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A NOTE ON DIGITAL SEQUENCE HYPERGRAPHS AND 2-GRAPH CONGRUENCE ARITHMETIC

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Abstract: In this paper we present some aspects of hypergraphs, related to digital arithmetic. It leads us to the notion of digital infinite sequence hyperedges on finite decimal digits vertex sets which responsively exposes hypergraphic presentation of division algorithm leading to most common notion of Greatest Common Divisor. The notion of so-called digital hyperstar reveals some interesting phenomena of hypergraphic approach of division algorithm, in particular, the remainder and dividend petals invariant property together with hypergraphic illustrative approach of Euclidean algorithm. Finally, the paper discusses as a particular case of hypergraph viz. 2-graph exposure of congruence arithmetic using the operations addition and multiplication separately giving rise to similar pattern of complete directed graphs of four vertices using two arcs. Also the same concept leads to the notion of incidence matrix giving weightage on the resultant sum of weights on arcs leading to an induced complete directed graph etc. together with the cancellation property of congruence relation.

Keywords and Phrases: Hypergraph, Digital sequence, Division Algorithm, Hyperstar, Congruence modulo.

2020 Mathematics Subject Classification: 68R10, 94C15.

1. Introduction

In many computer applications, division is less frequently used than the other three operations. As a result, some microprocessors designed for digital signal processing (DSP) or embedded processor applications do not have a divide instruction.

They also usually omit floating point support as well. Division is considered as the most complicated process and can consume resources. Kaplan [11] has dealt with, in his code generation work, a division function which is a basic binary division function. He has included in some reference on higher performance algorithms. His integer division algorithm includes a so called radix 2 division algorithm. One computation state is needed for each binary digit. There are radix 4, 8, 16 and 256 algorithms which are faster but more difficult to implement. Some high radix division cited are as follows: (1) Digital computer Arithmetic (2) Computer Arithmetic (3) High radix division with approx. Quotient estimation. Also it is explained that [11] division is the process of repeated subtraction. Like the long division we learned in grade school, a binary division algorithm works from the high order digits to the low order digits and generates(division result) with each step. The division algorithm is divided into two steps: (a)Shift the upper bits of the dividend (the no. we are dividing into) into the remainder. (b) Subtract the divisor from the value in the remainder. The high order bit of the result become a bit of the quotient (div. result) [11].

What has been described above is the principal motivation towards our approach keeping in note the graph theoretic exposition of such algorithms. Hypergraphs are generalisations of graphs, sometimes known as 2-graphs. These graphs are widely studied by researchers like Berge [3, 4, 5], Ausiello [1], Bretto [6] etc. Hypegraphs model many practical problems in different branches of sciences thus appearing as a very useful tool to resolve various optimization problems. However keeping in note of its pure mathematical vigour mathematicians like Erdos [8], [9] and others have already delved into various types of number theoretic aspects of graph and hypergraph theories. Now, during this digital age, there is ample scope of coordinating various types of digital sequence of numbers of decimal and binary systems. Using this digital sequence hypergraph, it may, with great expectation lead to the area of cryptographic advancements etc. Keeping all these in note, in this paper, our endeavour would be to highlight the connection between some number theoretic notions and hypergraph together with 2-graphs in particular.

2. Preliminaries

Definition 2.1. *(Hypergraph) A hypergraph is a pair $H = (V, E)$, where $V = \{v_1, v_2, ..., v_n\}$ is the set of vertices(or nodes) and $E = \{E_1, E_2, ..., Em\}$ with $E_i \subseteq V$ for $i = 1, 2, ..., m$ is the set of hyperedges* [9].

Clearly, when $|E_i| = 2, i = 1, 2, \ldots, m$ the hypergraph is a standard graph. While the size of a standard graph is uniquely defined by n and m, the size of a hypergraph depends also on the cardinality of its hyperedges; we define the size of

H as the sum of the cardinalities of its hyperedges;

$$size(H) = \sum_{E_i \in E} |E_i|$$

The notions of hyperarc, directed hypergraph etc. are suggested by Gallo et. al. [10] as follows.

Definition 2.2. *(Hyperarc) A directed hyperedge or a hyperarc is an ordered pair $E = (X, Y)$ with $X, Y \subseteq V$, $X \cap Y = \phi$, where X is the tail of E while Y is its head.*

Definition 2.3. *(Directed Hypergraph) A directed hypergraph H is a hypergraph with directed hyperedges as hyperarcs. As an extension of the above notions, we would like to redefine our relevant corresponding notions to suit our purpose.*

Definition 2.4. *(Sequence Hyperedge) In a hypergraph $H = (V, E)$ an edge $E_i (\subseteq E)$ is a sequence hyperedge if E_i is of the type $< v_i > = < v_1, v_2, ..., v_n >$ (an ordered figure set or a finite sequence).*

Definition 2.5. *(Sequence Hyperarc or a Petal) A sequence directed hyperedge or a sequence hyperarc(or a petal) is a hyperarc $E = (X, Y)$, where each of X and Y is a finite sequence of vertices in V.*

The following discussion paves the track or way towards the justification of our above definitions. We now note the decimal representation of our well known number system. For this system we usually take ten digits viz. $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ to denote any number available. Here digits in consideration play their respective place value and in this sense a number is expressible in the form $x = 10^n a_n + 10^{n-1} a_{n-1} + ... + 10a_2 + 10a_1$ where $a_i \in \{0, 1, 2, ..., 9\}$ and the usual decimal presentation of x is chosen as $a_n, a_{n-1}, ..., a_1$ or, to make it free from any misreading one may also denote it as $\underline{a_n}\ \ \underline{a_{n-1}}...\ \ \underline{a_1}$. We agree to denote such a number in sequence form as $< a_n, a_{n-1}, ..., , a_1 >$. In our discussion our vertex set of hypergraph would consist of 10 vertices labelled as $0, 1, ..., 9$ the usual decimal digits. Now, we introduce the following notions:

Definition 2.6. *(Sequence directed hypergraph) A sequence directed hypergraph is a hypergraph $H = (V, E)$ where E contains a class of finite sequences of the type $E_i = < a_i >$.*

As we are concerned in our discussion with the above type of sequence presentation of decimal digits we write such presentation as a digital sequence of numbers. We have already mentioned that the decimal expression of a number is of the type,

$$a = 10^n a_n + 10^{n-1} a_{n-1} + ... + 10a_2 + a_1$$

And if we denote a_r for the rth digit in the above number, then the decimal expression for a appears as

$$a = \underline{a_n}\ \underline{a_{n-1}} ...\ \underline{a_1}$$

We agree to denote its digital sequence hyperarc form as

$$< a_n, a_{n-1}, ..., a_1 >$$

We now define magnitude of this hyperarc as,

$$| < a_n, a_{n-1}, ...a_1 > | = \underline{a_n}\ \underline{a_{n-1}} ...\ \underline{a_1} = 10^n a_n + 10^{n-1} a_{n-1} + ... + 10a_2 + a_1$$

Definition 2.7. *(Distinct digital sequence) Two digital sequences* $X =< x_n, ..., x_1 >$ *and* $Y =< y_m, ..., y_1 >$ *are said to be distinct if* $|X| \neq |Y|$ *and we denote it by* $X \cap Y = \phi$ *[Being different from usual set empty-intersection].*

For example, the digital sequences $< 1, 2, 3 >$ and $< 3, 1, 2 >$ are distinct as $123 \neq 312$. In this sense, $< 1, 2, 3 > \cap < 3, 1, 2 >= \phi$. Keeping this in note we introduce the notion of a digital sequence directed hypergraph.

Definition 2.8. *(Digital sequence directed hypergraph) A digital sequence directed hypergraph is a sequence directed hypergraph* $H = (V, E)$ *with* $E_i (\in E)$, $E_i = X \cap Y$ *where each of* X *and* Y *is a digital sequence with* $|X| \neq |Y|$ *and* $V = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Definition 2.9. *(Digital hyperstar) A digital hyperstar* $H(< d_1, d_2, ...d_t >)$ *centered at* $< d_1, d_2, ...d_t >$ *is the family of digital sequence hyperarcs (or digital sequence directed hyperedges)* $\{E_j\}, j \in J$ *with degree* $d(< d_1, d_2, ..., d_t >)$*=no. of digital sequence hyperarcs.*

3. Graph theoretic aspects of congruence modulo n

Definition 3.1. *(Congruence modulo n):* [7] *Let* $a, b, n \in \mathbb{Z}$, $n \geq 0$. *Then,* $'a'$ *is said to be congruent to* $'b'$ *modulo* $'n'$ *if and only if* $n|a - b$ *and is written as* $a \equiv b(mod\ n)$. *To make* $a \equiv b(mod\ n)$ *graph theoretically distinct from* $b \equiv a(mod\ n)$ *we use the notion of directed edge or arc as follows.*

If $a \equiv b(mod\ n)$ and $b \equiv a(mod\ n)$ then the corresponding arc (a, b) respectively are as follows:

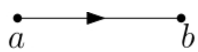


Figure 1

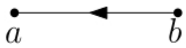


Figure 2

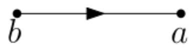


Figure 3

The arc representation for $b \equiv a(mod\ n)$ can also depicted as,

Here, $'a'$ is termed as the tail of the arc if the arc is $e = (a, b)$ and is denoted by $T(e)$.

And $'b'$ is termed as the head of the arc and is denoted by $H(e)$. Our discussion here will be the vertex set V with infinite number of elements viz. the set of integers. For a positive integer n, an integer b is adjacent a if and only if

$$a \equiv b(mod\ n)$$

And the corresponding arc is denoted as $e = (a, b)$.

Definition 3.2. *(Incidence value of a vertex at an arc)* [2, 10] *For directed graph, the incidence value v at e, denoted by $i(v/e)$ for an arc e is defined as*

$$i(v/e) = \begin{cases} -1 & if\ v\ =\ T(e) \\ 1 & if\ v\ =\ H(e) \\ 0 & if\ v \notin e\ or\ is\ a\ loop \end{cases}$$

We agree to write the directed arc $e = (a, b)$ or as
$e =$

$$\begin{pmatrix} a \\ b \end{pmatrix}$$

and the incidence matrix for this is

	$i(v/e)$
a	-1
b	1

Figure 4

Definition 3.3. *(Weight of an arc at a vertex) The weight of a congruence arc $e = (a, b)$ at the head or tail is denoted by $w(e/a) = w(e/b) = k$, where k can be* *chosen according to the context of discussion.*

Remark 3.1. *To introduce a weight to a congruence arc $e = (a, b)$ at the vertex $'a'$ or $'b'$ so that it resembles a close connection with congruence modulo n, the nearest natural way of defining it would be $w(e/a) = w(e/b) = |k|$, where k is obtained from the fact that $a - b = nk$ for some integer k. The existence of such a weight is possible only if it forms a congruence modulo n arc. If no such congruence relation is known between any two vertices, then we consider the weight between these two vertices to be 0. It is to be noted that for $n = 1$, we always have a complete graph for any set of vertices and $|a - b|$ would be the weight of the arc joining the vertices a and b.*

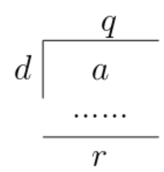
Definition 3.4. *(Complete directed graph) Complete directed graphs are simple directed graphs where each pair of vertices is joined by a symmetric pair of directed arrows. It is equivalent to an undirected complete graph with the edges replaced by pairs of inverse arrows.*

4. Results and Illustrations

4.1. Hypergraphic approach of Division Algorithm

Definition 4.1. *(Division Algorithm): For an integer a, and an integer $d \geq 0$, there exist two integers q and r such that $a = dq + r, 0 \leq r \leq d$.*

The demonstration technique of the above algorithm takes the following form,



$\ldots (*)$

Figure 5

Digital expression for d, q and r : As mentioned already digital expression of d, q and r take the form,

$$a = \underline{a_n}\ \underline{a_{n-1}} \ldots \ \underline{a_1}$$
$$d = \underline{d_t}\ \underline{d_{t-1}} \ldots \ \underline{d_1}$$
$$r = \underline{r_p}\ \underline{r_{p-1}} \ldots \ \underline{r_1}$$

Hence the above mentioned model (*) takes the form

From now on, for simplicity, we will mean $\underline{1}\ \underline{2}\ \underline{3}$ by 123 etc. in usual decimal notation. Now, we note systematic algorithm break up of above division responsi-

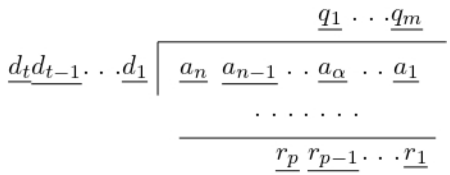


Figure 6

ble for corresponding **digital sequence hypergraphic model** takes the following form,

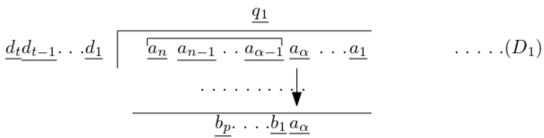


Figure 7

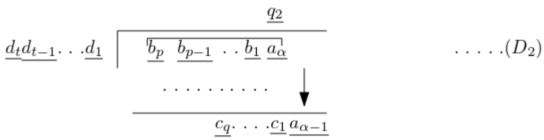


Figure 8

.. .. ..

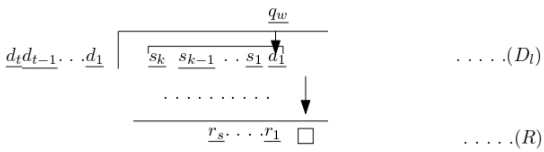
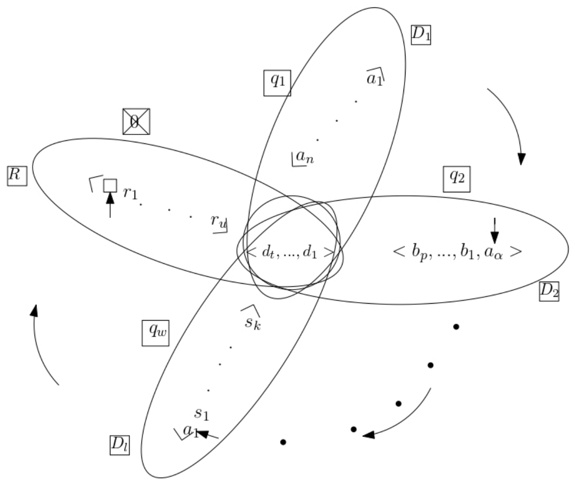


Figure 9

Now, the division algorithm hypergraph model is as follows.

In our discussion in the sequence hypergraph of division algorithm a hyperarc $E =< X, Y >, X \cap Y = \phi$ consists of two digital sequences $X =< d_t, ..., d_1 >$ and Y, X being the divisor digital sequence and Y the corresponding dividend digital sequence wherever possible with preceding remainder.



$....(M)$

Figure 10

This sequential family of hyperarcs of digital vertex set has the divisor sequence hyperarc as its core (or tail) and thus it appears as a directed digital sequence hyperstar where each of the hyperarc would be termed as petal.

Thus, it is a directed sequence $< D_1, D_2, ..., D_t >$ of hyperarcs with the divisor sequence as its core (or tail). We note that each of the petals D_i appear as a dividend sequence with some a_i of the sequence $< a_n, ..., a_1 >$ or a remainder sequence only (viz. R_α). Each of the petals will be termed as (R, D) petals. Now we present an important result about the size of the hyperstar.

Theorem 4.1. *If in the division algorithm hypergraph model $a_k \in D_p$ then the number of (R, D) petals $N(RDP) = k + p$ (is invariant for any values of k and p).*
Proof. The model (M) (previously discussed) indicates that the number of petals beginning from D_p onwards in which $a_k, a_{k-1}, ..., a_1$ occur upto the remainder petal R is $k + 1$. And the number of petals preceding D_p are $D_1, D_2, ..., D_{p-1}$, in total, $p - 1$. So, the total number of petals $= (k + 1) + (p - 1) = k + p$. Thus, it appears as an invariant quantity which is nothing but the degree of the digital hyperstar.

Now, we present some illustrations for proper justification of what we have presented here which includes some 2-graph examples also together with its hypergraph mode.
Illustration(i) : Star 2-graph and star hypergraph presentation of $6396 \div 3 = 2132$.
Note : A brief explanation of the Theorem 4.1 with the help of the aforesaid illustration.
The digit $a_4 = 6$ takes place in the first dividend D_1, the digit $a_3 = 3$ takes place

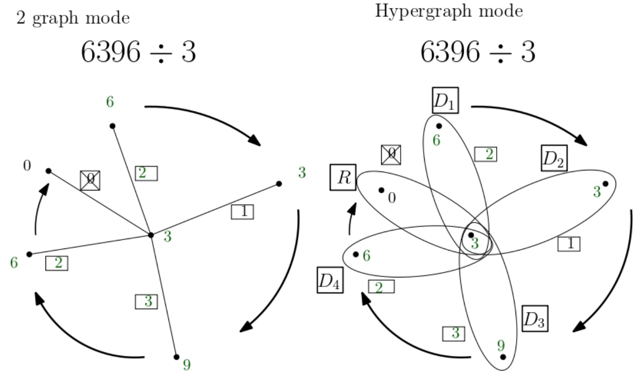


Figure 11

in the second dividend D_2, digit $a_2 = 9$ takes place in the third dividend D_3, the digit $a_1 = 6$ takes place in the fourth dividend D_4. In each case we observe $4+1 = 3+2 = 2+3 = 1+4 = 5$ =no. of petals, an invariant quantity.
Illustration(**ii**) : Digital hyperstar presentation $249675 \div 123 = 2029$

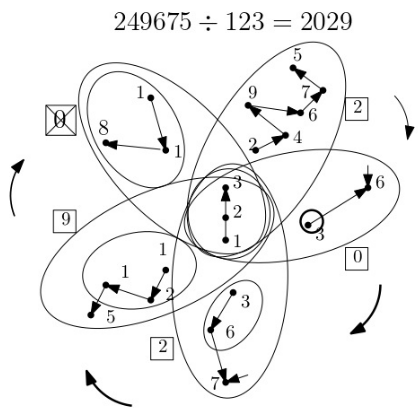


Figure 12

5. Hypergraphic approach of Euclidean Algorithm

By repeated use of Division Algorithm we set the following sequence of divisions

$$\begin{gathered} a_0 = q_1 a_1 + a_2 \\ a_1 = q_2 a_2 + a_2 \\ \cdots\ \cdots\ \cdots\ \cdots \\ a_{k-2} = q_k a_{k-1} + a_{k-2} \\ a_{k-1} = q_k a_k + 0 \end{gathered}$$

Here, $a_1 > a_2 > ... > a_{k-1} > a_k = 0$, finally $(a_1, a_0) = a_k$.
We here obatin a sequence of Division Algorithm and as already mentioned we therefore obtain a sequence of hypergraph stars to obtain the G.C.D of two numbers. Thus, it appears as a sequence algorithm of hyperstar that may be termed as the Digital sequence G.C.D hyperstar and finally we obtain a two petal hyperstar with G.C.D at its core and 0 in one of its petals. Thus the digital sequence of G.C.D hyperstar with the last term as the G.C.D hyperstar with 0(zero) as the remainder.

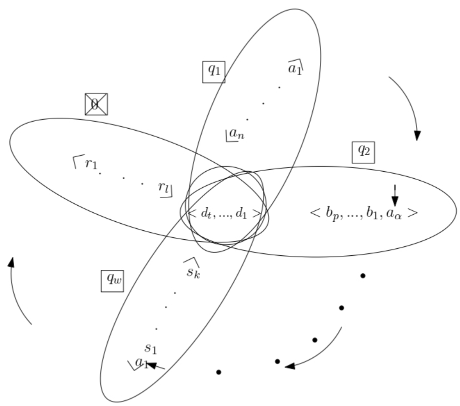


Figure 13

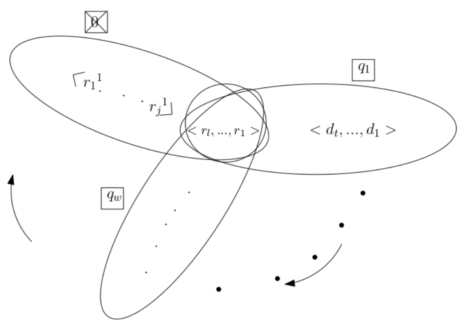


Figure 14

The above star graph with core as the last remainder as the divisor and corresponding remainder as 0 gives us the hyperstar with G.C.D hyperstar with G.C.D=

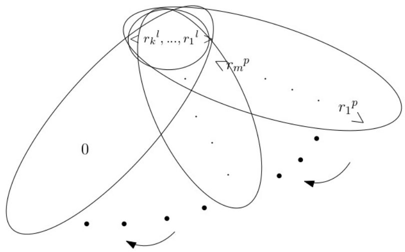


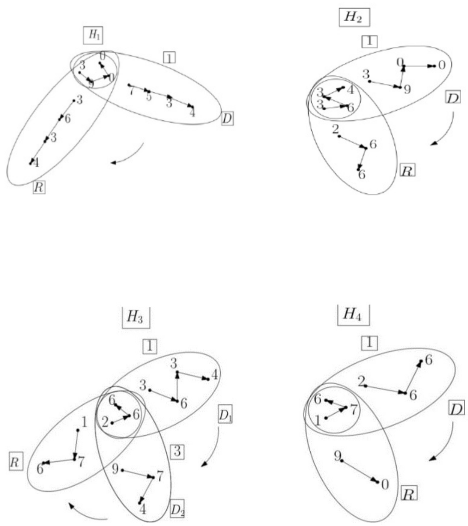
Figure 15

$| < r^l_k ... r^l_1 > |$.

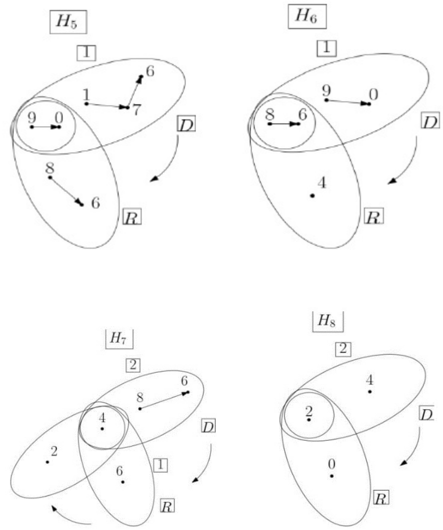
Illustration **:** Digital sequence of G.C.D hyperstars of 3900 and 7534.

$$3900 = 10^3 \times\ 3 + 10^2 \times\ 9 + 10\ \times\ 0 + 0 = \underline{3}\ \underline{9}\ \underline{0}\ \underline{0} = < 3, 9, 0, 0 >$$
$$7534 = 10^3\ \times\ 7 + 10^2\ \times\ 5 + 10\ \times\ 3 + 4 = \underline{7}\ \underline{5}\ \underline{3}\ \underline{4} = < 7, 5, 3, 4 >$$

In the following finite sequence of digital sequence of digital sequence G.C.D hyperstars $< H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8 >$



the last one H_8 is the G.C.D hyperstar. Here, we observe the descending sequence of weights of remainders, $| < 3, 9, 0, 0 > | > | < 3, 6, 3, 4 > | > | < 2, 6, 6 > | > | < 1, 7, 6 > | > | < 9, 0 > | > | < 8, 6 > | > | < 4 > | > | < 2 > | > | < 0 > |$. Thus, we obtain the G.C.D hyperstar.



6. Congruence modulo 2-graphs

Throughout our discussion here by 'congruence' or by 'congruence mod' we will mean 'congruence mod n' where n is a positive integer. Now, we try to exhibit 2-graph approach of some basic congruence properties.

In $\mathbb{Z}$ a sequence of arc $_{i=2}^{n} < (a_{i-1}, a_i) >= (a_1, a_2), (a_2, a_3), ..., (a_{n-1}, a_n)$ is termed as a transitive sequence of arcs.

Note 1: If $_{i=2}^{n} < (a_{i-1}, a_i) >$ is a transitive sequence of arcs then (a_1, a_n) is an arc.

Since, $(i/a_1)(f) = -1$, $(i/a_\alpha)(f) = 0$ for $\alpha \neq 1, ..., n, (i/a_n)(f) = 1$. The above

	$i(a_\alpha/e_\beta); \alpha = 1, ..., n, \beta = 1..., n-1$					*transitive sum e_i's for f*
	(a_1, a_2) e_1	(a_2, a_3) e_2	(a_3, a_4) e_3	$\cdots\cdots$	(a_{n-1}, a_n) e_{n-1}	$\sum i(a_\alpha/e_\beta)$
a_1	-1	0	0	$\cdots\cdot$	0	-1
a_2	1	-1	0	$\cdots\cdot$	0	0
a_3	0	1	-1	$\cdots\cdot$	0	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
a_{n-1}	0	0	0	$\cdots$ 1	-1	0
a_n	0	0	0	$\cdots\cdot$	1	1

Figure 16

table is sufficient to show that (a_1, a_n) is an arc where a_n is the head having weight 1 and a_1 is its tail having weight -1.

So, f is the arc (a_1, a_n).

Note 2: As a corollary to the above we get if $e = (a, b)$ and $f = (b, a)$ are two arcs then (a, a) and (b, b) are two loops at a and b respectively.

Since,

$$(i/a)(g) = (i/a)(e) + (i/a)(f) = (-1) + 1 = 0$$
$$(i/b)(g) = (i/b)(e) + (i/b)(f) = 1 + (-1) = 0$$

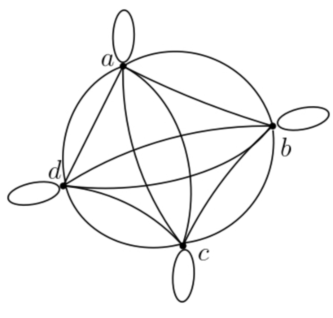
Now, by definition, $(i/a)(g) = 0$ implies that the arc is a loop at a.
$(i/b)(g) = 0$ implies that the arc is a loop at b.
Hence, (a, a) and (b, b) are loops at a and b respectively.

Theorem 6.1. *The resulting graph of a transitive arcs sequence* $_{i=2}^{m} < (a_{i-1}, a_i) >$ *of* m *congruence vertices is with* $2^m - 1$ *cycles including loops at each vertex.*

Proof. The above two notes viz. **Note 1** and **Note 2** reveals that there are cycles of l(loop), 2, ...upto m.
No. of cycles of length 1(loop)$={}^mC_1$.
No. of cycles of length 2$= {}^mC_2$. .. ..
No. of cycles of length $m$$={}^mC_m$.
So, the total no. of cycles is equal to $C_1 + C_2 + ... + C_m = 2^m - 1$.

As an illustration we exhibit the case taking $m = 4$.

$$C_1 + C_2 + C_3 + C_4 = 2^4 - 1 = 16 - 1 = 15$$



Theorem 6.2. *If* (a, b) *and* (c, d) *are two congruence arcs, then it induces a complete directed graph of four vertices using the arcs.*

Proof. Given the arcs,

Here, $w(e/a) = k_1, w(f/a) = 0, w(e/b) = k_1, w(f/b) = 0, w(e/c) = 0, w(f/c) = k_2, w(e/d) = 0, w(f/d) = k_2$. In order to estimate the weight of an arc g combining

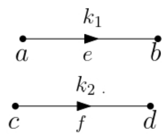


Figure 17

the weights of the arcs e and f, we define (e, f) weight of the arc as follows, $w^{(e,f)}(x+y) = w(e/x)+w(f/x)+w(e/y)+w(f/y)$, where $x, y \in \{a, b, c, d\}$. Now, for any $z, t \in \{a, b, c, d\}$ if $w^{(e,f)}(x+y) = w^{(e,f)}(z+t)$, then the arc g can be obtained from the congruence relation by taking one of $x + y$ and $z + t$ as tail and other as head. We note that, in the above construction, $w^{(e,f)}(a+c) = k_1+k_2 = w^{(e,f)}(b+d)$ and so two arcs can be formed as follows:

Figure 18

between the vertices $a + c$ and $b + d$. Similarly, arcs can be obtained between the vertices $a + d$ and $b + c$ with weights $w^{(e,f)}(a + d) = k_1 + k_2 = w^{(e,f)}(b + c)$. The following table reveals the above discussion, giving a proper visualisation of the probable complete directed graph:

The above table leads to the construction of the following complete directed graph having the 4 vertices $a+c$, $b+c$, $a+d$ and $b+d$, where all the arcs are with the same weight.

Also, the vertices $c + d$ and $a + b$ remain as isolated points as the (e, f) weights of $w^{(e,f)}(c + d)$ and $w^{(e,f)}(a + b)$ at that points are not matching with the weights of rest of the vertices in question. We note that if we remove the isolated vertices, then a complete directed graph can be obtained from this graph.

Theorem 6.3. *If (a, b) and (c, d) are two congruence arcs then it induces another complete directed graph of four vertices using the arcs.*

Proof. Given the arcs,

Here, $w(e/a) = k_1, w(f/a) = 0, w(e/b) = k_1, w(f/b) = 0, w(e/c) = 0, w(f/c) = k_2, w(e/d) = 0, w(f/d) = k_2$.

In the above context of the arcs e and f we define (e, f) weight $w^{(e,f)}$ of an arc g at ac as the product of the total (e, f) weight of a and total (e, f) weight of c.

In symbol, $w^{(e,f)}(ac) = [w(e/a) + w(f/a)][w(e/c) + w(f/c)] = (k_1 + 0)(0 + k_2) = (k_1 + 0)(0 + k_2) = k_1k_2$.

	$w(e/x)$	$w(f/x)$	
a	k_1	0	
b	k_1	0	
c	0	k_2	
d	0	k_2	
			$w^{(e,f)}(y+z)$
$a+c$	$-$	$-$	k_1+k_2
$b+d$	$-$	$-$	k_1+k_2
$a+d$	$-$	$-$	k_1+k_2
$b+c$	$-$	$-$	k_1+k_2
$c+d$	$-$	$-$	$2k_2$
$a+b$	$-$	$-$	$2k_1$

Figure 19

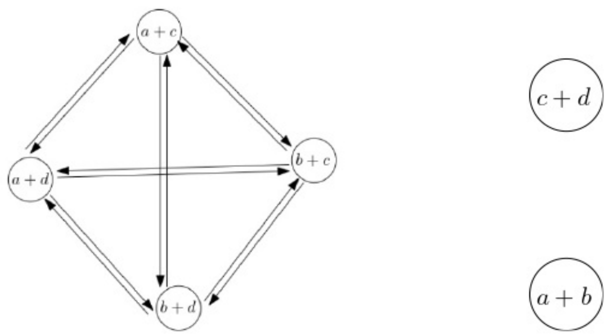


Figure 20

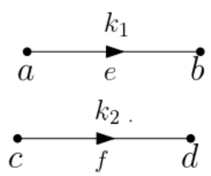


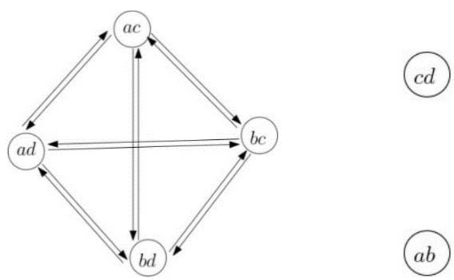
Figure 21

Similarly, $w^{(e,f)}(ad) = k_1k_2, w^{(e,f)}(bd) = k_1k_2, w^{(e,f)}(bc) = k_1k_2$.
Now, we contrive the following table:

	$w(e/x)$	$w(f/x)$	
a	k_1	0	
b	k_1	0	
c	0	k_2	
d	0	k_2	
			$w^{(e,f)}(yz)$
ac	–	–	k_1k_2
bd	–	–	k_1k_2
ad	–	–	k_1k_2
bc	–	–	k_1k_2
cd	–	–	${k_2}^2$
ab	–	–	${k_1}^2$

Figure 22

Using the same argumentation as already done in the previous result, the following complete directed graph having vertices ac, bc, ad and bd gets revealed.



Theorem 6.4. *Existence of the arc* (ma, mc) *implies existence of the arc* $((m,n)a, (m,n))c$ *and as corollary to this if* $(m,n) = 1$ *then existence of an arc* $(a,b) \implies$ *existence of an arc* (a,c).

Proof. Given the arc $e = (ma, mc)$ with weight $w = k$ leads us to the existence of an edge (m_1ma, m_1mc) with weight m_1k. As we are dealing with congruence mod n, always we have an edge $w = (n_1a - n_1c)$. As $gcd(m, n) = mm_1 + nn_1 = d$(say), we get $m_1ma + n_1na = gcd(m, n)a$ and $m_1mc + n_1nc = gcd(m, n)c$.
Thus, we get two vertices each of weight $w + k$ and a definite arc (da, dc).
Now, if $(m, n) = d = 1$ then existence of $(ma, mc) \implies$ existence of $(da, dc) \implies$ existence of (a, c). And the whole fact is revealed from the following table:

	$w(e/)$	$w(f/)$	$w(g_i/)$	
ma	k	0	0	
mc	k	0	0	
m_1ma	0	m_1k	0	$\longrightarrow$ *[by preceding proposition]*
m_1mc	0	m_1k	0	
n_1na	0	0	w	$\longrightarrow$ *[congruence mod n]*
n_1nc	0	0	w	
$m_1ma + n_1na = gcd(m, n)a$	0	0	$w + k$	$\longrightarrow$ *[by preceding proposition]*
$m_1mc + n_1nc = gcd(m, n)c$	0	0	$w + k$	

Figure 23

7. Scope and conclusion

The main notable aspects to be emphasized in this paper are of two folds:

(1). The hypergraph in discussion contains the finite vertex set viz. $\{0, 1, 2, ..., 9\}$ where the choice of digital sequence hyperedges permit places towards the existence of hyperedges with infinite size, each edge being a set of finite or infinite cardinality. This broader aspect of hypergraph which we have called here as digital sequence hypergraph leaves ample scope for development of Euclidean algorithmic character of any types of number system whether it is decimal or binary(viz. $\{0, 1\}$ as the vertex set) . In this way such type of digital sequence hypergraph may appear as a responsible factor along the track in discussion.

(2). The notion of weights discussed in 2-graphs congruence characteristics reveals multifacet exposure of graph theoretic congruence arithmetic.

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