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# NEIGHBOURLY PSEUDO IRREGULAR NEUTROSOPHIC FUZZY GRAPHS

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**Abstract:** The idea of pseudo regular and irregular $N_G$($N_G$ - Neutrosophic Fuzzy Graph), neighbourly pseudo irregular $N_G$ are made known here. We represent some theorems and results of these graphs.

**Keywords and Phrases:** Pseudo regular and irregular $N_G$($N_G$ - Neutrosophic Fuzzy Graph), neighbourly pseudo irregular $N_G$.

**2020 Mathematics Subject Classification:** 05C78.

## 1. Introduction and Preliminaries

Divya and J. Malarvizhi introduced the notions and fundamental operations on neutrosophic fuzzy graph [1]. S. Sivabala and N. R. Santhi Maheswari introduced Neighbourly and highly irregular neutrosophic fuzzy graph [3]. These ideas encourage us to introduce Neighbourly pseudo irregular neutrosophic fuzzy graphs.

**Definition 1.1.** *"A neutrosophic fuzzy graph with underlying set $V$ is defined to be a pair $N_G = (A, B)$, where*
*(i) The functions $T_A, I_A, F_A : V \to [0, 1]$ denote the degree of truth membership, degree of indeterminacy membership and the degree of falsity membership of the element $v_i \in V$ respectively and $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$.*
*(ii) $E \subseteq V \times V$ where the functions $T_B, I_B, F_B : V \times V \to [0, 1]$ are defined by*

$$T_B(v_i, v_j) \leq T_A(v_i).T_A(v_j)$$
$$I_B(v_i, v_j) \leq I_A(v_i).I_A(v_j)$$
$$F_B(v_i, v_j) \leq F_A(v_i).F_A(v_j)$$

*for all* $v_i, v_j \in V$*, where . means ordinary multiplication denotes the degrees of truth membership, indeterminacy membership and falsity membership of the edge* $(v_i, v_j) \in E$ *respectively, where* $0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$ *for all* $(v_i, v_j) \in E$ $(i, j = 1, 2, ..., n)$*."* [1]

**Definition 1.2.** *The degree of a vertex x in* $N_G$ *defined by* $d_{N_G}(x) = (deg_T(x), deg_I(x), deg_F(x))$*, where* $deg_T(x) = \sum_{xy\in E} T_B(xy)$*,* $deg_I(x) = \sum_{xy\in E} I_B(xy)$*,* $deg_F(x) = \sum_{xy\in E} F_B(xy)$*.* [3]

## 2. Pseudo degree of a vertex in $N_G$

**Definition 2.1.** *The* $2-$ *degree of a vertex v in* $N_G$ *is defined by* $t_{N_G}(v) = \sum d_{N_G}(u)$*, where* $d_{N_G}(u)$ *is the degree of the vertex u which is adjacent with the vertex v.*

**Definition 2.2.** *A pseudo degree of a vertex v in* $N_G$ *is denoted by* $pd_{N_G}(v)$ *and is defined by* $pd_{N_G}(v) = \frac{t_{N_G}(v)}{d^*_{N_G}(v)}$*, where* $d^*_{N_G}(v)$ *is the number of edges incident at v.*

**Definition 2.3.** *Let* $N_G$ *be a neutrosophic fuzzy graph on* $G(V, E)$*. The total pseudo degree of a vertex v in* $N_G$ *is defined by* $tpd_{N_G}(v) = pd_{N_G}(v) + (T_A, I_A, F_A)(v), \forall$ *v in* $V$*.*

## 3. Pseudo Regular $N_G$

**Definition 3.1.** *If all the vertices of* $N_G$ *have same pseudo degree, then* $N_G$ *is pseudo regular* $N_G$ *simply says* $PRN_G$*.*

**Definition 3.2.** *If all the vertices of* $N_G$ *have same total pseudo degree,then* $N_G$ *is totally pseudo regular* $N_G$ *simply says* $TPRN_G$

**Example 3.3.** Consider $N_G$ .

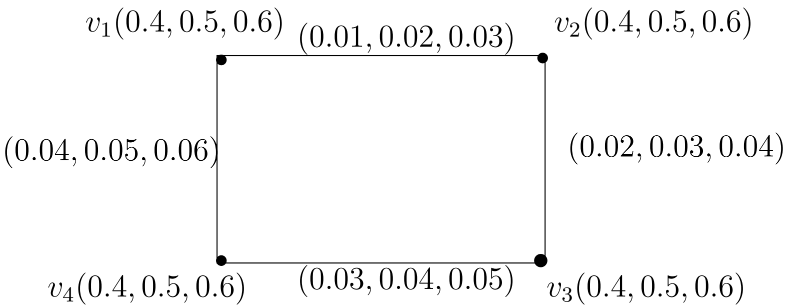


Fig.1

In this Fig.1, $pd_{N_G}(v_1) = (0.05, 0.07, 0.09)$; $pd_{N_G}(v_2) = (0.05, 0.07, 0.09)$;
$pd_{N_G}(v_3) = (0.05, 0.07, 0.09)$; $pd_{N_G}(v_4) = (0.05, 0.07, 0.09)$;
$tpd_{N_G}(v_1) = (0.45, 0.57, 0.69)$; $tpd_{N_G}(v_2) = (0.45, 0.57, 0.69)$;
$tpd_{N_G}(v_3) = (0.45, 0.57, 0.69)$; $tpd_{N_G}(v_4) = (0.45, 0.57, 0.69)$.
Here, every vertices having same pseudo degree and same total pseudo degree. Hence the $N_G$ is $PRN_G$ and $TPRN_G$.

**Remark 3.4.** *Every $PRN_G$ need not be $TPRN_G$.*

**Example 3.5.** Consider $N_G$

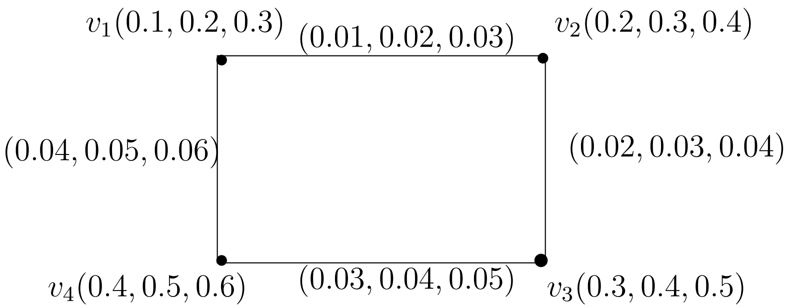


Fig.2

In this Fig.2, $pd_{N_G}(v_1) = (0.05, 0.07, 0.09) = pd_{N_G}(v_2) = pd_{N_G}(v_3) = pd_{N_G}(v_4)$;
$tpd_{N_G}(v_1) = (0.15, 0.27, 0.39)$; $tpd_{N_G}(v_2) = (0.25, 0.37, 0.49)$;
$tpd_{N_G}(v_3) = (0.35, 0.47, 0.59)$; $tpd_{N_G}(v_4) = (0.45, 0.57, 0.69)$.
Here, every vertices having same pseudo degree but every vertices having distinct total pseudo degrees. Therefore $N_G$ is $PRN_G$ but not $TPRN_G$.

**Remark 3.6.** *Every $TPRN_G$ need not be $PRN_G$.*

**Example 3.7.** In Fig 3, $pd_{N_G}(v_1) = (0.02, 0.04, 0.06) = pd_{N_G}(v_3)$;
$pd_{N_G}(v_2) = (0.01, 0.02, 0.03)$.
$tpd_{N_G}(v_1) = (0.22, 0.34, 0.46) = tpd_{N_G}(v_2) = tpd_{N_G}(v_3)$.

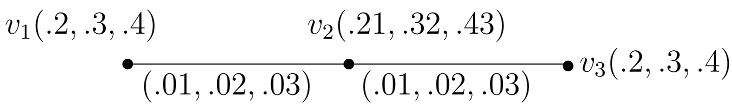


Fig.3

Here, every vertices having same total pseudo degrees. Therefore this graph is $TPRN_G$ . But pseudo degree of $v_2$ is distinct from the vertices $v_1$ and $v_3$. Therefore this is not a $PRN_G$.

**Theorem 3.8.** *Let $N_G$ be a cycle of length $n$ with $(T_A, I_A, F_A)$ and $(T_B, I_B, F_B)$ are constant , then $N_G$ is $PRN_G$ and $TPRN_G$.*

**Proof.** We take $N_G$ with $(T_B, I_B, F_B)$ is constant. i.e. $(T_B, I_B, F_B)(v_iv_j) = (c_T, c_I, c_F)$, $v_iv_j \in E, i \neq j$. Then $t_{N_G}(v_i) = \sum d_{N_G}(v_j) = 4(T_B, I_B, F_B)(v_iv_j)$. This implies that $pd_{N_G}(v_i) = \frac{t_{N_G}(v_i)}{d^*_{N_G}(v_i)} = \frac{4(T_B,I_B,F_B)(v_iv_j)}{2} = 2(T_B, I_B, F_B)(v_iv_j) = 2(c_T, c_I, c_F)$, also a constant $\forall v_iv_j \in E, i \neq j$. Therefore every vertices have same pseudo degree. Hence $N_G$ is $PRN_G$.
Now, we take $(T_A, I_A, F_A)(v_i) = (c_{T_2}, c_{I_2}, c_{F_2})$, for all $v_i \in V$ where $(c_{T_2}, c_{I_2}, c_{F_2})$ is constant. Then $tpd_{N_G}(v_i) = pd_{N_G}(v_i) + (T_A, I_A, F_A)(v_i)$, for all $v_i \in V$. This implies that $tpd_{N_G}(v_i) = (c_{T_1}, c_{I_1}, c_{F_1}) + (c_{T_2}, c_{I_2}, c_{F_2})$ = constant, $\forall\ v_i \in V$. Hence $N_G$ is $TPRN_G$.

## 4. Pseudo irregular $N_G$.

**Definition 4.1.** *$N_G$ is said to be pseudo irregular $N_G(PIN_G)$ if the adjacent vertices of the vertex $v \in N_G$ having distinct pseudo degrees in $N_G$.*

**Definition 4.2.** *If the adjacent vertices of the vertex $v \in N_G$ having distinct total pseudo degrees, then $N_G$ is totally pseudo irregular $N_G(TPIN_G)$*

**Example 4.3.** Consider $N_G$

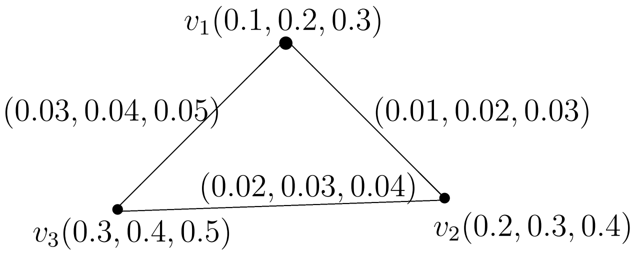


Fig.4

In this Fig. 4, $pd_{N_G}(v_1) = (0.04, 0.06, 0.08)$;
$pd_{N_G}(v_2) = (0.045, 0.065, 0.085)$; $pd_{N_G}(v_3) = (0.035, 0.055, 0.075)$;
$tpd_{N_G}(v_1) = (0.14, 0.26, 0.38)$; $tpd_{N_G}(v_2) = (0.245, 0.365, 0.485)$;
$tpd_{N_G}(v_3) = (0.335, 0.455, 0.575)$.
Hence this $N_G$ is both $PIN_G$ and $TPIN_G$.

**Remark 4.4.** *Every $PIN_G$ need not be $TPIN_G$.*

**Example 4.5.** Consider $N_G$

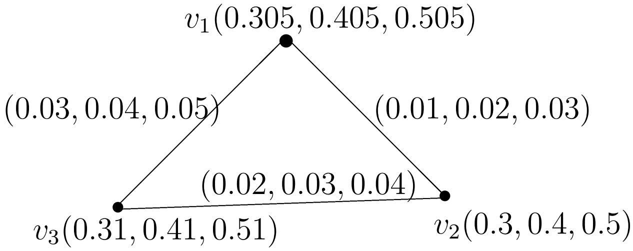


Fig.5

In Fig.5, $pd_{N_G}(v_1) = (0.04, 0.06, 0.08)$; $pd_{N_G}(v_2) = (0.045, 0.065, 0.085)$; $pd_{N_G}(v_3) = (0.035, 0.055, 0.075)$; $tpd_{N_G}(v_1) = (0.345, 0.465, 0.585)$; $tpd_{N_G}(v_2) = (0.345, 0.465, 0.585)$; $tpd_{N_G}(v_3) = (0.345, 0.465, 0.585)$.
Hence this $N_G$ is $PIN_G$ but not $TPIN_G$.

**Remark 4.6.** *Every $TPIN_G$ need not be $PIN_G$.*

**Example 4.7.** Consider $N_G$

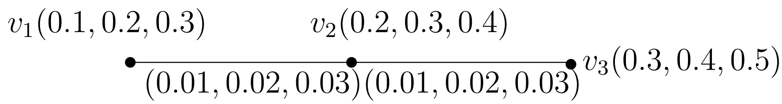


Fig.6

In this graph, $pd_{N_G}(v_1) = (0.02, 0.04, 0.06)$; $pd_{N_G}(v_2) = (0.01, 0.02, 0.03)$; $pd_{N_G}(v_3) = (0.02, 0.04, 0.06)$. $tpd_{N_G}(v_1) = (0.12, 0.24, 0.36)$
$tpd_{N_G}(v_2) = (0.21, 0.32, 0.43)$; $tpd_{N_G}(v_3) = (0.32, 0.44, 0.56)$.
Hence this $N_G$ is $TPIN_G$ but not $PIN_G$.

**Theorem 4.8.** *Let $N_G$ with constant $(T_A, I_A, F_A)$, then the following conditions are equivalent*
*(i) $PIN_G$ (ii) $TPIN_G$.*

**Proof.** Let $N_G$ with $(T_A, I_A, F_A)$ is constant. i.e. $(T_A, I_A, F_A)(v_i) = (p, q, r), \forall v_i \in V$, where $(p, q, r)$ is constant. Suppose $N_G$ is $PIN_G$. Then at least one vertex of $N_G$ which is adjacent to distinct pseudo degrees of the vertices. Let $v_1$ and $v_2$ be the adjacent vertices of $v_3$ with distinct pseudo degrees $(l_1, m_1, n_1)$ and $(l_2, m_2, n_2)$ respectively. Then $(l_1, m_1, n_1) \neq (l_2, m_2, n_2)$. This implies that $(l_1, m_1, n_1) + (p, q, r) \neq (l_2, m_2, n_2) + (p, q, r)$. $\implies$ $(l_1, m_1, n_1) + (T_A, I_A, F_A)(v_1)$ $\neq (l_2, m_2, n_2)) + (T_A, I_A, F_A)(v_2)$. $\implies$ $tpd_{N_G}(v_1) \neq tpd_{N_G}(v_2)$. Therefore $v_1$ and $v_2$ be the adjacent vertices of $v_3$ in $N_G$ with distinct total pseudo degrees. Hence

$N_G$ is $TPIN_G$.
Hence $(i) \implies (ii)$ is hold.
Conversely, Suppose $N_G$ is $TPIN_G$. Suppose $v_1$ and $v_2$ are the adjacent vertices of $v_3$ having the total pseudo degrees $(tx_1, ty_1, tz_1)$ and $(tx_2, ty_2, tz_2)$ respectively are distinct. Then $tpd_{N_G}(v_1) \neq tpd_{N_G}(v_2)$.
This implies that $(l_1, m_1, n_1) + (T_A, I_A, F_A)(v_1) \neq (l_2, m_2, n_2)) + (T_A, I_A, F_A)(v_2)$.
Since $(T_A, I_A, F_A)(v_i) = (p, q, r)$, we have $(l_1, m_1, n_1) + (p, q, r) \neq (l_2, m_2, n_2) + (p, q, r)$. This implies that $(l_1, m_1, n_1) \neq (l_2, m_2, n_2)$. Therefore $v_1$ and $v_2$ be the adjacent vertices of $v_3$ in $N_G$ with distinct pseudo degrees. Hence $N_G$ is $PIN_G$ .
Hence (i) $\implies$ (ii) is hold.

**Remark 4.9.** *Converse of the above theorem need not be true.*

**Example 4.10.** Refer example 5.3.

## 5. Neighbourly Pseudo irregular $N_G$

**Definition 5.1.** *Any two adjacent vertices in $N_G$ having distinct pseudo degrees, then $N_G$ is called neighbourly pseudo irregular $N_G$ simply $NPIN_G$.*

**Definition 5.2.** *Any two adjacent vertices in $N_G$ having the total pseudo degrees are distinct, then $N_G$ is called neighbourly totally pseudo irregular $N_G$ simply $NTPIN_G$.*

**Example 5.3.** Refer Example 5.3

**Remark 5.4.** *Every $NPIN_G$ need not be $NTPIN_G$.*

**Example 5.5.** Refer Example 4.7

**Remark 5.6.** *Every $NTPIN_G$ need not be $NPIN_G$.*

**Example 5.7.** Consider $N_G$.

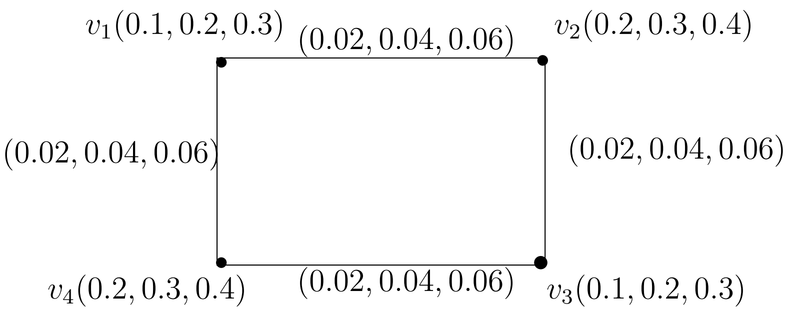


Fig.7

In this Fig.7, $pd_{N_G}(v_1) = (0.04, 0.08, 0.12)$; $pd_{N_G}(v_2) = (0.04, 0.08, 0.12)$;

$pd_{N_G}(v_3) = (0.04, 0.08, 0.12)$; $pd_{N_G}(v_4) = (0.04, 0.08, 0.12)$;
$tpd_{N_G}(v_1) = (0.14, 0.28, 0.42)$; $tpd_{N_G}(v_2) = (0.24, 0.38, 0.52)$;
$tpd_{N_G}(v_3) = (0.14, 0.28, 0.42)$; $tpd_{N_G}(v_4) = (0.24, 0.38, 0.52)$.
Hence this $N_G$ is $NTPIN_G$ but not $NPIN_G$.

**Remark 5.8.** *Every $PIN_G$ need not $NPIN_G$.*

**Example 5.9.** Consider $N_G$ .

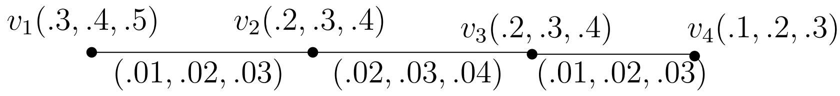


Fig.8

In this graph, $pd_{N_G}(v_1) = (0.03, 0.05, 0.07)$; $pd_{N_G}(v_2) = (0.02, 0.035, 0.05)$;
$pd_{N_G}(v_3) = (0.02, 0.035, 0.05)$; $pd_{N_G}(v_4) = (0.03, 0.05, 0.07)$.
Therefore this $N_G$ is $PIN_G$ but not $NPIN_G$.

**Remark 5.10.** *Every $NPIN_G$ need not be $PIN_G$.*

**Example 5.11.** Consider $N_G$.

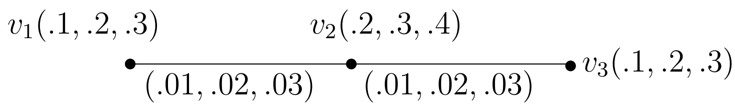


Fig.9

In this graph, $pd_{N_G}(v_1) = (0.02, 0.04, 0.06)$; $pd_{N_G}(v_2) = (0.01, 0.02, 0.03)$;
$pd_{N_G}(v_3) = (0.02, 0.04, 0.06)$.
Therefore this $N_G$ is $NPIN_G$ but not $PIN_G$.

**Remark 5.12.** *Every $TPIN_G$ need not be $NTPIN_G$.*

**Example 5.13.** Consider $N_G$ .

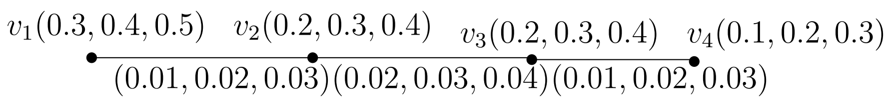


Fig.10

In this graph, $tpd_{N_G}(v_1) = (0.33, 0.45, 0.57)$; $tpd_{N_G}(v_2) = (0.22, 0.335, 0.45)$;
$tpd_{N_G}(v_3) = (0.22, 0.335, 0.45)$; $tpd_{N_G}(v_3) = (0.13, 0.25, 0.37)$.

Therefore this $N_G$ is $TPIN_G$ but not $NTPIN_G$.

**Remark 5.14.** *Every $NTPIN_G$ need not be $TPIN_G$.*

**Example 5.15.** Refer example 6.11, $tpd_{N_G}(v_1) = (0.12, 0.24, 0.36)$; $tpd_{N_G}(v_2) = (0.21, 0.32, 0.43)$; $tpd_{N_G}(v_3) = (0.12, 0.24, 0.36)$. Therefore this $N_G$ is $NTPIN_G$ but not $TPIN_G$.

**Theorem 5.16.** *Let $N_G = (A, B)$ with $(T_A, I_A, F_A)$ is constant, then the following two conditions are equivalent (i) $NPIN_G$ (ii) $NTPIN_G$.*
**Proof.** Proof is similar to the theorem 5.8.

**Remark 5.17.** *Converse of the above theorem need not be true.*

**Example 5.18.** Refer Example 5.3.

**Theorem 5.19.** *Let $N_G$ be an even cycle of length $n$ and $(T_A, I_A, F_A)(v_i)$ are distinct for every $i$. If the membership values of the alternate edges are same, then $N_G$ is $NPIN_G$ and $NTPIN_G$.*
**Proof.** Let $(T_A, I_A, F_A)(v_i) = (l_i, m_i, n_i), \forall\, v_i$ and $(l_1, m_1, n_1) \neq (l_2, m_2, n_2) \neq ... \neq (l_n, m_n, n_n)$. Suppose the membership values of the alternate edges are same.
$(T_B, I_B, F_B)(e_j) = (c_{T_1}, c_{I_1}, c_{F_1})$ if j is odd
$(T_B, I_B, F_B)(e_j) = (c_{T_2}, c_{I_2}, c_{F_2})$ if j is even
$\implies pd_{N_G}(v_i) = (c_{T_1}, c_{I_1}, c_{F_1}) + (c_{T_2}, c_{I_2}, c_{F_2}),\ i = 1, 2, ..., n.$
$\implies pd_{N_G}(v_i) = constant$. Therefore $N_G$ is $NPIN_G$. This implies that
$tpd_{N_G}(v_i) = pd_{N_G}(v_i) + (T_A, I_A, F_A)(v_i), \forall v_i \in V$.
$\implies tpd_{N_G}(v_i) = pd_{N_G}(v_i) + (l_i, m_i, n_i)$, i= 1,2,...,n.
where $(l_1, m_1, n_1) \neq (l_2, m_2, n_2) \neq ... \neq (l_n, m_n, n_n)$. Hence $N_G$ is $NTPIN_G$.

**Observation 5.20.** *Let $N_G$ be a cycle of length $n$. If the membership values of the alternate vertices and edges are same, then $N_G$ is $NTPIN_G$*

**Theorem 5.21.** *Let $N_G$ be a cycle of length $n \geq 5$. If the edges having membership values are $(l_1, m_1, n_1), (l_2, m_2, n_2), ..., (l_n, m_n, n_n)$ such that $(l_1, m_1, n_1) < (l_2, m_2, n_2) < ... < (l_n, m_n, n_n)$. Then $N_G$ is $NPIN_G$.*
**Proof.** Let $N_G$ be a cycle of length $n \geq 5$. If the edges having membership values are $(l_1, m_1, n_1), (l_2, m_2, n_2), ..., (l_n, m_n, n_n)$ such that $(l_1, m_1, n_1) < (l_2, m_2, n_2) < ... < (l_n, m_n, n_n)$. Then
$d_{N_G}(v_i) = (l_n, m_n, n_n) + (l_1, m_1, n_1)$ if i = 1
$d_{N_G}(v_i) = (l_{i-1}, m_{i-1}, n_{i-1}) + (l_i, m_i, n_i)$ if i = 2, 3, ..., n
This implies that
$pd_{N_G}(v_i) = \frac{d_{N_G}(v_2)+d_{N_G}(v_n)}{2}$ if n = 1

$pd_{N_G}(v_i) = \frac{d_{N_G}(v_{i-1})+d_{N_G}(v_{i+1})}{2}$ if i = 2,3,..,n-1
$pd_{N_G}(v_i) = \frac{d_{N_G}(v_{n-1})+d_{N_G}(v_1)}{2}$ if i = n
This implies that
$pd_{N_G}(v_i) = \frac{(l_2,m_2,n_2)+(l_3,m_3,n_3)+(l_{n-1},m_{n-1},n_{n-1})+(l_n,m_n,n_n)}{2}$ if i = 1
$pd_{N_G}(v_i) = \frac{(l_n,m_n,n_n)+(l_1,m_1,n_1)+(l_2,m_2,n_2)+(l_3,m_3,n_3)}{2}$ if i = 2
$pd_{N_G}(v_i) = \frac{(l_{i-2},m_{i-2},n_{i-2})+(l_{i-1},m_{i-1},n_{i-1})+(l_i,m_i,n_i)+(l_{i+1},m_{i+1},n_{i+1})}{2}$ if i=3,...,n-1
$pd_{N_G}(v_i) = \frac{(l_1,m_1,n_1)+(l_n,m_n,n_n)+(l_{n-1},m_{n-1},n_{n-1})+(l_{n-2},m_{n-2},n_{n-2})}{2}$ if i = n.
Since $(l_1, m_1, n_1) < (l_2, m_2, n_2) < ... < (l_n, m_n, n_n)$, we have every pair of adjacent vertices having pseudo degrees are distinct.
Hence $N_G$ is $NPIN_G$.

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