma_t^2(x) + \kappa_t^2(x) \leq 1$ *and* $0 \leq \theta^2_{\gamma_t}(x) + \theta^2_{\kappa_t}(x) \leq 1$. *The degree of hesitancy function* $H_t = \eta_t(x).e^{i2\pi\theta_{\eta_t}(x)}$, *such that* $\eta_t(x) = \sqrt{1 - \gamma_t^2(x) - \kappa_t^2(x)}$ *and* $\theta_{\eta_t}(x) = \sqrt{1 - \theta^2_{\gamma_t}(x) - \theta^2_{\kappa_t}(x)}$. *Then* $\mathcal{F}_t = (\gamma_t.e^{i2\pi\theta_{\gamma_t}}, \kappa_t.e^{i2\pi\theta_{\kappa_t}})$ *is called CPyFN.*

Definition 2.3. [10] *A CIVPyFS represented as* $\mathcal{F}_t = \left\{x, \left[\underline{P}_{\mathcal{F}_t}(x), \overline{P}_{\mathcal{F}_t}(x)\right]\left[\underline{Q}_{\mathcal{F}_t}(x), \overline{Q}_{\mathcal{F}_t}(x)\right]/x \in U\right\}$ *where* $\left[\underline{P}_{\mathcal{F}_t}(x), \overline{P}_{\mathcal{F}_t}(x)\right] : U \to \{\underline{z}_1, \overline{z}_1 : \underline{z}_1, \overline{z}_1 \in \mathcal{F}_t : |\underline{z}_1|, |\overline{z}_1| \leq 1\}$ *and* $\left[\underline{Q}_{\mathcal{F}_t}(x), \overline{Q}_{\mathcal{F}_t}(x)\right] : U \to \{\underline{z}_2, \overline{z}_2 : \underline{z}_2, \overline{z}_2 \in \mathcal{F}_t : |\underline{z}_2|, |\overline{z}_2| \leq 1\}$. *Have* $\underline{P}_{\mathcal{F}_t}(x) = \underline{z}_1 = \underline{\gamma}_t(x).e^{i2\pi\underline{\theta}_{\gamma_t}(x)}$, $\overline{P}_{\mathcal{F}_t}(x) = \overline{z}_1 = \overline{\gamma_t}(x).e^{i2\pi\overline{\theta}_{\gamma_t}(x)}$ *and* $\underline{Q}_{\mathcal{F}_t}(x) = \underline{z}_2 = \underline{\kappa_t}(x).e^{i2\pi\underline{\theta}_{\kappa_t}(x)}$, $\overline{Q}_{\mathcal{F}_t}(x) = \overline{z}_2 = \overline{\kappa_t}(x).e^{i2\pi\overline{\theta}_{\kappa_t}(x)}$, *satisfying the* *condition* $0 \leq (\overline{\gamma_t}(x))^2 + (\overline{\kappa_t}(x))^2 \leq 1$ *and* $0 \leq (\overline{\theta}_{\gamma_t}(x))^2 + (\overline{\theta}_{\kappa_t}(x)^2 \leq 1$. *The hesitancy function is given by* $H_t = \left[\underline{\vartheta_t}(x), \overline{\vartheta_t}(x)\right].e^{i2\pi[\underline{\theta}_{\vartheta_t}(x),\overline{\theta}_{\vartheta_t}(x)]}$, *where* $\underline{\vartheta_t}(x) = \sqrt{1-(\overline{\gamma_t}(x))^2-(\overline{\kappa_t}(x))^2}$, $\overline{\vartheta_t}(x) = \sqrt{1-(\underline{\gamma_t}(x))^2-(\underline{\kappa_t}(x))^2}$ *and* $\underline{\theta}_{\vartheta_t}(x) = \sqrt{1-(\overline{\theta}_{\gamma_t}(x))^2-(\overline{\theta}_{\kappa_t}(x))^2}, \overline{\theta}_{\vartheta_t}(x) = \sqrt{1-(\underline{\theta}_{\gamma_t}(x))^2+(\underline{\theta}_{\kappa_t}(x))^2}$. *Therefore, mathematically CIVPyFS* $\mathcal{F}_t$ *defined on* U *can be represented as* $\mathcal{F}_t = \left\{x, \left[\underline{\gamma_t}(x), \overline{\gamma_t}(x)\right].e^{i2\pi[\underline{\theta}_{\gamma_t}(x),\overline{\theta}_{\gamma_t}(x)]}, \left[\underline{\kappa_t}(x), \overline{\kappa_t}(x)\right].e^{i2\pi[\underline{\theta}_{\kappa_t}(x),\overline{\theta}_{\kappa_t}(x)]}/x \in U\right\}$. *The amplitude terms* $\left[\underline{\gamma_t}(x), \overline{\gamma_t}(x), \underline{\kappa_t}(x), \overline{\kappa_t}(x)\right] \subset [0,1]$ *and the real valued phase terms lie within the interval* $\left[\underline{\theta}_{\gamma_t}(x), \overline{\theta}_{\gamma_t}(x), \underline{\theta}_{\kappa_t}(x), \overline{\theta}_{\kappa_t}(x)\right] \subset [0,1]$ *subject to the condition,* $(\overline{\gamma_t}(x))^2 + (\overline{\kappa_t}(x))^2 \leq 1, (\overline{\theta}_{\gamma_t}(x))^2 + (\overline{\theta}_{\kappa_t}(x))^2 \leq 1$. *Furthermore,* $\mathcal{F}_t = \left\langle \left[\underline{\gamma_t}, \overline{\gamma_t}\right].e^{i2\pi[\underline{\theta}_{\gamma_t},\overline{\theta}_{\gamma_t}]}, \left[\underline{\kappa_t}, \overline{\kappa_t}\right].e^{i2\pi[\underline{\theta}_{\kappa_t},\overline{\theta}_{\kappa_t}]}\right\rangle$ *is called CIVPyFN.*

3. Complex cubic Pythagorean fuzzy set (CCPyFS)

In this section, we define a new concept CCPyFS and discuss some of its properties.

Definition 3.1. *Let* U *be the universal set. A Complex cubic Pythagorean fuzzy set(CCPyFS) represents as*

$$\mathtt{F}_t = \left\{\dot{z}, \left(\left\langle \left[\underline{P}_{\mathtt{F}_t}(\dot{z}), \overline{P}_{\mathtt{F}_t}(\dot{z})\right], \left[\underline{Q}_{\mathtt{F}_t}(\dot{z}), \overline{Q}_{\mathtt{F}_t}(\dot{z})\right]\right\rangle, \langle P_{\mathtt{F}_t}(\dot{z}), Q_{\mathtt{F}_t}(\dot{z})\rangle\right) : \dot{z} \in U\right\}.$$

Have the degrees of CIVPyFS are given by, $\underline{P}_{\mathtt{F}_t}(\dot{z}) = \underline{\upsilon}_t(\dot{z})e^{i2\pi\underline{\theta}_{\upsilon_t}(\dot{z})}$, $\overline{P}_{\mathtt{F}_t}(\dot{z}) = \overline{\upsilon}_t(\dot{z})e^{i2\pi\overline{\theta}_{\upsilon_t}(\dot{z})}$ *and* $\underline{Q}_{\mathtt{F}_t}(\dot{z}) = \underline{\varphi}_t(\dot{z})e^{i2\pi\underline{\theta}_{\varphi_t}(\dot{z})}$, $\overline{Q}_{\mathtt{F}_t}(\dot{z}) = \overline{\varphi}_t(\dot{z})e^{i2\pi\overline{\theta}_{\varphi_t}(\dot{z})}$. *These the degrees of CPyFS are* $P_{\mathtt{F}_t}(\dot{z}) = \upsilon_t(\dot{z})e^{i2\pi\theta_{\upsilon_t}(\dot{z})}$, $Q_{\mathtt{F}_t}(\dot{z}) = \varphi_t(\dot{z})e^{i2\pi\theta_{\varphi_t}(\dot{z})}$. *Therefore, mathematically CCPyFS* $\mathtt{F}_t$ *defined on* U *can be represent as*

$$\mathtt{F}_t = \left(\left\langle \left[\underline{\upsilon}_t(\dot{z}), \overline{\upsilon}_t(\dot{z})\right] e^{i2\pi[\underline{\theta}_{\upsilon_t}(\dot{z}),\overline{\theta}_{\upsilon_t}(\dot{z})]}, \left[\underline{\varphi}_t(\dot{z}), \overline{\varphi}_t(\dot{z})\right] e^{i2\pi[\underline{\theta}_{\varphi_t}(\dot{z}),\overline{\theta}_{\varphi_t}(\dot{z})]}\right\rangle, \left\langle \upsilon_t(\dot{z})e^{i2\pi\theta_{\upsilon_t}(\dot{z})}, \varphi_t(\dot{z})e^{i2\pi\theta_{\varphi_t}(\dot{z})}\right\rangle\right).$$

satisfying the condition $0 \leq (\overline{\upsilon}_t(\dot{z}))^2 + (\overline{\varphi}_t(\dot{z}))^2 \leq 1$, $0 \leq (\overline{\theta}_{\upsilon_t(\dot{z})})^2 + (\overline{\theta}_{\varphi_t(\dot{z})})^2 \leq 1$ *and*

$$0 \leq (\overline{\upsilon}_t(\dot{z}))^2 + (\overline{\varphi}_t(\dot{z}))^2 \leq 1,\ 0 \leq (\overline{\theta}_{\upsilon_t(\dot{z})})^2 + (\overline{\theta}_{\varphi_t(\dot{z})})^2 \leq 1. \quad (1)$$

Then the indeterminacy function can be represented as,

$\mathtt{H}_t = \left\langle \left[\underline{\vartheta}_t e^{i2\pi\underline{\theta}_{\vartheta_t}(\dot{z})}, \overline{\vartheta}_t e^{i2\pi\overline{\theta}_{\vartheta_t}(\dot{z})}\right], \vartheta_t e^{i2\pi\theta_{\vartheta_t}(\dot{z})}\right\rangle$ such that the amplitude terms

$\underline{\vartheta}_t(z) = \sqrt{1-(\overline{\upsilon}_t(\dot{z}))^2-(\overline{\varphi}_t(\dot{z}))^2}$, $\overline{\vartheta}_t(z) = \sqrt{1-(\underline{\upsilon}_t(\dot{z}))^2-(\underline{\varphi}_t(\dot{z}))^2}$ and

$\vartheta_t(z) = \sqrt{1-(\upsilon_t(\dot{z}))^2-(\varphi_t(\dot{z}))^2}$ and the phase terms are

$\underline{\theta}_{\vartheta_t}(\dot{z}) = \sqrt{1-(\overline{\theta}_{\upsilon_t}(\dot{z}))^2-\overline{\theta}_{\varphi_t}(\dot{z})}$, $\overline{\theta}_{\vartheta_t}(\dot{z}) = \sqrt{1-(\underline{\theta}_{\upsilon_t}(\dot{z}))^2-\underline{\theta}_{\varphi_t}(\dot{z})}$,

$\theta_{\vartheta_t}(\dot{z}) = \sqrt{1-(\theta_{\upsilon_t}(\dot{z}))^2-\theta_{\varphi_t}(\dot{z})}$. Furthermore

$\mathtt{F}_t = \left(\left\langle [\underline{\upsilon}_t, \overline{\upsilon}_t]\, e^{i2\pi[\underline{\theta}_{\upsilon_t}, \overline{\theta}_{\upsilon_t}]}, \left[\underline{\varphi}_t, \overline{\varphi}_t\right] e^{i2\pi[\underline{\theta}_{\varphi_t}, \overline{\theta}_{\varphi_t}]} \right\rangle, \left\langle \upsilon_t e^{i2\pi\theta_{\upsilon_t}}, \varphi_t(\dot{z}) e^{i2\pi\theta_{\varphi_t}} \right\rangle\right)$
is called CCPyFN.

Example 3.2. Let us consider an example in CIVIFS form
$\mathtt{F}_1 = \left(\left\langle [0.1, 0.2]\, e^{i2\pi[0.1,0.3]}, [0.3, 0.4]\, e^{i2\pi[0.2,0.4]} \right\rangle, \left\langle 0.4e^{i2\pi(0.2)}, 0.5e^{i2\pi(0.6)} \right\rangle\right)$. It is clear that $0 \leq 0.2+0.4 \leq 1$, $0 \leq 0.2+0.4 \leq 1$ and the fuzzy values are $0 \leq 0.4+0.5 \leq 1$, $0 \leq 0.2 + 0.6 \leq 1$ which satisfy condition (1). Another example is CIVIFS form
$\mathtt{F}_2 = \left(\left\langle [0.6, 0.7]\, e^{i2\pi[0.3,0.4]}, [0.5, 0.6]\, e^{i2\pi[0.5,0.7]} \right\rangle, \left\langle 0.4e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.5)} \right\rangle\right)$. Now $0.7+0.6 \neq 1$, $0.4+0.7 \neq 1$ and $0.4+0.8 \neq 1$, $0.7+0.5 \neq 1$. This set does not satisfy the given condition of CIVIFS. This proves that the given number is not CIVPyFS, but it is a CCPyFS an it satisfies the condition. However, it is evident that CCPyFS can satisfy the condition $0.7^2 + 0.6^2 \leq 1$, $0.4^2 + 0.7^2 \leq 1$ and $0.4^2 + 0.8^2 \leq 1$, $0.7^2 + 0.5^2 \leq 1$.

Definition 3.3. Let $\mathtt{F}_t = \left(\left\langle [\underline{\upsilon}_t, \overline{\upsilon}_t]\, e^{i2\pi[\underline{\theta}_{\upsilon_t}, \overline{\theta}_{\upsilon_t}]}, \left[\underline{\varphi}_t, \overline{\varphi}_t\right] e^{i2\pi[\underline{\theta}_{\varphi_t}, \overline{\theta}_{\varphi_t}]} \right\rangle, \left\langle \upsilon_t e^{i2\pi\theta_{\upsilon_t}}, \varphi_t e^{i2\pi\theta_{\varphi_t}} \right\rangle\right)$, $\mathtt{F}_1 = \left(\left\langle [\underline{\upsilon}_1, \overline{\upsilon}_1]\, e^{i2\pi[\underline{\theta}_{\upsilon_1}, \overline{\theta}_{\upsilon_1}]}, \left[\underline{\varphi}_1, \overline{\varphi}_1\right] e^{i2\pi[\underline{\theta}_{\varphi_1}, \overline{\theta}_{\varphi_1}]} \right\rangle, \left\langle \upsilon_1 e^{i2\pi\theta_{\upsilon_1}}, \varphi_1 e^{i2\pi\theta_{\varphi_1}} \right\rangle\right)$ and $\mathtt{F}_2 = \left(\left\langle [\underline{\upsilon}_2, \overline{\upsilon}_2]\, e^{i2\pi[\underline{\theta}_{\upsilon_2}, \overline{\theta}_{\upsilon_2}]}, \left[\underline{\varphi}_2, \overline{\varphi}_2\right] e^{i2\pi[\underline{\theta}_{\varphi_2}, \overline{\theta}_{\varphi_2}]} \right\rangle, \left\langle \upsilon_2 e^{i2\pi\theta_{\upsilon_2}}, \varphi_2 e^{i2\pi\theta_{\varphi_2}} \right\rangle\right)$ be CCPyFNs, then

$i)\mathtt{F}^c = \left(\left\langle \left[\underline{\varphi}_t, \overline{\varphi}_t\right] e^{i2\pi[\underline{\theta}_{\varphi_t}, \overline{\theta}_{\varphi_t}]}, [\underline{\upsilon}_t, \overline{\upsilon}_t]\, e^{i2\pi[\underline{\theta}_{\upsilon_t}, \overline{\theta}_{\upsilon_t}]} \right\rangle, \left\langle \varphi_t e^{i2\pi\theta_{\varphi_t}}, \upsilon_t e^{i2\pi\theta_{\upsilon_t}} \right\rangle\right)$

$$ii)\mathtt{F}_1 \wedge \mathtt{F}_2 = \left(\left\langle [min[\underline{\upsilon}_1 \underline{\upsilon}_2], min[\overline{\upsilon}_1, \overline{\upsilon}_2]]\, e^{i2\pi[min(\underline{\theta}_{\upsilon_1}, \underline{\theta}_{\upsilon_2}), min(\overline{\theta}_{\upsilon_1}, \overline{\theta}_{\upsilon_2})]}, \left[max[\underline{\varphi}_1 \underline{\varphi}_2], max[\overline{\varphi}_1, \overline{\varphi}_2]\right] e^{i2\pi[max(\underline{\theta}_{\varphi_1}, \underline{\theta}_{\varphi_2}), max(\overline{\theta}_{\varphi_1}, \overline{\theta}_{\varphi_2})]} \right\rangle, \left\langle min\,[\upsilon_1, \upsilon_2]\, e^{i2\pi[min(\theta_{\upsilon_1}, \theta_{\upsilon_2})]}, max\,[\varphi_1, \varphi_2]\, e^{i2\pi[max(\theta_{\varphi_1}, \theta_{\varphi_2})]} \right\rangle\right)$$

$$iii)\mathtt{F}_1 \vee \mathtt{F}_2 = \left(\left\langle [max[\underline{\upsilon}_1 \underline{\upsilon}_2], max[\overline{\upsilon}_1, \overline{\upsilon}_2]]\, e^{i2\pi[max(\underline{\theta}_{\upsilon_1}, \underline{\theta}_{\upsilon_2}), max(\overline{\theta}_{\upsilon_1}, \overline{\theta}_{\upsilon_2})]}, \left[min[\underline{\varphi}_1 \underline{\varphi}_2], min[\overline{\varphi}_1, \overline{\varphi}_2]\right] e^{i2\pi[min(\underline{\theta}_{\varphi_1}, \underline{\theta}_{\varphi_2}), min(\overline{\theta}_{\varphi_1}, \overline{\theta}_{\varphi_2})]} \right\rangle, \left\langle max\,[\upsilon_1, \upsilon_2]\, e^{i2\pi[max(\theta_{\upsilon_1}, \theta_{\upsilon_2})]}, min\,[\varphi_1, \varphi_2]\, e^{i2\pi[min(\theta_{\varphi_1}, \theta_{\varphi_2})]} \right\rangle\right)$$

$$iv)\mathtt{F}_1 \bigoplus \mathtt{F}_2 = \left(\left\langle \left[\sqrt{\underline{\upsilon}_1^2 + \underline{\upsilon}_2^2 - \underline{\upsilon}_1^2 \underline{\upsilon}_2^2}, \sqrt{\overline{\upsilon}_1^2 + \overline{\upsilon}_2^2 - \overline{\upsilon}_1^2 \overline{\upsilon}_2^2}\right] e^{i2\pi\left[\sqrt{\underline{\theta}_{\upsilon_1}^2 + \underline{\theta}_{\upsilon_2}^2 - \underline{\theta}_{\upsilon_1}^2 \underline{\theta}_{\upsilon_2}^2}, \sqrt{\overline{\theta}_{\upsilon_1}^2 + \overline{\theta}_{\upsilon_2}^2 - \overline{\theta}_{\upsilon_1}^2 \overline{\theta}_{\upsilon_2}^2}\right]}, \left[[\underline{\varphi}_1 \underline{\varphi}_2], [\overline{\varphi}_1 \overline{\varphi}_2]\right] e^{i2\pi[(\underline{\theta}_{\varphi_1} \underline{\theta}_{\varphi_2}), (\overline{\theta}_{\varphi_1} \overline{\theta}_{\varphi_2})]} \right\rangle, \left\langle \sqrt{\upsilon_1^2 + \upsilon_2^2 - \upsilon_1^2 \upsilon_2^2}\, e^{i2\pi\sqrt{\theta_{\upsilon_1}^2 + \theta_{\upsilon_2}^2 - \theta_{\upsilon_1}^2 \theta_{\upsilon_2}^2}}, [\varphi_1 \varphi_2] e^{i2\pi[\theta_{\varphi_1} \theta_{\varphi_2}]} \right\rangle\right)$$

$v)\mathtt{F}_1 \bigotimes \mathtt{F}_2 = \left(\left\langle [[\underline{\upsilon}_1 \underline{\upsilon}_2], [\overline{\upsilon}_1 \overline{\upsilon}_2]]\, e^{i2\pi[(\underline{\theta}_{\upsilon_1} \underline{\theta}_{\upsilon_2}), (\overline{\theta}_{\upsilon_1} \overline{\theta}_{\varphi_2})]}, \right.\right.$

$\left[\sqrt{\underline{\varphi}_1^2+\underline{\varphi}_2^2-\underline{\varphi}_1^2\underline{\varphi}_2^2}, \sqrt{\overline{\varphi}_1^2+\overline{\varphi}_2^2-\overline{\varphi}_1^2\overline{\varphi}_2^2}\right] e^{i2\pi\left[\sqrt{\underline{\theta}_{\varphi_1}^2+\underline{\theta}_{\varphi_2}^2-\underline{\theta}_{\varphi_1}^2\underline{\theta}_{\varphi_2}^2}, \sqrt{\overline{\theta}_{\varphi_1}^2+\overline{\theta}_{\varphi_2}^2-\overline{\theta}_{\varphi_1}^2\overline{\theta}_{\varphi_2}^2}\right]}\Big\rangle,$
$\Big\langle[\upsilon_1\upsilon_2]e^{i2\pi[\theta_{\upsilon_1}\theta_{\upsilon_2}]}, \sqrt{\varphi_1^2+\varphi_2^2-\varphi_1^2\varphi_2^2}e^{i2\pi\sqrt{\theta_{\varphi_1}^2+\theta_{\varphi_2}^2-\theta_{\varphi_1}^2\theta_{\varphi_2}^2}}, \Big\rangle\Big)$

$vi)\lambda.\mathtt{F} = \Big(\Big\langle \left[\sqrt{1-(1-\underline{\upsilon}^2)^\lambda}, \sqrt{1-(1-\overline{\upsilon}^2)^\lambda}\right] e^{i2\pi\left[\sqrt{1-(1-\underline{\theta}_\upsilon^2)^\lambda}, \sqrt{1-(1-\overline{\theta}_\upsilon^2)^\lambda}\right]},$
$\left[(\underline{\varphi})^\lambda, (\overline{\varphi})^\lambda\right] e^{i2\pi\left[(\underline{\theta}_\varphi)^\lambda, (\overline{\theta}_\varphi)^\lambda\right]}\Big\rangle, \Big\langle\sqrt{1-(1-\upsilon^2)^\lambda}e^{i2\pi\sqrt{1-(1-\theta_\upsilon^2)^\lambda}}, (\varphi)^\lambda e^{i2\pi(\theta_\varphi)^\lambda}\Big\rangle\Big),$
$\lambda > 0.$

$vii)\mathtt{F}^\lambda = \Big(\Big\langle \left[(\underline{\upsilon})^\lambda, (\overline{\upsilon})^\lambda\right] e^{i2\pi\left[(\underline{\theta}_\upsilon)^\lambda, (\overline{\theta}_\upsilon)^\lambda\right]}, \left[\sqrt{1-(1-\underline{\varphi}^2)^\lambda}, \sqrt{1-(1-\overline{\varphi}^2)^\lambda}\right]$
$e^{i2\pi\left[\sqrt{1-(1-\underline{\theta}_\varphi^2)^\lambda}, \sqrt{1-(1-\overline{\theta}_\varphi^2)^\lambda}\right]}\Big\rangle, \Big\langle(\upsilon)^\lambda e^{i2\pi(\theta_\upsilon)^\lambda}, \sqrt{1-(1-\varphi^2)^\lambda}e^{i2\pi\sqrt{1-(1-\theta_\varphi^2)^\lambda}}\Big\rangle\Big).$

Definition 3.4. *For any CCPyFN,*
$\mathtt{F} = \Big(\Big\langle [\underline{\upsilon}, \overline{\upsilon}]\, e^{i2\pi[\underline{\theta}_\upsilon, \overline{\theta}_\upsilon]}, [\underline{\varphi}, \overline{\varphi}]\, e^{i2\pi[\underline{\theta}_\varphi, \overline{\theta}_\varphi]}\Big\rangle, \Big\langle\upsilon e^{i2\pi\theta_\upsilon}, \varphi_t e^{i2\pi\theta_\varphi}\Big\rangle\Big)$ *we define the score function* (Q) *as*

$$Q(\mathtt{F}) = \frac{1}{6}\left[(\underline{\upsilon}^2+\overline{\upsilon}^2+\upsilon^2)-(\underline{\varphi}^2+\overline{\varphi}^2+\varphi^2)+\frac{1}{2\pi}\left[(\underline{\theta}_\upsilon^2+\overline{\theta}_\upsilon^2+\theta_\upsilon^2)-(\underline{\theta}_\varphi^2+\overline{\theta}_\varphi^2+\theta_\varphi^2)\right]\right].$$

It is clear that $Q(\mathcal{F}) \in [-1, 1]$ *, and the accuracy function* $\c{S}$ *of* $\mathtt{F}$ *is defined as*

$$\c{S}(\mathtt{F}) = \frac{1}{6}\left[(\underline{\upsilon}^2+\overline{\upsilon}^2+\upsilon^2)+(\underline{\varphi}^2+\overline{\varphi}^2+\varphi^2)+\frac{1}{2\pi}\left[(\underline{\theta}_\upsilon^2+\overline{\theta}_\upsilon^2+\theta_\upsilon^2)+(\underline{\theta}_\varphi^2+\overline{\theta}_\varphi^2+\theta_\varphi^2)\right]\right].$$

Based on these function, a comparison method for any two CCPyFNs $\mathtt{F}_1, \mathtt{F}_2$ *is defined as follows.*

Definition 3.5. *Let* $\mathtt{F}_1, \mathtt{F}_2$ *be two CCPyFNs corresponding to CCPyFs, then the compression between the CCPyFNs is done as follows : if* $Q(\mathtt{F}_1) < Q(\mathtt{F}_2)$ *then* $\mathtt{F}_1$ *is inferior to* $\mathtt{F}_2$ *and if* $Q(\mathtt{F}_1) = Q(\mathtt{F}_2)$ *then, if* $\c{S}(\mathtt{F}_1) < \c{S}(\mathtt{F}_2)$ *then* $\mathtt{F}_1$ *is inferior to* $\mathtt{F}_2$ *and if* $\c{S}(\mathtt{F}_1) = \c{S}(\mathtt{F}_2)$ *then* $\mathtt{F}_1$ *and* $\mathtt{F}_2$ *have the same information indicted by* $\mathtt{F}_1 \sim \mathtt{F}_2$.

Theorem 3.6. *All the operational results in Definition 3.2 are in CCPyFN forms.*

Proof. (i) Since
$\mathtt{F}_t = \Big(\Big\langle [\underline{\upsilon}_t, \overline{\upsilon}_t]\, e^{i2\pi[\underline{\theta}_{\upsilon_t}, \overline{\theta}_{\upsilon_t}]}, \left[\underline{\varphi}_t, \overline{\varphi}_t\right] e^{i2\pi[\underline{\theta}_{\varphi_t}, \overline{\theta}_{\varphi_t}]}\Big\rangle, \Big\langle\upsilon_t e^{i2\pi\theta_{\upsilon_t}}, \varphi_t e^{i2\pi\theta_{\varphi_t}}\Big\rangle\Big)$ is an CCPyFN, so it is satisfies the equation (1) and hence
$\mathtt{F}^c = \Big(\Big\langle \left[\underline{\varphi}_t, \overline{\varphi}_t\right] e^{i2\pi[\underline{\theta}_{\varphi_t}, \overline{\theta}_{\varphi_t}]}, [\underline{\upsilon}_t, \overline{\upsilon}_t]\, e^{i2\pi[\underline{\theta}_{\upsilon_t}, \overline{\theta}_{\upsilon_t}]}\Big\rangle, \Big\langle\varphi_t e^{i2\pi\theta_{\varphi_t}}, \upsilon_t e^{i2\pi\theta_{\upsilon_t}}\Big\rangle\Big)$ also satisfies this condition. Thus $\mathtt{F}^c$ is CCPyFN.

(ii) Since $\mathtt{F}_1$ and $\mathtt{F}_2$ are CCPyFNs, so $\mathtt{F}_1, \mathtt{F}_2$ satisfy the condition (1), i.e., $min[\overline{\upsilon_1}, \overline{\upsilon_2}]+max[\overline{\varphi_1}, \overline{\varphi_2}] \leq 1$, $min[\overline{\theta}_{\upsilon_1}, \overline{\theta}_{\upsilon_2}]+max[\overline{\theta}_{\varphi_1}, \overline{\theta}_{\varphi_2}] \leq 1$ and $min[\upsilon_1, \upsilon_2]+max[\varphi_1, \varphi_2] \leq 1$, $min[\theta_{\upsilon_1}, \theta_{\upsilon_2}]$
$+max[\theta_{\varphi_1}, \theta_{\varphi_2}] \leq 1$. Then $\mathtt{F}_1 \wedge \mathtt{F}_2$ satisfy the condition (1),i.e.,$\mathtt{F}_1 \wedge \mathtt{F}_2$ is an CCPyFN.

(iii) Similar to (ii), we can prove that $\mathtt{F}_1 \vee \mathtt{F}_2$ is an CCPyFN.

(iv) Since both $\mathtt{F}_1$ and $\mathtt{F}_2$ satisfy the condition (1), it follows that, $\sqrt{\underline{\upsilon}_1^2+\underline{\upsilon}_2^2-\underline{\upsilon}_1^2\underline{\upsilon}_2^2} = \sqrt{\underline{\upsilon}_1^2(1-\underline{\upsilon}_2^2)+\underline{\upsilon}_2^2} \geq \sqrt{\underline{\upsilon}_2^2} \geq \underline{\upsilon}_2 \geq 0$ and $\underline{\varphi}_1, \underline{\varphi}_2 \geq 0$ and the lower phase terms $\sqrt{\underline{\theta}_{\upsilon_1}^2+\underline{\theta}_{\upsilon_2}^2-\underline{\theta}_{\upsilon_1}^2\underline{\upsilon}_{\upsilon_2}^2} = \sqrt{\underline{\theta}_{\upsilon_1}^2(1-\underline{\theta}_{\upsilon_2}^2)+\underline{\theta}_{\upsilon_2}^2} \geq \sqrt{\underline{\theta}_{\upsilon_2}^2} \geq \underline{\theta}_{\upsilon_2} \geq 0$ and $\underline{\theta}_{\varphi_1}^2, \underline{\theta}_{\varphi_1}^2 \geq 0$. Also $\overline{\upsilon}_1^2+\overline{\upsilon}_2^2-\overline{\upsilon}_1^2\overline{\upsilon}_2^2+\overline{\varphi}_1^2\overline{\varphi}_2^2 \leq \overline{\upsilon}_1^2+\overline{\upsilon}_2^2-\overline{\upsilon}_1^2\overline{\upsilon}_2^2+(1-\overline{\upsilon}_1^2)(1-\overline{\upsilon}_2^2) = 1$ and the upper phase terms $\overline{\theta}_{\upsilon_1}^2+\overline{\theta}_{\upsilon_2}^2-\overline{\theta}_{\upsilon_1}^2\overline{\theta}_{\upsilon_2}^2+\overline{\theta}_{\varphi_1}^2\overline{\theta}_{\varphi_2}^2 \leq \overline{\theta}_{\upsilon_1}^2+\overline{\theta}_{\upsilon_2}^2-\overline{\theta}_{\upsilon_1}^2\overline{\theta}_{\upsilon_2}^2+(1-\overline{\theta}_{\upsilon_1}^2)(1-\overline{\theta}_{\upsilon_2}^2) = 1$. Then the fuzzy values are $\upsilon_1^2+\upsilon_2^2-\upsilon_1^2\upsilon_2^2+\varphi_1^2\varphi_2^2 \leq \upsilon_1^2+\upsilon_2^2-\upsilon_1^2\upsilon_2^2+(1-\upsilon_1^2)(1-\upsilon_2^2) = 1$ and the fuzzy upper phase terms $\theta_{\upsilon_1}^2+\theta_{\upsilon_2}^2-\theta_{\upsilon_1}^2\theta_{\upsilon_2}^2+\theta_{\varphi_1}^2\theta_{\varphi_2}^2 \leq \theta_{\upsilon_1}^2+\theta_{\upsilon_2}^2-\theta_{\upsilon_1}^2\theta_{\upsilon_2}^2+(1-\theta_{\upsilon_1}^2)(1-\theta_{\upsilon_2}^2) = 1$. Therefore, the value of $\mathtt{F}_1 \bigoplus \mathtt{F}_2$ satisfy the condition of equation (1) and hence it is a CCPyFN. In the similar way (v) can be proved.

(vi) Since $\sqrt{1-(1-\underline{\upsilon}^2)^\lambda} \geq 0, \sqrt{1-(1-\overline{\upsilon}^2)^\lambda} \geq 0, \sqrt{1-(1-\upsilon^2)^\lambda} \geq 0,$ $(\underline{\varphi})^\lambda, (\overline{\varphi})^\lambda, (\varphi)^\lambda \geq 0$ and the phase terms $\sqrt{1-(1-\underline{\theta}_\upsilon^2)^\lambda} \geq 0, \sqrt{1-(1-\overline{\theta}_\upsilon^2)^\lambda} \geq 0, \sqrt{1-(1-\theta_\upsilon^2)^\lambda} \geq 0, (\underline{\theta}_\varphi)^\lambda, (\overline{\theta}_\varphi)^\lambda, (\theta_\varphi)^\lambda \geq 0$ and $1-(1-\overline{\upsilon}^2)^\lambda+(\overline{\varphi}^2)^\lambda \leq 1-(1-\overline{\upsilon}^2)^\lambda+(1-\overline{\upsilon}^2)^\lambda = 1$ and the phase terms $1-(1-\overline{\theta}_\upsilon^2)^\lambda+(\overline{\theta}_\varphi^2)^\lambda \leq 1-(1-\overline{\theta}_\upsilon^2)^\lambda+(1-\overline{\theta}_\upsilon^2)^\lambda = 1$. Always satisfy the fuzzy values $1-(1-\upsilon^2)^\lambda+(\varphi^2)^\lambda \leq 1-(1-\upsilon^2)^\lambda+(1-\upsilon^2)^\lambda = 1$ and the phase terms $1-(1-\theta_\upsilon^2)^\lambda+(\theta_\varphi^2)^\lambda \leq 1-(1-\theta_\upsilon^2)^\lambda+(1-\theta_\upsilon^2)^\lambda = 1$.Thus, the value of $\lambda.\mathtt{F}$ is a CCPyFN.

(vii) Can be proved similarly.

Theorem 3.7. *Let $\lambda, \lambda_1, \lambda_2 \geq 0$. Then*

$i)\mathtt{F}_1 \bigoplus \mathtt{F}_2 = \mathtt{F}_2 \bigoplus \mathtt{F}_1$ $\quad ii)\mathtt{F}_1 \bigotimes \mathtt{F}_2 = \mathtt{F}_2 \bigotimes \mathtt{F}_1$

$iii)\lambda.(\mathtt{F}_1 \bigoplus \mathtt{F}_2) = \lambda.\mathtt{F}_1 \bigoplus \lambda.\mathtt{F}_2$ $\quad vi)(\mathtt{F}_1 \bigotimes \mathtt{F}_2)^\lambda = \mathtt{F}_1^\lambda \bigotimes \mathtt{F}_2^\lambda$

$v)\lambda_1.\mathtt{F} \bigoplus \lambda_2.\mathtt{F} = (\lambda_1+\lambda_2).\mathtt{F}$ $\quad vi)\mathtt{F}^{\lambda_1} \bigotimes \mathtt{F}^{\lambda_2} = \mathtt{F}^{\lambda_1+\lambda_2}$

Proof. Straight forward.

4. Complex cubic Pythagorean fuzzy aggregation operators

In this section, we discuss some operators for aggregating CCPyFNs.

Definition 4.1. *Let* $\mathtt{F}_t = \left(\left\langle [\underline{\upsilon}_t, \overline{\upsilon}_t]\, e^{i2\pi[\underline{\theta}_{\upsilon_t}, \overline{\theta}_{\upsilon_t}]}, \left[\underline{\varphi}_t, \overline{\varphi}_t\right] e^{i2\pi[\underline{\theta}_{\varphi_t}, \overline{\theta}_{\varphi_t}]}\right\rangle, \left\langle \upsilon_t e^{i2\pi\theta_{\upsilon_t}}, \varphi_t e^{i2\pi\theta_{\varphi_t}}\right\rangle\right), (t=1,2,...n)$ *a collection of CCPyFNs and let $CCPyFWA_\chi : \aleph^n \to \aleph$.*

If $CCPyFWA_\chi(\mathtt{F}_1, \mathtt{F}_2, ...\mathtt{F}_n) = \chi_1.\mathtt{F}_1 \bigoplus \chi_2.\mathtt{F}_1 \bigoplus, ... \bigoplus \chi_n.\mathtt{F}_n$ (2)

where $\aleph$ is the collection of all CCPyFNs, χ_t is the weight of $\mathtt{F}_t (t=1,2,...n)$, $\chi_t \in [0,1]$ and $\sum_{t=1}^{n} \chi_t = 1$ then the function CCPyFWA is called an CCPyF weighted averaging operator. In particular, if $\chi_t = \frac{1}{n}$ for all j then $CCPyFWA_\chi$ operator reduces to CIVPyF averaging operator(CCPyFA) $CCPyFA(\mathtt{F}_1, \mathtt{F}_2, ...\mathtt{F}_n) = \frac{1}{n}(\mathtt{F}_1 \bigoplus \mathtt{F}_2 \bigoplus ...... \bigoplus \mathtt{F}_n)$

Theorem 4.2. *Let* $\mathtt{F}_t = \left(\left\langle [\underline{\upsilon}_t, \overline{\upsilon}_t]\, e^{i2\pi[\underline{\theta}_{\upsilon_t}, \overline{\theta}_{\upsilon_t}]}, \left[\underline{\varphi}_t, \overline{\varphi}_t\right] e^{i2\pi[\underline{\theta}_{\varphi_t}, \overline{\theta}_{\varphi_t}]}\right\rangle, \left\langle \upsilon_t e^{i2\pi\theta_{\upsilon_t}}, \varphi_t e^{i2\pi\theta_{\varphi_t}}\right\rangle\right), (t = 1, 2, ...n)$ *be a collection of CCPyFNs then the aggregated value by using equation (2) is also an CCPyFNs denoted as*

$$CCPyFWA_\chi(\mathtt{F}_1, \mathtt{F}_2, ...\mathtt{F}_n) = \left(\left\langle \left[\sqrt{1-\prod_{t=1}^{n}(1-\underline{\upsilon}_t^2)^{\chi_t}}, \sqrt{1-\prod_{t=1}^{n}(1-\overline{\upsilon}_t^2)^{\chi_t}}\right] e^{i2\pi\left[\sqrt{1-\prod_{t=1}^{n}(1-\underline{\theta}_{\upsilon_t}^2)^{\chi_t}}, \sqrt{1-\prod_{t=1}^{n}(1-\overline{\theta}_{\upsilon_t}^2)^{\chi_t}}\right]}, \left[\prod_{t=1}^{n}\underline{\varphi}_t^{\chi_t}, \prod_{t=1}^{n}\overline{\varphi}_t^{\chi_t}\right] e^{i2\pi\left[\prod_{t=1}^{n}\underline{\theta}_{\varphi_t}^{\chi_t}, \prod_{t=1}^{n}\overline{\theta}_{\varphi_t}^{\chi_t}\right]}\right\rangle, \left\langle \sqrt{1-\prod_{t=1}^{n}(1-\upsilon_t^2)^{\chi_t}}\, e^{i2\pi\sqrt{1-\prod_{t=1}^{n}(1-\theta_{\upsilon_t}^2)^{\chi_t}}}, \prod_{t=1}^{n}\varphi_t^{\chi_t} e^{i2\pi\prod_{t=1}^{n}\theta_{\varphi_t}^{\chi_t}}\right\rangle\right) \quad (3)$$

where χ_t *is the weight of* $\mathtt{F}_t(t = 1, 2, ...n)$, $\chi_t \in [0, 1]$ *and* $\sum_{t=1}^{n}\chi_t = 1$

Proof. We prove equation (3), when $n = 2$. $\text{CCPyFWA}_\chi \mathtt{F}_1, \mathtt{F}_2) = \chi_1\mathtt{F}_1 \bigoplus \chi_1\mathtt{F}_1$. According to Theorem (3.1), we can see that both $\chi_1\mathtt{F}_1$ and $\chi_1\mathtt{F}_1$ are CCPyFNs, and the value of $\chi_1\mathtt{F}_1 \bigoplus \chi_1\mathtt{F}_1$ is an CCPyFN. By definition (3.2) (vi), we have

$$\chi_1.\mathtt{F}_1 = \left(\left\langle \left[\sqrt{1-(1-\underline{\upsilon_1}^2)^{\chi_1}}, \sqrt{1-(1-\overline{\upsilon_1}^2)^{\chi_1}}\right] e^{i2\pi\left[\sqrt{1-(1-\underline{\theta}_{\upsilon_1}^2)^{\chi_1}}, \sqrt{1-(1-\overline{\theta}_{\upsilon_1}^2)^{\chi_1}}\right]}, \left[(\underline{\varphi_1})^{\chi_1}, (\overline{\varphi_1})^{\chi_1}\right] e^{i2\pi(\underline{\theta}_{\varphi_1})^{\chi_1}, (\overline{\theta}_{\varphi_1})^{\chi_1}}\right\rangle, \left\langle \sqrt{1-(1-\upsilon_1^2)^{\chi_1}}\, e^{i2\pi\sqrt{1-(1-\theta_{\upsilon_1}^2)^{\chi_1}}}, (\varphi_1)^{\chi_1} e^{i2\pi(\theta_{\varphi_1})^{\chi_1}}\right\rangle\right),$$

$$\chi_2.\mathtt{F}_2 = \left(\left\langle \left[\sqrt{1-(1-\underline{\upsilon_2}^2)^{\chi_2}}, \sqrt{1-(1-\overline{\upsilon_2}^2)^{\chi_2}}\right] e^{i2\pi\left[\sqrt{1-(1-\underline{\theta}_{\upsilon_2}^2)^{\chi_2}}, \sqrt{1-(1-\overline{\theta}_{\upsilon_2}^2)^{\chi_2}}\right]}, \left[(\underline{\varphi_2})^{\chi_2}, (\overline{\varphi_2})^{\chi_2}\right] e^{i2\pi(\underline{\theta}_{\varphi_2})^{\chi_2}, (\overline{\theta}_{\varphi_2})^{\chi_2}}\right\rangle, \left\langle \sqrt{1-(1-\upsilon_2^2)^{\chi_2}}\, e^{i2\pi\sqrt{1-(1-\theta_{\upsilon_2}^2)^{\chi_2}}}, (\varphi_2)^{\chi_2} e^{i2\pi(\theta_{\varphi_2})^{\chi_2}}\right\rangle\right).$$

Then $\text{CCPyFWA}_\chi(\mathtt{F}_1, \mathtt{F}_2) = \chi_1\mathtt{F}_1 \bigoplus \chi_1\mathtt{F}_2$.

$$= \left(\left\langle \left[\sqrt{1-(1-\underline{\upsilon}_1^2)^{\chi_1}\,(1-\underline{\upsilon}_2^2)^{\chi_2}}, \sqrt{1-(1-\overline{\upsilon}_1^2)^{\chi_1}\,(1-\overline{\upsilon}_2^2)^{\chi_2}}\right] e^{i2\pi\left[\sqrt{1-\left(1-\underline{\theta}_{\upsilon_1}^2\right)^{\chi_1}\left(1-\underline{\theta}_{\upsilon_2}^2\right)^{\chi_2}}, \sqrt{1-\left(1-\overline{\theta}_{\upsilon_1}^2\right)^{\chi_1}\left(1-\overline{\theta}_{\upsilon_2}^2\right)^{\chi_2}}\right]}, \left[\underline{\varphi}_1^{\chi_1}\underline{\varphi}_2^{\chi_2}, \overline{\varphi}_1^{\chi_1}\overline{\varphi}_2^{\chi_2}\right] e^{i2\pi\left[(\underline{\theta}_{\varphi_1}^{\chi_1}\underline{\theta}_{\varphi_2}^{\chi_2}), (\overline{\theta}_{\varphi_1}^{\chi_1}\overline{\theta}_{\varphi_2}^{\chi_2})\right]}\right\rangle, \left\langle \sqrt{1-(1-\upsilon_1^2)^{\chi_1}\,(1-\upsilon_2^2)^{\chi_2}}\, e^{i2\pi\sqrt{1-\left(1-\theta_{\upsilon_1}^2\right)^{\chi_1}\left(1-\theta_{\upsilon_2}^2\right)^{\chi_2}}}, \varphi_1^{\chi_1}\varphi_2^{\chi_2} e^{i2\pi(\theta_{\varphi_1}^{\chi_1}\theta_{\varphi_2}^{\chi_2})}\right\rangle\right)$$

Thus, result $n = 2$ is true.

Equation (3) holds, then the result is true for $n = r$, i.e.,

$$\text{CCPyFWA}_\chi(\mathtt{F}_1, \mathtt{F}_2, ...\mathtt{F}_r) = \left(\left\langle \left[\sqrt{1-\prod_{t=1}^{r}(1-\underline{\upsilon}_t^2)^{\chi_t}}, \sqrt{1-\prod_{t=1}^{r}(1-\overline{\upsilon}_t^2)^{\chi_t}}\right] e^{i2\pi\left[\sqrt{1-\prod_{t=1}^{r}(1-\underline{\theta}_{\upsilon_t}^2)^{\chi_t}}, \sqrt{1-\prod_{t=1}^{r}(1-\overline{\theta}_{\upsilon_t}^2)^{\chi_t}}\right]}, \left[\prod_{t=1}^{r}\underline{\varphi}_t^{\chi_t}, \prod_{t=1}^{r}\overline{\varphi}_t^{\chi_t}\right] e^{i2\pi\left[\prod_{t=1}^{r}\underline{\theta}_{\varphi_t}^{\chi_t}, \prod_{t=1}^{r}\overline{\theta}_{\varphi_t}^{\chi_t}\right]}\right\rangle, \left\langle \sqrt{1-\prod_{t=1}^{r}(1-\upsilon_t^2)^{\chi_t}}\, e^{i2\pi\sqrt{1-\prod_{t=1}^{r}(1-\theta_{\upsilon_t}^2)^{\chi_t}}}, \prod_{t=1}^{r}\varphi_t^{\chi_t} e^{i2\pi\prod_{t=1}^{r}\theta_{\varphi_t}^{\chi_t}}\right\rangle\right)$$

Then, when $n = r + 1$, by using Definition 3.2 (iv) and (vi) we get,

$$\mathrm{CCPyFWA}_\chi(\mathtt{F}_1,\mathtt{F}_2,......\mathtt{F}_{r+1})=\mathrm{CCPyFWA}\ _\chi(\mathtt{F}_1,\mathtt{F}_2,......\mathtt{F}_r)\bigoplus\chi_{r+1}.\mathtt{F}_{r+1}=$$

$$\Bigg(\Big\langle\left[\sqrt{1-\prod_{t=1}^{r}(1-\underline{\upsilon}_t^2)^{\chi_t}},\sqrt{1-\prod_{t=1}^{r}(1-\overline{\upsilon}_t^2)^{\chi_t}}\right]e^{i2\pi\left[\sqrt{1-\prod_{t=1}^{r}(1-\underline{\theta}_{\upsilon_t}^2)^{\chi_t}},\sqrt{1-\prod_{t=1}^{r}(1-\overline{\theta}_{\upsilon_t}^2)^{\chi_t}}\right]},$$

$$\left[\prod_{t=1}^{r}\underline{\varphi}_t^{\chi_t},\prod_{t=1}^{r}\overline{\varphi}_t^{\chi_t}\right]e^{i2\pi\left[\prod_{t=1}^{r}\underline{\theta}_{\varphi_t}^{\chi_t},\prod_{t=1}^{r}\overline{\theta}_{\varphi_t}^{\chi_t}\right]}\Big\rangle,\Big\langle\sqrt{1-\prod_{t=1}^{r}(1-\upsilon_t^2)^{\chi_t}}e^{i2\pi\sqrt{1-\prod_{t=1}^{r}(1-\theta_{\upsilon_t}^2)^{\chi_t}}},$$

$$\prod_{t=1}^{r}\varphi_t^{\chi_t}e^{i2\pi\prod_{t=1}^{r}\theta_{\varphi_t}^{\chi_t}}\Big\rangle\Bigg)\bigoplus\Bigg(\Big\langle\left[\sqrt{1-(1-\underline{\upsilon}_{r+1}^2)^{\chi_{r+1}}},\sqrt{1-(1-\overline{\upsilon}_{r+1}^2)^{\chi_{r+1}}}\right]$$

$$e^{i2\pi\left[\sqrt{1-(1-\underline{\theta}_{\upsilon_{r+1}}^2)^{\chi_{r+1}}},\sqrt{1-(1-\overline{\theta}_{\upsilon_{r+1}}^2)^{\chi_{r+1}}}\right]},\left[\underline{\varphi}_{r+1}^{\chi_{r+1}},\overline{\varphi}_{r+1}^{\chi_{r+1}}\right]e^{i2\pi\left[(\underline{\theta}_\varphi)_{r+1}^{\chi_{r+1}},(\overline{\theta}_\varphi)_{r+1}^{\chi_{r+1}}\right]}\Big\rangle,$$

$$\Big\langle\sqrt{1-(1-\upsilon_{r+1}^2)^{\chi_{r+1}}}.e^{i2\pi\sqrt{1-(1-\theta_{\upsilon_{r+1}}^2)^{\chi_{r+1}}}},\varphi_{r+1}^{\chi_{r+1}}e^{i2\pi(\theta_\varphi)_{r+1}^{\chi_{r+1}}}\Big\rangle\Bigg)$$

$$=\Bigg(\Big\langle\left[\sqrt{1-\prod_{t=1}^{r+1}(1-\underline{\upsilon}_t^2)^{\chi_t}},\sqrt{1-\prod_{t=1}^{r+1}(1-\overline{\upsilon}_t^2)^{\chi_t}}\right]e^{i2\pi\left[\sqrt{1-\prod_{t=1}^{r+1}(1-\underline{\theta}_{\upsilon_t}^2)^{\chi_t}},\sqrt{1-\prod_{t=1}^{r+1}(1-\overline{\theta}_{\upsilon_t}^2)^{\chi_t}}\right]},$$

$$\left[\prod_{t=1}^{r+1}\underline{\varphi}_t^{\chi_t},\prod_{t=1}^{r+1}\overline{\varphi}_t^{\chi_t}\right]e^{i2\pi\left[\prod_{t=1}^{r+1}\underline{\theta}_{\varphi_t}^{\chi_t},\prod_{t=1}^{r+1}\overline{\theta}_{\varphi_t}^{\chi_t}\right]}\Big\rangle,\Big\langle\sqrt{1-\prod_{t=1}^{r+1}(1-\upsilon_t^2)^{\chi_t}}.e^{i2\pi\sqrt{1-\prod_{t=1}^{r+1}(1-\theta_{\upsilon_t}^2)^{\chi_t}}},$$

$$\prod_{t=1}^{r+1}\varphi_t^{\chi_t}.e^{i2\pi\prod_{t=1}^{r+1}\theta_{\varphi_t}^{\chi_t}}\Big\rangle\Bigg)$$ i.e., when $n=r+1$, equation (3) also holds.

Next in need to show CCPyWA_χ is an CCPyFN. As $\mathtt{F}_t=\Big(\Big\langle[\underline{\upsilon}_t,\overline{\upsilon}_t]\,e^{i2\pi[\underline{\theta}_{\upsilon_t},\overline{\theta}_{\upsilon_t}]},\left[\underline{\varphi}_t,\overline{\varphi}_t\right]e^{i2\pi[\underline{\theta}_{\varphi_t},\overline{\theta}_{\varphi_t}]}\Big\rangle,\Big\langle\upsilon_te^{i2\pi\theta_{\upsilon_t}},\varphi_te^{i2\pi\theta_{\varphi_t}}\Big\rangle\Big)$ for all t is an CCPyFN, thus $0\le\underline{\upsilon}_t,\overline{\upsilon}_t,\underline{\varphi}_t,\overline{\varphi}_t,\upsilon_t,\varphi_t\le1$ then satisfy the condition $\overline{\upsilon}_t^2+\overline{\varphi}_t^2\le1$, $0\le\upsilon_t^2+\varphi_t^2\le1$ and the phase terms are $0\le\underline{\theta}_{\upsilon_t},\overline{\theta}_{\upsilon_t},\underline{\theta}_{\varphi_t},\overline{\theta}_{\varphi_t},\theta_{\upsilon_t},\theta_{\varphi_t}\le1$ and the condition$\overline{\theta}_{\upsilon_t}^2+\overline{\theta}_{\varphi_t}^2\le1$, $0\le\theta_{\upsilon_t}^2+\theta_{\varphi_t}^2\le1$. And hence $0\le\sqrt{1-\prod_{t=1}^{n}(1-\underline{\upsilon}_t^2)^{\chi_t}}\le1$, $0\le\prod_{t=1}^{n}\underline{\varphi}_t^{\chi_t}\le1$ and $0\le\sqrt{1-\prod_{t=1}^{r}(1-\underline{\theta}_{\upsilon_t}^2)^{\chi_t}}\le1, 0\le\prod_{t=1}^{r}\underline{\theta}_{\varphi_t}^{\chi_t}\le1.$ and $0\le\sqrt{1-\prod_{t=1}^{n}(1-\overline{\upsilon}_t^2)^{\chi_t}}\le1$ and $0\le\prod_{t=1}^{n}\overline{\varphi}_t^{\chi_t}\le1$ and the phase terms are, $0\le\sqrt{1-\prod_{t=1}^{r}(1-\overline{\theta}_{\upsilon_t}^2)^{\chi_t}}\le1,0\le\prod_{t=1}^{r}\overline{\theta}_{\varphi_t}^{\chi_t}\le1.$ Similarly, the fuzzy values are $0\le\sqrt{1-\prod_{t=1}^{n}(1-\upsilon_t^2)^{\chi_t}}\le1$ and $0\le\prod_{t=1}^{n}\varphi_t^{\chi_t}\le1$ then the fuzzy phase terms are, $0\le\sqrt{1-\prod_{t=1}^{r}(1-\theta_{\upsilon_t}^2)^{\chi_t}}\le1,0\le\prod_{t=1}^{r}\theta_{\varphi_t}^{\chi_t}\le1$ Again, $\left(\sqrt{1-\prod_{t=1}^{n}(1-\overline{\upsilon}_t^2)^{\chi_t}}\right)^2+\left(\prod_{t=1}^{n}\overline{\varphi}_t^{\chi_t}\right)^2=1-\prod_{t=1}^{n}(1-\overline{\upsilon}_t^2)^{\chi_t}+\left(\prod_{t=1}^{n}\overline{\varphi}_t^{2\chi_t}\right)\le1-\left(\prod_{t=1}^{n}\overline{\varphi}_t^{2\chi_t}\right)+\left(\prod_{t=1}^{n}\overline{\varphi}_t^{2\chi_t}\right)=1$ and the phase terms are,

$$\left(\sqrt{1-\prod_{t=1}^{n}(1-\overline{\theta}_{\upsilon_t}^2)^{\chi_t}}\right)^2 + \left(\prod_{t=1}^{n}\overline{\theta}_{\varphi_t}^{\chi_t}\right)^2 = 1-\prod_{t=1}^{n}(1-\overline{\theta}_{\upsilon_t}^2)^{\chi_t} + \prod_{t=1}^{n}\overline{\theta}_{\varphi_t}^{2\chi_t} \leq 1-\left(\prod_{t=1}^{n}\overline{\theta}_{\varphi_t}^{2\chi_t}\right) + \left(\prod_{t=1}^{n}\overline{\theta}_{\varphi_t}^{2\chi_t}\right) = 1.$$

Hence, also satisfy the fuzzy results $\left(\sqrt{1-\prod_{t=1}^{n}(1-\upsilon_t^2)^{\chi_t}}\right)^2 + \left(\prod_{t=1}^{n}\varphi_t^{\chi_t}\right)^2 = 1-\prod_{t=1}^{n}(1-\upsilon_t^2)^{\chi_t} + \left(\prod_{t=1}^{n}\varphi_t^{2\chi_t}\right) \leq 1-\left(\prod_{t=1}^{n}\varphi_t^{2\chi_t}\right) + \left(\prod_{t=1}^{n}\varphi_t^{2\chi_t}\right) = 1$ and the phase terms are, $\left(\sqrt{1-\prod_{t=1}^{n}(1-\theta_{\upsilon_t}^2)^{\chi_t}}\right)^2 + \left(\prod_{t=1}^{n}\theta_{\varphi_t}^{\chi_t}\right)^2 = 1-\prod_{t=1}^{n}(1-\theta_{\upsilon_t}^2)^{\chi_t} + \prod_{t=1}^{n}\theta_{\varphi_t}^{2\chi_t} \leq 1-\left(\prod_{t=1}^{n}\theta_{\varphi_t}^{2\chi_t}\right) + \left(\prod_{t=1}^{n}\theta_{\varphi_t}^{2\chi_t}\right) = 1$ Hence, CCPyWA_χ is a CCPyFN and therefore proof is completed.

Definition 4.3. *Let* $CCPyFWG_\chi : \aleph^n \to \aleph$. *If*

$$CCPyFWG_\chi(\mathtt{F}_1, \mathtt{F}_2, ...\mathtt{F}_n) = \mathtt{F}_1^{\chi_1} \bigotimes \mathtt{F}_1^{\chi_2} \bigotimes, ... \bigotimes \mathtt{F}_n^{\chi_n} \tag{4}$$

then the function $CCPyFWG_\chi$ *is called an CCPyF weighted geometric operator. In particular, if* $\chi = (\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})^T$ *then the* $CCPyFWG_\chi$ *operator reduces to an complex cubic Pythagorean fuzzy geometric operator* $CCPyFWG_\chi(\mathtt{F}_1, \mathtt{F}_2, ...\mathtt{F}_n) = (\mathtt{F}_1 \bigotimes \mathtt{F}_1 \bigotimes, ... \bigotimes \mathtt{F}_n)^{\frac{1}{n}}$

Theorem 4.4. *The aggregated value by using equation (4) is also an CCPyFN, and*

$$CCPyFWG_\chi(\mathtt{F}_1, \mathtt{F}_2, ...\mathtt{F}_n) = \left(\left\langle \left[\prod_{t=1}^{n}\underline{\upsilon}_t^{\chi_t}, \prod_{t=1}^{n}\overline{\upsilon}_t^{\chi_t}\right] e^{i2\pi\left[\prod_{t=1}^{n}\underline{\theta}_{\upsilon_t}^{\chi_t}, \prod_{t=1}^{n}\overline{\theta}_{\upsilon_t}^{\chi_t}\right]},\right.\right.$$
$$\left.\left[\sqrt{1-\prod_{t=1}^{n}(1-\underline{\varphi}_t^2)^{\chi_t}}, \sqrt{1-\prod_{t=1}^{n}(1-\overline{\varphi}_t^2)^{\chi_t}}\right] e^{i2\pi\left[\sqrt{1-\prod_{t=1}^{n}(1-\underline{\theta}_{\varphi_t}^2)^{\chi_t}}, \sqrt{1-\prod_{t=1}^{n}(1-\overline{\theta}_{\varphi_t}^2)^{\chi_t}}\right]}\right\rangle,$$
$$\left.\left\langle \prod_{t=1}^{n}\upsilon_t^{\chi_t} e^{i2\pi\prod_{t=1}^{n}\theta_{\upsilon_t}^{\chi_t}}, \sqrt{1-\prod_{t=1}^{n}(1-\varphi_t^2)^{\chi_t}}\, e^{i2\pi\sqrt{1-\prod_{t=1}^{n}(1-\theta_{\varphi_t}^2)^{\chi_t}}} \right\rangle\right) \tag{5}$$

Proof: The proof of this theorem is similar to Theorem (4.1).

5. Multi-criteria Decision Making(MCDM) using Complex Cubic Pythagorean Fuzzy Numbers(CCPyFNs)

Let $\mathtt{F} = \{\mathtt{F}_1, \mathtt{F}_2....\mathtt{F}_n\}$ be alternatives and $\mathtt{C} = \{\mathtt{C}_1, \mathtt{C}_2......\mathtt{C}_n\}$ be the parameters. Each alternative can be represented in CCPyFNs form as $\mathtt{F}_t = \left(\left\langle [\underline{\upsilon}_t, \overline{\upsilon}_t]\, e^{i2\pi[\underline{\theta}_{\upsilon_t}, \overline{\theta}_{\upsilon_t}]}, \left[\underline{\varphi}_t, \overline{\varphi}_t\right] e^{i2\pi[\underline{\theta}_{\varphi_t}, \overline{\theta}_{\varphi_t}]}\right\rangle, \left\langle \upsilon_t e^{i2\pi\theta_{\upsilon_t}}, \varphi_t e^{i2\pi\theta_{\varphi_t}}\right\rangle\right)$, where $\left\langle [\underline{\upsilon}_t, \overline{\upsilon}_t]\, e^{i2\pi[\underline{\theta}_{\upsilon_t}, \overline{\theta}_{\upsilon_t}]}, \upsilon_t e^{i2\pi\theta_{\upsilon_t}}\right\rangle$ represents the membership value given by the decision maker(DM) of the alternative $\mathtt{F}_t$ corresponding to the parameter $\mathtt{C}_j$. Similarly, $\left\langle \left[\underline{\varphi}_t, \overline{\varphi}_t\right] e^{i2\pi[\underline{\theta}_{\varphi_t}, \overline{\theta}_{\varphi_t}]}, \varphi_t e^{i2\pi\theta_{\varphi_t}}\right\rangle$ represents the non-membership value given by the DM for the alternative $\mathtt{F}_t$ corresponding to the parameter $\mathtt{C}_j$. The values given by the DM are tabulated. So based on this, we apply the aggregated operators CCPyFWA and CCPyFWG. Finally, rank the alternatives by using score value definition and select the best alternatives. Let us summarize the steps for computing MCDM as below.

Step 1. Let $\mathtt{F}_t = \{\mathtt{F}_1, \mathtt{F}_2....\mathtt{F}_n\}$ be alternatives and let $\mathtt{C}_j = \{\mathtt{C}_1, \mathtt{C}_2......\mathtt{C}_n\}$ be parameters.

Step 2. By using Definitions 4.1 and 4.2 of CCPyFWA or CCPyFWG, we aggregate the values.

Step 3. Compute the score value by using Definition 3.3.

Step 4. Rank all the alternatives in descending order and select the best alternative.

Case studies. In this section, we discuses two case studies. In case-I, we study the selection of best cotton by using CCPyFWA operator and in case-II, we use CCPyFWG operator.

Case I. The aim of this study is to select the best quality cotton to make fabrics. Let the expert check the quality of cotton received from various cotton vendors $\mathtt{F}_t, (t = 1, 2, 3, 4)$. Let the parameters be e_1 = staple length, e_2 = grade, e_3 = micronaire and e_4 = strength. Let us assume the weight of e_1, e_2, e_3 and e_4 be (0.2,0.4,0.3,0.1)respectively.

The Method. To find out the best quality cotton, we use the following method.

Step 1. The decision maker provides the information for the alternative as below

	e_1
$\mathtt{F}_1$	$\left(\left\langle [0.8, 0.9]\, e^{i2\pi[0.6,0.7]}, [0.2, 0.3]\, e^{i2\pi[0.1,0.3]} \right\rangle, \left\langle 0.4e^{i2\pi(0.5)}, 0.3e^{i2\pi(0.31)} \right\rangle\right)$
$\mathtt{F}_2$	$\left(\left\langle [0.5, 0.6]\, e^{i2\pi[0.6,0.7]}, [0.3, 0.4]\, e^{i2\pi[0.5,0.6]} \right\rangle, \left\langle 0.7e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.3)} \right\rangle\right)$
$\mathtt{F}_3$	$\left(\left\langle [0.7, 0.9]\, e^{i2\pi[0.5,0.6]}, [0.1, 0.3]\, e^{i2\pi[0.4,0.5]} \right\rangle, \left\langle 0.3e^{i2\pi(0.4)}, 0.2e^{i2\pi(0.3)} \right\rangle\right)$
$\mathtt{F}_4$	$\left(\left\langle [0.4, 0.8]\, e^{i2\pi[0.5,0.6]}, [0.3, 0.5]\, e^{i2\pi[0.1,0.5]} \right\rangle, \left\langle 0.6e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.4)} \right\rangle\right)$

	e_2
$\mathtt{F}_1$	$\left(\left\langle [0.5, 0.6]\, e^{i2\pi[0.8,0.9]}, [0.4, 0.5]\, e^{i2\pi[0.1,0.2]} \right\rangle, \left\langle 0.8e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.4)} \right\rangle\right)$
$\mathtt{F}_2$	$\left(\left\langle [0.3, 0.5]\, e^{i2\pi[0.6,0.7]}, [0.1, 0.15]\, e^{i2\pi[0.2,0.3]} \right\rangle, \left\langle 0.7e^{i2\pi(0.8)}, 0.1e^{i2\pi(0.5)} \right\rangle\right)$
$\mathtt{F}_3$	$\left(\left\langle [0.4, 0.7]\, e^{i2\pi[0.3,0.5]}, [0.1, 0.2]\, e^{i2\pi[0.15,0.21]} \right\rangle, \left\langle 0.5e^{i2\pi(0.7)}, 0.2e^{i2\pi(0.3)} \right\rangle\right)$
$\mathtt{F}_4$	$\left(\left\langle [0.6, 0.71]\, e^{i2\pi[0.7,0.9]}, [0.3, 0.4]\, e^{i2\pi[0.2,0.3]} \right\rangle, \left\langle 0.8e^{i2\pi(0.9)}, 0.21e^{i2\pi(0.2)} \right\rangle\right)$

	e_3
F_1	$\left(\left\langle [0.6, 0.7]\, e^{i2\pi[0.5,0.6]}, [0.3, 0.4]\, e^{i2\pi[0.1,0.2]} \right\rangle, \left\langle 0.8e^{i2\pi(0.5)}, 0.3e^{i2\pi(0.4)} \right\rangle\right)$
F_2	$\left(\left\langle [0.4, 0.5]\, e^{i2\pi[0.8,0.9]}, [0.1, 0.2]\, e^{i2\pi[0.3,0.4]} \right\rangle, \left\langle 0.5e^{i2\pi(0.6)}, 0.2e^{i2\pi(0.3)} \right\rangle\right)$
F_3	$\left(\left\langle [0.8, 0.91]\, e^{i2\pi[0.7,0.8]}, [0.3, 0.4]\, e^{i2\pi[0.15,0.7]} \right\rangle, \left\langle 0.4e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.5)} \right\rangle\right)$
F_4	$\left(\left\langle [0.35, 0.4]\, e^{i2\pi[0.6,0.71]}, [0.15, 0.2]\, e^{i2\pi[0.2,0.4]} \right\rangle, \left\langle 0.7e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.2)} \right\rangle\right)$

	e_4
F_1	$\left(\left\langle [0.5, 0.7]\, e^{i2\pi[0.4,0.5]}, [0.15, 0.21]\, e^{i2\pi[0.3,0.4]} \right\rangle, \left\langle 0.8e^{i2\pi(0.7)}, 0.5e^{i2\pi(0.3)} \right\rangle\right)$
F_2	$\left(\left\langle [0.3, 0.4]\, e^{i2\pi[0.6,0.7]}, [0.11, 0.24]\, e^{i2\pi[0.05,0.1]} \right\rangle, \left\langle 0.5e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.2)} \right\rangle\right)$
F_3	$\left(\left\langle [0.4, 0.5]\, e^{i2\pi[0.7,0.8]}, [0.3, 0.39]\, e^{i2\pi[0.4,0.5]} \right\rangle, \left\langle 0.6e^{i2\pi(0.5)}, 0.2e^{i2\pi(0.3)} \right\rangle\right)$
F_4	$\left(\left\langle [0.6, 0.8]\, e^{i2\pi[0.5,0.7]}, [0.4, 0.5]\, e^{i2\pi[0.3,0.4]} \right\rangle, \left\langle 0.9e^{i2\pi(0.7)}, 0.6e^{i2\pi(0.5)} \right\rangle\right)$

Step 2. By using Definition 4.1 of CCPyFWA, we aggregate the values. The overall values corresponding to each alternative F_t are given below.

$$F_1 = \left(\left\langle [0.64, 0.81]\, e^{i2\pi[0.56,0.66]}, [0.19, 0.35]\, e^{i2\pi[0.28,0.48]} \right\rangle, \left\langle 0.56e^{i2\pi(0.47)}, 0.36e^{i2\pi(0.31)} \right\rangle\right)$$
$$F_2 = \left(\left\langle [0.41, 0.61]\, e^{i2\pi[0.61,0.75]}, [0.14, 0.22]\, e^{i2\pi[0.15,0.24]} \right\rangle, \left\langle 0.69e^{i2\pi(0.76)}, 0.16e^{i2\pi(0.37)} \right\rangle\right)$$
$$F_3 = \left(\left\langle [0.61, 0.74]\, e^{i2\pi[0.71,0.82]}, [0.18, 0.28]\, e^{i2\pi[0.18,0.41]} \right\rangle, \left\langle 0.59e^{i2\pi(0.57)}, 0.26e^{i2\pi(0.35)} \right\rangle\right)$$
$$F_4 = \left(\left\langle [0.41, 0.56]\, e^{i2\pi[0.59,0.7]}, [0.17, 0.29]\, e^{i2\pi[0.15,0.24]} \right\rangle, \left\langle 0.67e^{i2\pi(0.54)}, 0.35e^{i2\pi(0.26)} \right\rangle\right)$$

Step 3. By using Definition 3.3, we find score value
$Q(F_1) = 0.28$, $Q(F_2) = 0.42$, $Q(F_3) = 0.39$, $Q(F_4) = 0.33$.
Step 4. Rank the alternatives based on score value, $Q(F_2) > Q(F_3) > Q(F_4) > Q(F_1)$ and the best alternative is $Q(F_2)$.

Case-II

Step 1. Same as in Case-I

Step 2. By using Definition 4.2 of CCPyFWG, we aggregate the values. The over all values corresponding to each alternative F_t are listed below.

$$F_1 = \left(\left\langle [0.59, 0.75]\, e^{i2\pi[0.55,0.65]}, [0.23, 0.36]\, e^{i2\pi[0.39,0.51]} \right\rangle, \left\langle 0.47e^{i2\pi(0.46)}, 0.45e^{i2\pi(0.31)} \right\rangle\right)$$
$$F_2 = \left(\left\langle [0.38, 0.59]\, e^{i2\pi[0.52,0.68]}, [0.22, 0.3]\, e^{i2\pi[0.16,0.25]} \right\rangle, \left\langle 0.65e^{i2\pi(0.73)}, 0.19e^{i2\pi(0.4)} \right\rangle\right)$$
$$F_3 = \left(\left\langle [0.52, 0.62]\, e^{i2\pi[0.67,0.78]}, [0.22, 0.31]\, e^{i2\pi[0.22,0.31]} \right\rangle, \left\langle 0.53e^{i2\pi(0.56)}, 0.27e^{i2\pi(0.38)} \right\rangle\right)$$
$$F_4 = \left(\left\langle [0.38, 0.51]\, e^{i2\pi[0.56,0.68]}, [0.23, 0.32]\, e^{i2\pi[0.27,0.36]} \right\rangle, \left\langle 0.61e^{i2\pi(0.5)}, 0.41e^{i2\pi(0.29)} \right\rangle\right)$$

Step 3. By using Definition 3.3, we evaluate score value
$Q(F_1) = 0.2$, $Q(F_2) = 0.33$, $Q(F_3) = 0.32$, $Q(F_4) = 0.22$.
Step 4. Rank the alternatives based on score value, $Q(F_2) > Q(F_3) > Q(F_4) > Q(F_1)$ and the best alternative is $Q(F_2)$.

It is clear from Case-I and Case-II that the alternative $Q(F_2)$ provides best quality cotton to making fabrics.

6. Conclusion

In this article, we have studied the concept of CCPyFS. We have discussed two aggregation operators namely CCPyFWA and CCPyFWG. Also, we have discussed score value to facilitate the ranking of the alternatives. Finally, a MCDM method is illustrated with case studies.

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