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BIANCHI TYPE-V BULK VISCOUS UNIVERSE WITH CONSTANT DECELERATION PARAMETER

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Abstract: In this article, we have obtained some exact solutions of the Einstein field equations with time-varying in existence of bulk viscosity in Bianchi type-V cosmological models. We have taken cosmic matter which follows a bar tropic equation of state. We see that the role of deceleration parameter (q) is important to describe the different phases of the universe. Finally, we conclude that universe decelerates due to positive value of q while universe accelerates due to negative value of q. Some physical properties and behaviors of parameters on the solutions of field equations are discussed in this paper.

Keywords and Phrases: Bianchi Type–V, Constant Deceleration Parameter, Hubble's parameter, cosmological constant.

2020 Mathematics Subject Classification: 83C15, 83F05.

1. Introduction

In general theory of relativity, "cosmological models" are generally established beneath the postulation that the matter content of the cosmos is sufficiently explained by a perfect fluid. The review of relativistic cosmological models generally has the energy-momentum tensor of matter by a perfect fluid. Now days, the "cosmological constant" problem is one of the serious parameter to study about universe in cosmology [19, 35]. In the circumstance of "quantum theory", "cosmological term" links to the "energy density of vacuum" and measures the energy of empty space which offers a repulsive force opposes the gravity between the galaxies. The energy represents the counts as mass by "Einstein's mass energy equivalent formula" which is used to energy counts of mass. But current studies recommend that the "cosmological term has a very small value of the order of 10^{-58} cm^{-2} [3]". Linde [13] has recommended that Λ is the function of temperature and is associated to the spontaneous symmetry breaking procedure. So, it possibly will be a function of time in a "spatially homogeneous expanding universe [36]". The newest measurements of the "Hubble's parameter [6, 22]" argument to an fundamental weakness of the "standard (photon conserving) FRW cosmology" in such a way that models without a "cosmological constant" give the impression to be effectively ruled out [2, 14]. In the evaluation of "Hubble's constant (H_0)", without a cosmological constant it is difficult to lead for the "FRW models" to the time of cosmos larger than that of stars [5, 7]. Many more researchers are attracted to work upon realistic models with viscosity. Misner [16, 17] recommended that the dissipation due to "neutrino viscosity" may significantly decrease "the anisotropy of the black body radiation". The anomalously high entropy per baryon can be described by the viscosity mechanism in cosmology [37, 38]. Bulk viscosity connected with the grand unified theory phase transition [12], may show the way to an inflationary scenario [9, 20, 34]. Murphy [18] investigated a homogeneous and isotropic model filled with a fluid with pressure and bulk viscosity. The solution given by him hints to the decision that the "Big Bang singularity" occurs in the past. Santos et al. [30] have founded the solutions for "open, closed and flat universe" under the postulation that "the bulk viscosity (ζ) is a power function of energy density (ρ)".

The purpose of this manuscript is to study and look into "Bianchi type -V universe" in view of bulk viscous fluid distribution in principal of relativity. Here accurate solution of Einstein field equ. has been founded by formulating a law for "Hubble's parameter (H)" that produces a constant value of "deceleration parameter (q)". Some physical properties and behaviors of parameters on the solutions of field equations are also explained.

2. Model with Field Equations

Let us regard as the homogeneous & isotropic Bianchi type-V universe represented by

$$ds^2 = -dt^2 + A^2dx^2 + e^{2mx}(B^2dy^2 + C^2dz^2), \tag{1}$$

where A, B, C are metric functions of time t, m is a constant.
The "energy momentum tensor" for cosmic matter having bulk viscous fluid is

assumed as

$$T_i^j = (\rho + \bar{p})u_i u^j + \bar{p}\delta_i^j, \tag{2}$$

where

$$\bar{p} = p - \zeta u^i_{;i}. \tag{3}$$

Here $\rho, p, \bar{p}$ and ζ are the energy density, isotropic pressure, effective pressure and bulk viscous coefficient respectively and u_i is four velocity vector of the fluid satisfying the relation

$$u_i u^i = -1 \tag{4}$$

We assume that the "non-vacuum component of matter" obey the "equation of state" as

$$p = \omega\rho, \qquad \text{where} \;\; 0 \leq \omega \leq 1. \tag{5}$$

The Einstein field equations are

$$R_i^j - \frac{1}{2}R\delta_i^j = -8\pi G T_i^j + \Lambda(t)\delta_i^j, \tag{6}$$

where G, R_i^j and R are the gravitational constant, Ricci tenser and Ricci scale factor respectively.
Cosmological term Λ is considered to vary as

$$\Lambda = \frac{\lambda}{R} \qquad (\text{where} \;\; \lambda \text{ is constant}). \tag{7}$$

In the "co-moving system of coordinates", the field equation (6) for the line element (1) and matter allocation (2) are given by

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\lambda^2}{A^2} = -8\pi G\bar{p} + \Lambda. \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\lambda^2}{A^2} = -8\pi G\bar{p} + \Lambda. \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\lambda^2}{A^2} = -8\pi G\bar{p} + \Lambda. \tag{10}$$

$$\frac{\dot{A}\dot{B}}{A} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\lambda^2}{A^2} = 8\pi G\rho + \Lambda. \tag{11}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{2\dot{A}}{A} = 0. \tag{12}$$

3. Solution of Field Equations

The "average scale factor R" is given by

$$R^3 = ABC. \tag{13}$$

$$\therefore \quad \frac{3\dot{R}}{R} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{14}$$

Using equation (13) in equations (8)-(12), we have

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R} \tag{15}$$

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R} - \frac{\lambda_1}{R^3} \tag{16}$$

$$\frac{\dot{C}}{C} = \frac{\dot{R}}{R} + \frac{\lambda_1}{R^3}, \tag{17}$$

where λ_1 is a constant of integration.
Integrating (15), (16) and (17), we get

$$A = k_1 R, \; B = k_2 R \exp\left[-\lambda_1 \int \frac{dt}{R^3}\right] \quad \text{and} \quad C = k_3 R \exp\left[\lambda_1 \int \frac{dt}{R^3}\right], \tag{18}$$

where k_1, k_2, k_3 are constants of integration, satisfying the relation $k_1 k_2 k_3 = 1$ for $k_1 = 1$, $k_3 = k_2^{-1}$.
Then equation (18) reduces to

$$A = R, \quad B = k_2 R \exp\left[-\lambda_1 \int \frac{dt}{R^3}\right], \quad C = k_2^{-1} R \exp\left[\lambda_1 \int \frac{dt}{R^3}\right]. \tag{19}$$

In resemblance with FRW model, we introduce a "generalized Hubble's parameter (H)" and "generalized deceleration parameter (q)" as

$$H = \frac{\dot{R}}{R}. \tag{20}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{21}$$

Anisotropy parameter $\bar{A}$ is given by

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right). \tag{22}$$

Volume expansion (θ) & shear scalar (σ) are following

$$\theta = u^i_{;i}, \qquad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}. \tag{23}$$

Here semicolon stands for covariant differentiation and σ_{ij} being shear tensor. For the Bianchi Type -V universe, θ and σ_i^j can be written as

$$\theta = \frac{3\dot{R}}{R} = 3H. \tag{24}$$

$$\sigma^2 = \frac{\lambda_1^2}{R^6}. \tag{25}$$

Using H, σ and q, equations (8)-(9) can be recast as

$$8\pi G\bar{p} - \Lambda = \frac{m^2}{R^2} + H^2(2q-1) - \sigma^2. \tag{26}$$

$$8\pi G\rho + \Lambda = \frac{-3m^2}{R^2} + 3H^2 - \sigma^2. \tag{27}$$

From equation (27), we observe that

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{3m^2}{R^2\theta^2} - \frac{\Lambda}{\theta^2}, \tag{28}$$

therefore $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$.
Here, we hope that the expansion of the universe will tend to accelerate for positive value of Λ whereas in negative value of Λ, the expansion will slow down [23]. From (23) and (24), we obtain

$$\frac{d\theta}{dt} = -4\pi G(\rho + 3\bar{p}) - 2\sigma^2 - \frac{\theta^2}{3} + \Lambda \tag{29}$$

This is the Raychaudhari equation.
Therefore for $\Lambda \leq 0$ and $\zeta = 0$ the cosmos shall always be in shrinking phase provided the physically powerful energy state holds [24, 25, 26]. For this, we take

$$\frac{d\theta}{dt} \leq -\frac{1}{3}\theta^2 \tag{30}$$

On integration to gives

$$\left(\frac{1}{\theta}\right) \geq \left(\frac{1}{\theta_0} + \frac{t}{3}\right) \tag{31}$$

Here θ_0 is initial value.
If $\theta_0 < 0$ in starting, v will move away i.e. $\theta \to -\infty$ for $t \leq \left(\frac{3}{|\theta_0|}\right)$. A positive value of Λ will hold the rate reeducation [21, 27, 28]. Also $\dot{\sigma} = -3\sigma H$ suggesting that σ shrinkages in surfacing cosmos and for $R \to \infty, \sigma$ becomes insignificant. If we confine about dust only, then from (26) and (27), we have

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{m^2}{R^2} + \frac{k_1^2}{R^6} - \Lambda = 0$$

and

$$\frac{3\dot{R}^2}{R^2} = 8\pi G\rho + \frac{3m^2}{R^2} + \frac{k_1^2}{R^6} + \Lambda \tag{32}$$

We find that for $\Lambda \geq 0$, every term in the right hand of equation (32) is non-negative. Thus, there is no change in sign of $\dot{R}$ and we obtain ever-expanding cosmological models [1, 8, 15]. On the other hand, we can develop cosmos that expand and then re-contract for $\Lambda < 0$. Also $\rho < \rho_c$ for $\Lambda \geq 0$, where ρ_c is the "critical density" and defined as $\rho_c = \frac{3H}{8\pi G}$.
This minds that the "energy density" of all realized type of considerable substance is positive. In the early stage of a development if the "average scale factor" was nearly zero and the "energy density" of the cosmos is supposed to be very huge. Through "expansion phase of the universe", "the energy density" shrinkages and $R \to \infty$, correspond to an energy density (ρ) which is nearly zero.

4. Deceleration Parameter

The "deceleration parameter q" has a very serious role to explain the "dynamics of the universe" surrounded by the material quantities of interest in cosmology. As per the prediction, the cosmos at current is "decelerating" is conflicting to current observation of the elevated red shift of the "type I_a supernovae [21, 29]". The current observations expose that instead of slowing down; the "expanding universe" is fast-moving up. The cosmological models with "constant deceleration parameter" have been established the attention of many researchers [29, 31, 32, and 33] and it is pivot man recently.

Variation of "law for Hubble's parameter" has been initiated to study by many cosmologists [3, 4, 11] which was consistent with observations in beginning. We define a "law for variation of the Hubble's parameter" that produces constant value of "deceleration parameter".
Let us consider "Hubble's Parameter (H)" as

$$H = kR^{-n}, \tag{33}$$

provided $k > 0$ and $n \geq 0$ are constants.
In view of "Hubble's parameter, the deceleration parameter q" becomes constant i.e.

$$q = n - 1 \tag{34}$$

For $n > 1$, we find that the model reflects a "decelerating universe" and $n < 1$ relates to "accelerating phase of the universe". If $n = 1$, we get $H = 1/t$ and q is 0. Since $q = 0$ in the cosmological model, each galaxy moves with invariable speed. So for $n = 1$, we get well anisotropic "Milne model". We come to be $H = k$ and $q = -1$ for $n = 0$. We note that the "Hubble parameter being a large scale property of the universe" is constant at time t and so equal to present value Ho. So, the notable remark represent "accelerating phase of universe".
Integrating equation (33), we get

$$R = \{nk(t + t_1)\}^{\frac{1}{n}}, \quad \text{for} \;\; n \neq 0, \tag{35}$$

$$R = e^{k(t-t_0)}, \quad \text{for} \;\; n = 0, \tag{36}$$

where t_0 and t_1 are constants.
Using equation (35), equation (19) is taken the form

$$A = \{nk(t + t_1)\}^{\frac{1}{n}}. \tag{37}$$

$$B = k_2\{nk(t + t_1)\}^{\frac{1}{n}} \exp\left[\frac{-\lambda_1}{k(n-3)}\{nk(t + t_1)\}^{\frac{n-3}{n}}\right]. \tag{38}$$

$$C = k_2^{-1}\{nk(t + t_1)\}^{\frac{1}{n}} \exp\left[\frac{\lambda_1}{k(n-3)}\{nk(t + t_1)\}^{\frac{n-3}{n}}\right]. \tag{39}$$

Then line element (1) assumes the form

$$ds^2 = -dT^2 + (nkT)^{\frac{2}{n}}\left[dX^2 + \exp\left\{2mX - \frac{2\lambda_1}{k(n-3)}(nkT)^{\frac{n-3}{n}}\right\}dY^2 + \exp\left\{2mX + \frac{2\lambda_1}{k(n-3)}(nkT)^{\frac{n-3}{n}}\right\}dZ^2\right]. \tag{40}$$

Using equation (36)in equation (19), we have

$$A = \exp\{k(t - t_0)\}$$

$$B = k_2 \exp\{k(t - t_0) + \frac{\lambda_1}{3k}e^{-3k(t-t_0)}\}$$

$$C = k_2^{-1} \exp\{k(t - t_0) - \frac{\lambda_1}{3k}e^{-3k(t-t_0)}\}$$

Then metric (1) takes form

$$ds^2 = -dT^2 + e^{2kT}\left[dX^2 + \exp\left\{2mX + \frac{2\lambda_1}{k(n-3)}e^{-3kT}\right\}dY^2 + \exp\left\{2mX + \frac{2\lambda_1}{3k}e^{-3kT}\right\}dZ^2\right]. \tag{41}$$

5. Discussion

In the view of cosmological model (40), isotropic pressure (p), effective pressure ($\bar{p}$) and energy density (ρ) are

$$8\pi G\bar{p} = \frac{m^2}{(nkT)^{2/n}} - \frac{\lambda_1^2}{(nkT)^{6/n}} + \frac{\lambda^2}{(nkT)^{1/n}} + \frac{2n-3}{(nT)^2}. \tag{42}$$

$$8\pi G\rho = \frac{-3m^2}{(nkT)^2} + \frac{3}{(nT)^2} - \frac{\lambda^2}{(nkT)^{1/n}} - \frac{\lambda_1^2}{(nkT)^{6/n}}. \tag{43}$$

$$8\pi Gp = w\left\{\frac{3}{(nT)^2} - \frac{3m^2}{(nkT)^{2/n}} - \frac{\lambda_1}{(nkT)^{6/n}} - \frac{\lambda^2}{(nkT)^{1/n}}\right\}. \tag{44}$$

The bulk viscous coefficient ζ is given by

$$\zeta = \frac{3(1+w)-2n}{nT} - \frac{m^2(1+3w)}{k(nkT)^{(2/n)-1}} + \frac{\lambda_1^2}{k(nkT)^{(2/n)-1}} + \frac{\lambda^2(1+w)}{k(nkT)^{(2/n)-1}}. \tag{45}$$

Expansion scalar (θ), shear scalar (σ) and cosmological term (Λ) are given by

$$\theta = \frac{3}{nT}. \tag{46}$$

$$\sigma = \frac{\lambda_1}{(nkT)^{3/n}}. \tag{47}$$

$$\Lambda = \frac{\lambda}{(nkT)^{1/n}}. \tag{48}$$

The anisotropy parameter $\bar{A}$ is

$$\bar{A} = \frac{2\lambda_1^2}{3k^2} + (nkT)^{2(n-3)/n}. \tag{49}$$

For the cosmological model (41), $p, \bar{p}$ and ρ are given by

$$8\pi G\bar{p} = m^2e^{-2kT} - \lambda_1^2e^{-6kT} + \lambda^2e^{-kT} - 3k^2. \tag{50}$$

$$8\pi Gp = 3k^2 - \lambda^2e^{-kT} - 3m^2e^{-2kT} - \lambda_1^2e^{-6kT}. \tag{51}$$

$$8\pi G\rho = w\left\{3k^2 - \lambda^2e^{-kT} - 3m^2e^{-2kT} - \lambda_1^2e^{-6kT}\right\}. \tag{52}$$

And the bulk viscous coefficient is

$$8\pi G\zeta = (1+w)k - \frac{m^2(1+3w)e^{-2kT}}{3k} - \frac{(1+w)\lambda^2 e^{-kT}}{3k} + \frac{(1-w)e^{-6kT}}{3kT}. \quad (53)$$

Expansion scalar θ, shear scalar σ and cosmological term Λ are given by

$$\theta = k. \quad (54)$$

$$\sigma = \lambda_1 e^{-3kT} \quad (55)$$

$$\Lambda = \lambda^2 e^{-kT} \quad (56)$$

The anisotropy parameter $\bar{A}$ is

6. Conclusion

We observe that the model (40) is valid for $n \neq 0$ and $n \neq 3$. This model keeps singularity when $T = 0$. The expansion of the model starts with a "Big bang" from its "singular state" and keeps up to expand till $T = \infty$. At $T = 0, \rho, p, \bar{p}, \Lambda, \zeta, \theta$ and σ are all infinite. For infinitely large $T, \rho, p, \theta, \Lambda, \zeta$ and σ are all zero. We also observe that anisotropy parameters $\bar{A} \to 0$ as $T \to \infty$ for $n < 3$. Therefore, the model approaches isotropy for large values of T. Similarly for model (41), it is found that the model involve no "singularity". It begins evolving at $T = 0$ with $\rho, p, \zeta, \theta, \sigma, \bar{A}$ and Λ all finite. The "expansion scalar θ" is constant in this journey of the evolution of the cosmos and so the model, shows uniform expansion. As $T \to \infty$, effective pressure, isotropic pressure, energy density and bulk viscous coefficient become constant whereas $\bar{A}$ and σ become zero. So the model approaches isotropy for large values of T. We note that in the model, "Hubble's parameter" having large scale property of the cosmos is constant with time. From some year increasing signals have been found in favor of the present "accelerated expansion of our universe". Recent observation of type Ia supernovae [21, 29] powerfully suggest this acceleration. In this model (41), we yield $q = -1$ for $n = 0$. Thus the model characterizes an accelerating universe. The models given in this article possibly will provide a suitable explanation of the "evolution of universe". Finally, we conclude in view of discussions and results that the Bianchi Type-V model is consistent with the cosmological observations.

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