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FIBONACCI PRIME LABELING OF SNAKE GRAPH

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Abstract: Here we describe the Snake related graph into a Fibonacci prime Graph by the following condition, If there exist a one-to-one mapping between the vertex set and the fibonacci numbers then there is a mapping between edge set and natural numbers where the end points of the edges are relatively prime. This work is a continuation of S. Chandrakala, Dr. C. Sekar who introduced Fibonacci Prime Labeling. We represent Fibonacci Prime Labeling as *(FPL)*, Fibonacci Prime Graph as *(FPG)*.

Keywords and Phrases: Fibonacci Prime Labeling *(FPL)*, Fibonacci Prime Graph *(FPG)*.

2020 Mathematics Subject Classification: 05C78.

1. Introduction and Preliminaries

The finite, loopless and non-multiple edge, connected, bidirectional graph has been used in the current work. Let $G = (V', E')$ be a (p, q) graph where V', E', p and q denotes vertex set, edge set, the number of vertices, number of edges of the graph. Here we mentioned the Triangular Snake Graph as Δ_s^a, Double Triangular Snake graph as $D - \Delta_s^k$, Quadrilateral Snake Graph as Q_s^l, Double Quadrilateral Snake Graph as $D - Q_s^c$, $n-$ Polygonal Snake graph as $n - P_s^x$, where a, k, l, c, x

denotes number of times the cycles appears in the snake graph.

FPL was lauched by C. Sekar, S. Chandrakala [5] and they are also proved that the path, friendship graph, fan graph, star graph, dragon graph, umbrella graph, cycle related graph and crown graph are *FPG* [2],[5]. We refer Bondy and Murthy for notations and terminology [1]. This paper proves that some snake related graphs are admits *FPL*. A *FPL* of a graph $G = \left(V', E'\right)$ with $\left|V'\right|=m$ is an injective function $h : V' \to \{F_2, F_3, ..., F_{m+1}\}$, where F_m is the m-th Fibonacci number, that leads to a another function $h'' : E' \to N$ defined by $h''(vw) = \gcd(h(v), h(w)) = 1$ for all edges belong to the edge set $E'(G)$. Δ_s^a, $D - \Delta_s^k$, Q_s^l, $D - Q_s^c$, $n - P_s^x$ are satisfy the conditions of the Difference Perfect Square labeling, Mean Cordial labeling and Odd Prime labeling in the articles [3], [4], [6].

2. Main Results

Note 2.1. *From the Fibonacci numbers we get* $g.c.d\{F_m, F_k\} = 1$, *if* $g.c.d\{m, k\} = 1$ *or* $g.c.d\{m, k\} = 2$, *where* F_m, F_k *are distinct Fibonacci numbers and* m, k *are distinct integers,* $k = m + l, 1 \leq l \leq m - 1$ *and* $m \geq 3$.

Theorem 2.2. Δ_s^a *admits FPL.*

Proof. Δ_s^a, where $a \geq 2$ is a triangular snake graph.

Let $V'(\Delta_s^a) = \{v_1, v_2, ..., v_{2a-1}\}$ be the vertex set and

Let $E'(\Delta_s^a) = \{v_iv_{i+1}/1 \leq i \leq 2a - 2\} \cup \{v_iv_{i+2}/i \in [1, 2a - 3] - \{2i\}\}$ be the edge set.

Let $p = 2a - 1$, $q = 3a - 3$ indicates the number of nodes and links in Δ_s^a

Let us define a function $h' : V'(\Delta_s^a) \to \{F_2, F_3, ..., F_{2a}\}$ and the vertices of Δ_s^a are labeled with the Fibonacci numbers $F_2, F_3, ..., F_{2a}$

i.e. $h'(v_1) = F_2, h'(v_2) = F_3, ..., h'(v_{2a-1}) = F_{2a}$

$\Rightarrow h'(v_i) = F_{i+1}$, where $1 \leq i \leq 2a - 1$.

Then the function f induces the function $h^* : E'(\Delta_s^a) \to N$ is defined as $h^*(bd) = g.c.d\left(h'(b), h'(d)\right)$ for all edges in $E'(\Delta_s^a)$. Minimum degree of each vertex in Δ_s^a is 2. Let v_i be a vertex in Δ_s^a and it is 2−connected. Assume that the vertex v_i adjacent to the vertices v_{i+1}, v_{i+2}. Now the vertex v_i and the vertices adjacent to v_i are labeled as $F_{i+1}, F_{i+2}, F_{i+3}$.

$\Rightarrow h^*(v_iv_{i+1}) = g.c.d\left(h'(v_i), h'(v_{i+1})\right) = g.c.d(F_{i+1}, F_{i+2}) = 1, 1 \leq i \leq 2a - 2$

Similarly, $h^*(v_iv_{i+2}) = g.c.d(F_{i+1}, F_{i+3}) = 1$, where i is the odd number and not exceeded than $2a - 3$.

$\Rightarrow h^*(bd) = \gcd\left(h'(b), h'(d)\right) = 1$ for every edge belongs to $E'(\Delta_s^a)$

Therefore, Δ_s^a is a *FPG*.

Example 2.3. Consider the triangular snake graph Δ_s^4.

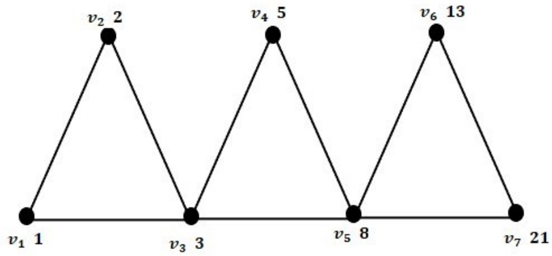

Figure 1: Δ_s^4 - *FPG*

Theorem 2.4. *$D-\Delta_s^k$ admits FPL.*
Proof. $D-\Delta_s^k$, $k \geq 2$, $V'\left(D-\Delta_s^k\right) = \{v_1, v_2, ..., v_{3k-2}\}$ is the vertex set, $E'\left(D-\Delta_s^k\right) = \{v_{3b-2}v_{3b-1} \cup v_{3b-2}v_{3b} \cup v_{3b-2}v_{3b+1} \cup v_{3b}v_{3b+1} \cup v_{3b-1}v_{3b+1} /\ 1 \leq b \leq k-1\}$ is the edge set and $\left|V'\left(D-\Delta_s^k\right)\right| = 3k-2$ $\left|E'\left(D-\Delta_s^k\right)\right| = 5k-5$ refers the number of points and lines.

Define a function $H : V'(D-\Delta_s^k) \to \{F_2, F_3, ..., F_{3k-1}\}$ and the vertices of $D-\Delta_s^k$ are labeled with the Fibonacci numbers $F_2, F_3, ..., F_{3k-1}$ i.e. $H(v_1) = F_2, H(v_2) = F_3, ..., H(v_{3k-2}) = F_{3k-1} \Rightarrow H(v_b) = F_{b+1}, where\ 1 \leq b \leq 3k-2.$

There exists the function $H' : E'\left(D-\Delta_s^k\right) \to N$ is defined by $H'(uv) = g.c.d\left(H(u), H(v)\right) \forall uv \in E'\left(D-\Delta_s^k\right)$.

Now, $H'(v_{3b-2}v_{3b-1}) = g.c.d\left(H(v_{3b-2}), H(v_{3b-1})\right) = g.c.d\left(F_{3b-1}, F_{3b}\right) = 1$
Similarly, $H'(v_{3b-2}v_{3b}) = g.c.d\left(H(v_{3b-2}), H(v_{3b})\right) = g.c.d\left(F_{3b-1}, F_{3b+1}\right) = 1$
$H'(v_{3b-1}v_{3b+1}) = g.c.d\left(H(v_{3b-1}), H(v_{3b+1})\right) = g.c.d\left(F_{3b}, F_{3b+2}\right) = 1$
$H'(v_{3b}v_{3b+1}) = g.c.d\left(H(v_{3b}), H(v_{3b+1})\right) = g.c.d\left(F_{3b+1}, F_{3b+2}\right) = 1$
$H'(v_{3b-2}v_{3b+1}) = g.c.d\left(H(v_{3b-2}), H(v_{3b+1})\right) = g.c.d\left(F_{3b-1}, F_{3b+2}\right) = 1$
$\Rightarrow H'(uv) = g.c.d\left(H(u), H(v)\right) \forall uv \in E'\left(D-\Delta_s^k\right)$
Therefore, $D-\Delta_s^k$ is a FPG.

Example 2.5. Consider the double triangular snake graph $D-\Delta_s^5$

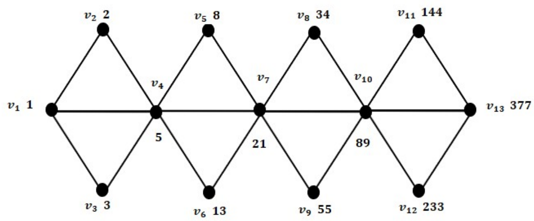

Figure 2: $D-\Delta_s^5$-FPG

Theorem 2.6. *Q_s^l exists as FPG.*

Proof. Q_s^l, where $l \geq 2$. The vertex set and edge set of Q_s^l are
$V^*\left(Q_n^l\right) = \{v_1, v_2, ..., v_{3l-2}\}$
$E^*\left(Q_s^l\right) = \{v_{3j-2}v_{3j-1} \cup v_{3j-1}v_{3j} \cup v_{3j-2}v_{3j+1} \cup v_{3j}v_{3j+1}/1 \leq j \leq l-1\}$
$p' = 3n-2$, $q' = 4n-4$ represents order and size of Q_s^l

Consider the function $M' : V\left(Q_s^l\right) \to \{F_2, F_3, ..., F_{3l-1}\}$ and the vertices of Q_s^l are labeled with the Fibonacci numbers $F_2, F_3, ..., F_{3l-1}$
i.e.$M'(v_1) = F_2, M'(v_2) = F_3, ..., M'(v_{3l-2}) = F_{3l-1}$
$\Rightarrow M'(v_j) = F_{j+1}$, where $1 \leq j \leq 3l-2$.

M' induces the function $M'' : E^*(Q_s^l) \to N$ is defined by
$M''(xy) = g.c.d\left\{M'(x), M'(y)\right\} \ \forall \, xy \in E'\left(Q_S^l\right)$.
Now, $M''(v_{3j-2}v_{3j-1}) = g.c.d\left(M'(v_{3j-2}), M'(v_{3j-1})\right) = g.c.d\left(F_{3j-1}, F_{3j}\right) = 1$,
$1 \leq j \leq l-1$
Similarly,
$M''(v_{3j-2}v_{3j}) = g.c.d\left\{M'(v_{3j-2}), M'(v_{3j})\right) = g.c.d\left(F_{3j-1}, F_{3j+1}\right) = 1$
$M''(v_{3j}v_{3j+1}) = g.c.d\left(M'(v_{3j}), M'(v_{3j+1})\right) = g.c.d\left(F_{3j+1}, F_{3j+2}\right) = 1$
$M''(v_{3j-2}v_{3j+1}) = g.c.d\left(M'(v_{3j-2}), M'(v_{3j+1})\right) = g.c.d\left(F_{3j-1}, F_{3j+2}\right) = 1$
$\Rightarrow M''(xy) = g.c.d\left(M'(x), M'(y)\right) = 1$, for each edge belongs to $E^*\left(Q_s^l\right)$
Hence Q_s^l exists as FPG.

Example 2.7. Consider the quadrilateral snake graph Q_s^6

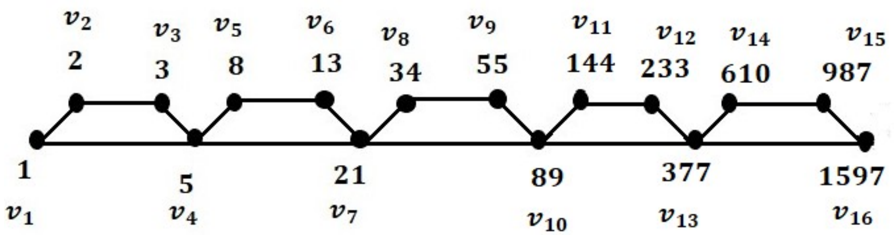

Figure 3: Q_s^6 - FPG

Theorem 2.8. *$D - Q_s^c$ allows FPL*

Proof. Suppose $D - Q_s^c, c \geq 2$ is a double quadrilateral graph,
$V\left(D - Q_s^c\right) = \{w_1, w_2, ..., w_{5c-4}\}$,
$E\left(D - Q_s^c\right) = \{w_{5i-4}w_{5i-3} \cup v_{5i-4}v_{5i-2} \cup w_{5i-1}w_{5i+1} \cup w_{5i}w_{5i+1} \cup w_{5i-2}w_{5i} \cup$
$w_{5i-3}w_{5i-1} \cup w_{5i-4}w_{5i+1}/1 \leq i \leq c-1\}$
are the points and lines of $D - Q_s^c$. Then $p^* = 5c-4$, $q^* = 7c-7$ signifies the

order and size of $D - Q_s^c$

Let us define a function $T^{'} : V\left(D - Q_s^c\right) \rightarrow \{F_2, F_3, ..., F_{5c-3}\}$ and the vertices of $D - Q_s^c$ are labeled with the Fibonacci numbers $F_2, F_3, ..., F_{5c-3}$
i.e. $T^{'}\left(w_1\right) = F_2, T^{'}\left(w_2\right) = F_3, ..., T^{'}\left(w_{5c-4}\right) = F_{5c-3}$
$\Rightarrow T^{'}\left(w_i\right) = F_{i+1}, where\; 1 \leq i \leq 5c - 4.$

Then the another function is exists $T^{''} : E\left(D - Q_s^c\right) \rightarrow N$ is defined by $T^{''}\left(xz\right) = g.c.d\left(T^{'}\left(x\right), T^{'}\left(z\right)\right), \forall xz \in E\left(D_{Q^s}^c\right).$

Now, $T^{''}\left(w_{5i-4}w_{5i-3}\right) = g.c.d\left(T^{'}\left(w_{5i-4}\right), T^{'}\left(w_{5i-3}\right)\right) = g.c.d\left(F_{5i-3}, F_{5i-2}\right) = 1,$
$1 \leq i \leq c - 1$
Similarly,

$$T^{''}\left(w_{5i-4}w_{5i-2}\right) = g.c.d\left(T^{'}\left(w_{5i-4}\right), T^{'}\left(w_{5i-2}\right)\right) = g.c.d\left(F_{5i-3}, F_{5i-1}\right) = 1$$
$$T^{''}\left(w_{5i-1}w_{5i+1}\right) = g.c.d\left(T^{'}\left(w_{5i-1}\right), T^{'}\left(w_{5i+1}\right)\right) = g.c.d\left(F_{5i}, F_{5i+2}\right) = 1$$
$$T^{''}\left(w_{5i}w_{5i+1}\right) = g.c.d\left(T^{'}\left(w_{5i}\right), T^{'}\left(w_{5i+1}\right)\right) = g.c.d\left(F_{5i+1}, F_{5i+2}\right) = 1$$
$$T^{''}\left(w_{5i-2}w_{5i}\right) = g.c.d\left(T^{'}\left(w_{5i-2}\right), T^{'}\left(w_{5i}\right)\right) = g.c.d\left(F_{5i-1}, F_{5i+1}\right) = 1$$
$$T^{''}\left(w_{5i-3}w_{5i-1}\right) = g.c.d\left(T^{'}\left(w_{5i-3}\right), T^{'}\left(w_{5i-1}\right)\right) = g.c.d\left(F_{5i-2}, F_{5i}\right) = 1$$
$$T^{''}\left(w_{5i-4}w_{5i+1}\right) = g.c.d\left(T^{'}\left(w_{5i-4}\right), T^{'}\left(w_{5i+1}\right)\right) = g.c.d\left(F_{5i-3}, F_{5i+2}\right) = 1$$

$\Rightarrow T^{''}\left(xz\right) = \text{gcd}\left(T^{'}\left(x\right)T^{'}\left(z\right)\right) = 1$ for all members of $E\left(D - Q_s^c\right)$.
Thus $D - Q_s^c$ proved as *FPG*.

Example 2.9. Consider the double quadrilateral snake graph $D - Q_s^3$

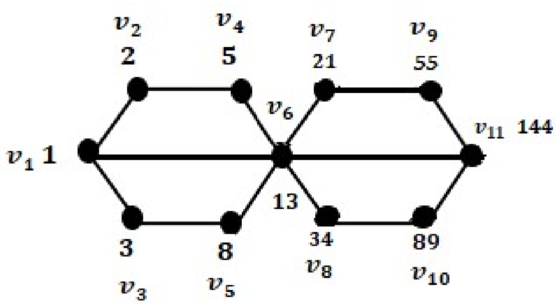

Figure 4: $D - Q_s^3$ - FPG

Theorem 2.10. *$n - P_s^x$ is a FPG*
Proof. Consider the graph $n - P_s^x, x \geq 2, n \geq 5$. The collection of vertices and edges are given below
$V^{''}\left(n - P_s^x\right) = \left\{v^{'}_{(n-1)m-(n-2)} / 1 \leq m \leq x\right\} \cup \left\{v^{'}_{(n-1)m-(n-2)+e} / 1 \leq m \leq x - 1,\right.$
$\left.1 \leq e \leq n - 2\right\}$
Let $E^{''}\left(n - P_s^x\right) = \left\{v^{'}_{(n-1)m-(n-2)}v^{'}_{(n-1)m+1} \cup v^{'}_{(n-1)m-(n-2)+1}v^{'}_{(n-1)m-(n-2)} \cup\right.$

$$v'_{(n-1)m}v'_{(n-1)m+1} \cup v'_{(n-1)m-(n-2)+(e-1)}v'_{(n-1)m-(n-2)+e}/1 \le m \le x-1, \\ 2 \le e \le n-2\}$$

$\left|V''(n-P_s^x)\right| = x(n-1)-(n-2)$, $\left|E''(n-P_s^x)\right| = n(x-1)$ denotes the number of points and lines of $n-P_s^x$

Case 1. If $n = 3, 4$

Suppose $n = 3$

$\Rightarrow V''(3-P_s^x) = \left\{v'_{2m-1}/1 \le m \le x\right\} \cup \left\{v'_{2m-1+e}/1 \le m \le x-1, e = 1\right\}$

$E''(3-P_s^x) = \Big\{v'_{2m-1}v'_{2m+1} \cup v'_{2m}v'_{2m-1} \cup v'_{2m}v'_{2m+1} \cup v'_{2(m-1)+e}v'_{2m+e-1}/$
$1 \le m \le x-1, e = 1\}$

When Substitutes $e = 1$ we get,

$E''(3-P_s^x) = \left\{v'_{2m-1}v'_{2m+1} \cup v'_{2m}v'_{2m-1} \cup v'_{2m}v'_{2m+1} \cup v'_{2m-1}v'_{2m}/1 \le m \le x-1\right\}$

Suppose $n = 4$

$\Rightarrow V''(4-P_s^x) = \left\{v'_{3m-2}/1 \le m \le x\right\} \cup \left\{v'_{3m-2+e}/1 \le m \le x-1, e = 1, 2\right\}$

$E''(4-P_s^x) = \Big\{v'_{3m-2}v'_{3m+1} \cup v'_{3m-2}v'_{3m-1} \cup v'_{3m}v'_{3m+1} \cup v'_{3(m-1)+e}v'_{3m+e-2}/$
$1 \le m \le x-1, e = 1, 2\}$

When Substitutes $e = 1, 2$ we get,

$$\Rightarrow E''(4-P_s^x) = \{v'_{3m-2}v'_{3m+1} \cup v'_{3m-2}v'_{3m-1} \cup v'_{3m}v'_{3m+1} \cup v'_{3m-2}v'_{3m-1} \\ \cup v'_{3m-1}v'_{3m}/1 \le m \le x-1\}$$

Edges are repeated in $3-P_s^x, 4-P_s^x$ Graphs.

Case 2. If $n > 4$

Suppose $n = 5$

$\Rightarrow V''(5-P_s^x) = \left\{v'_{4m-3}/1 \le m \le x\right\} \cup \left\{v'_{4m-3+e}/1 \le m \le x-1, 1 \le e \le 3\right\}$

$E''(5-P_s^x) = \Big\{v'_{4m-3}v'_{4m+1} \cup v'_{4m-2}v'_{4m-3} \cup v'_{4m}v'_{4m+1} \cup v'_{4(m-1)+e}v'_{4m+e-3}/$
$1 \le m \le x-1, 1 \le e \le 3\}$

When Substitutes $e = 1, 2, 3$ we get,

$$E''(5-P_s^x) = \{v'_{4m-3}v'_{4m+1} \cup v'_{4m-2}v'_{4m-3} \cup v'_{4m}v'_{4m+1} \cup v'_{4m-3}v'_{4m-4} \\ \cup v'_{4m-2}v'_{4m-1} \cup v'_{4m-1}v'_{4m}/1 \le m \le x-1\}$$

Edges are not repeated when the value of n is greater than four. Thus the generalized Vertex Set and Edge Set are true, when $n \ge 5$.

Consider the function $X' : V''(n-P_s^x) \to \left\{F_2, F_3, ..., F_{x(n-1)-(n-2)+1}\right\}$

$\Rightarrow X'\left(v'_m\right) = F_{m+1}, where\ 1 \le m \le x(n-1)-(n-2).$

Consequently the function exists $X^* : E''((n-P_s^x) \to N$ is defined as

$X^*(yz) = g.c.d\left\{X'(y), X'(z)\right\} \forall xz \in E''((n-P_s^x).$

$$\text{Now, } X^*\left(v'_{(n-1)m-(n-2)}v'_{(n-1)m+1}\right) = g.c.d\left(X'\left(v'_{(n-1)m-(n-2)}\right), X'\left(v'_{(n-1)m+1}\right)\right)$$
$$= g.c.d\left[x'\left(v'_{(n-1)m-(n-2)}\right), X'\left(v'_{(n-1)m-(n-2)+(n-1)}\right)\right]$$
$$= g.c.d\left[F_{(n-1)m-(n-2)+1}, F_{(n-1)m-(n-2)+(n-1)+1}\right]$$
$$= 1,\ 1 \leq m \leq x-1$$

Similarly, $X^*\left(v'_{(n-1)m-(n-2)+1}v'_{(n-1)m-(n-2)}\right)$

$$= g.c.d\left[\left(v'_{(n-1)m-(n-2)+1}\right), X'\left(v'_{(n-1)m-(n-2)}\right)\right]$$
$$= g.c.d\left[F_{(n-1)m-(n-2)+2}, F_{(n-1)m-(n-2)+1}\right] = 1$$

$$X^*\left(v'_{(n-1)m+1}v'_{(n-1)m}\right) = g.c.d\left[X'\left(v'_{(n-1)m+1}\right), X'\left(v'_{(n-1)m}\right)\right]$$
$$= g.c.d\left\{F_{(n-1)m+1}, F_{(n-1)m+2}\right\}$$
$$= 1$$

$$X^*\left(v'_{(n-1)m-(n-2)+(e-1)}v'_{(n-1)m-(n-2)+e}\right)$$

$$= g.c.d\left[X'\left(v'_{(n-1)m-(n-2)+(e-1)}\right), X'\left(v'_{(n-1)m-(n-2)+e}\right)\right]$$
$$= g.c.d\left\{F_{(n-1)m-(n-2)+e}, F_{(n-1)m-(n-2)+e+1}\right\} = 1$$

$\Rightarrow g.c.d\left[X'(y), X'(z)\right] = 1\ \forall yz \in E''(n-P_s^x)$.

$n-P_s^x$ admits FPL.

Example 2.11. Consider the 8−polygonal snake graph $8-P_s^3$

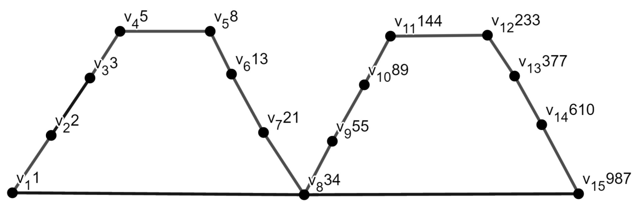

Figure 5: $8-P_s^3$ -FPG

3. Conclusion

In this paper, we proved that some snake graphs are admits Fibonacci Prime Labeling. The Triangular snake graph, Double Triangular snake graph, Quadrilateral Snake graph, Double Quadrilateral snake graph, $n-$Polygonal Snake graph, Double $n-$Polygonal snake graph, Alternate Triangular Snake graph are Fibonacci Prime Graphs.

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