

**A MATHEMATICAL MODEL FOR ENDURING EFFECT OF  
CHILDHOOD MALTREATMENT ON CORTISOL AND HEART  
RATE RESPONSES TO STRESS USING LOG-NORMAL  
DISTRIBUTIONS**

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**Abstract:** In this study, we implement the characteristics of hazard rate function of log-normal distribution model to analyze the impact of childhood maltreatment on later mental health problems and behavioural difficulties. Maltreated participants had higher cortisol responses to stress in comparison to controls. Finally, we conclude that the application part overlaps with a mathematical model. In the future, this paper will be advantageous in the medical field.

**Keywords and Phrases:** Log-normal distribution, Hazard rate function, CTQ, HR.

**2020 Mathematics Subject Classification:** 62Hxx, 62N05, 90B25.

## **1. Introduction**

Survival analysis is used to analyse data like clinical health for example, occurrence of sickness, recurrence of disease, recovery and death [12, 13]. A few of the points that is important to evaluate is the hazard scale, namely the probability density function ratio (pdf) and the survival function ( $s(t)$ ) [16, 17].

The log-normal distribution is the probability of a continuous random variable that has been converted from a normal distribution [3, 22]. Log-normal distribution can be applied in many fields of analysis for instantaneous hydrology, and can be

used to analyse extreme daily, weekly or annual rainfall values [2, 23, 24]. In addition, [19-21] the normal log distribution can also be used to model device management.

## 2. Mathematical Model and Assumptions

Log normal distribution is given [5, 14] as a positive integer  $T$  with a region of  $Rt = \{t|0 < t < a\}$  and  $Y = \ln T$  has a normal distribution with a mean  $\mu$  and a variance of  $\sigma^2$ . The probability distribution form of the random variable log-normal with the scale parameter  $\mu > 0$  and the shape parameter  $\sigma > 0$  is as continues to follow:

$$f(t) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma t}} \exp \left[ -\frac{1}{2} \left( \frac{\ln t - \mu}{\sigma} \right)^2 \right], & \text{for } t > 0 \\ 0, & \text{Otherwise} \end{cases} \quad (1)$$

The average and the variability will be as follows:

1.  $E(t) = \mu_t = \exp \left( t + \frac{\sigma^2}{2} \right)$
2.  $Var(t) = (e^{(2\mu + \sigma^2)})(e^{\sigma^2} - 1)$

The  $r^{th}$  moment of log-normal distribution [4] is:

$$\mu_t(r) = E [T^r] = \exp(r\mu + \frac{1}{2}r^2\sigma^2) \quad (2)$$

and the cumulative distribution function of log-normal is [11]:

$$F(t) = \varphi \left[ \frac{\ln t - \mu}{\sigma} \right], t \in (0, \infty) \quad (3)$$

Where  $\varphi$  is a cumulative distribution function from normal distribution. The survival function is defined [14]:

$$S(t) = 1 - F(t) \quad (4)$$

So that, the survival function of log-normal distribution is

$$S(t) = 1 - \varphi \left[ \frac{\ln t - \mu}{\sigma} \right] \quad (5)$$

And the hazard function of log-normal distribution is as follows:

$$h(t) = \frac{f(t)}{s(t)} \tag{6}$$

$$h(t) = \frac{\frac{1}{\sqrt{2\pi\sigma t}} e^{\frac{1}{2}\left(\frac{Int-\mu}{\sigma}\right)^2}}{1 - \varphi\left[\frac{Int-\mu}{\sigma}\right]} \tag{7}$$

The initial derivative of the pdf will be used to find the value of  $\eta(t)$ . The pdf for log-normal distribution is:

$$f(t) = \frac{1}{\sqrt{2\pi\sigma t}} e^{-\frac{1}{2}\left(\frac{Int-\mu}{\sigma}\right)^2} \tag{8}$$

To discover the derivative of the pdf of log-normal distributions, we can use the multiplicative rule:

$$f'(t) = u'v + uv' \tag{9}$$

so we have,

$$f'(t) = \frac{1}{\sqrt{2\pi\sigma t}} e^{-\frac{1}{2}\left(\frac{Int-\mu}{\sigma}\right)^2} \left[ -\frac{1}{t} - \frac{1}{\sigma^2 t} (Int - \mu) \right] \tag{10}$$

### 3. Applications

Relative consensus exists on the effects of childhood maltreatment on mental, behavioural and social functioning in childhood and adolescence [7, 8, 9]. Observational studies have also demonstrated that the detrimental impact of sexual abuse can continue in adolescence and effect several areas of society, like wellbeing and health, interpersonal relationships, work and serious acts [10, 11]. The hypothalamic-pituitary-adrenal (HPA) axis and the Sympathetic Nervous System (SNS) are hypothesised to play a key role in the interaction of early hardship and wellbeing [4, 15]. Although a flattened pattern of diurnal cortical activity has been recorded in children living with some accuracy in both maximum and minimum basal levels of cortisol [6, 18].

We welcomed respondents to participate in our research, which lasted about 3 hr 30 min. During this time, the members participated in the Trier Social Stress Test (TSST), a well-established, structured tension technique that creates an economic hazard by exposing the participants to a 5-minute simulated work interview in front of the “Panel of Clinical Experts”, treated with 5 minutes of simple arithmetic. Both the “panel-in” and “panel-out” approaches have been seen to generate reliable cortisol reactions in science laboratories [1]. The TSST was held in the early afternoon with all members. The CTQ-SF examines knowledge about mental,

physical and emotional violence and negligence that happened prior to age 18 and was chosen on the basis of its proven significance in population surveys. Cortisol was assessed by collecting five samples of saliva using passive drool.

Figure 1

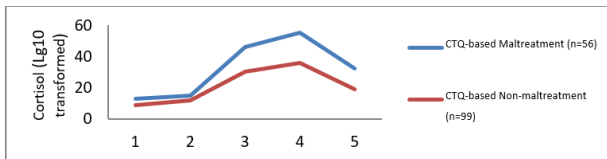


Figure 2

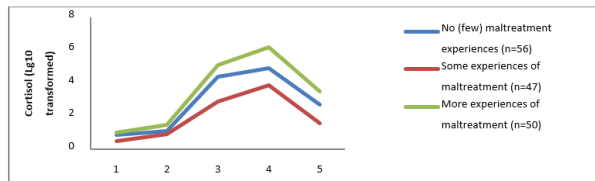


Figure 3

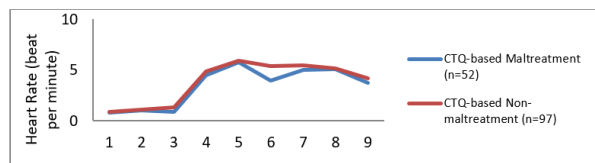


Figure 4

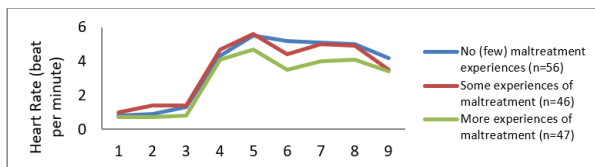


Figure 1 indicates the trends of release for the respondents for which the condition met the CTQ level for sexual abuse and for whom the experience did not meet

the CTQ threshold. The CTQ-based abused subjects had greater TSST reactions to cortisol.

Figure 2 indicates that although the three classes had comparable early levels of cortisol, participants who indicated more experience of sexual abuse had greater rate of cortisol to the TSST relative to the participants. As recorded in the Web Supplementary, respondents indicated, on average, a substantial improvement in HR and during TSST, that did not differ according to the CTQ-based child abuse categories (Figure 3). In comparison, no variations in HR could be observed depending on the nature of the maltreatment at the primary stage (Figure 4).

The current study looked at the relationship among childhood trauma, cortisol, and HR reactions to psychological factors in young adult. In accordance with previous observations, we observed signs of both primary and secondary cortisol reactions to stress in the sense of maltreatment.

#### 4. Mathematical Results

Figure 5

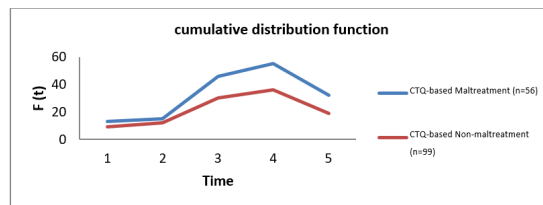


Figure 6

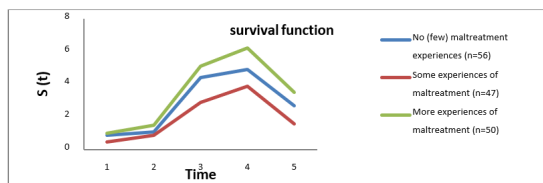


Figure 7

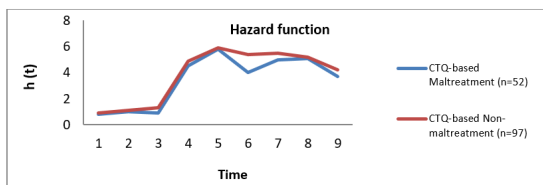
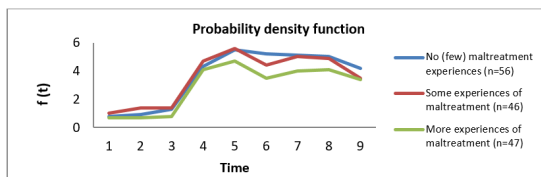


Figure 8



## 5. Conclusion

In this article, the log-normal distribution was used to analyse the impact of childhood maltreatment on cortisol and heart rate reactions to stress. For chosen medical results, we map the probability density function, cumulative distribution function, probability survival function, and hazard rate function of log normal distribution. It may be concluded that the properties of the log-normal distribution model are well adapted for the statistical study of medical results. It seems to be helpful to medical professionals.

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