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On summation and transformation formulae for basic hypergeometric series

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Abstract: In this paper, making use of some known WP-Bailey pairs and theorems for constructing new WP-Bailey pairs from a known WP-Bailey pair, we have established transformation formulae for basic hypergeometric series.
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1. Introduction, Notations and Definitions

As usual, for a and q complex numbers with $|q| < 1$, define

$$[a;q]_0 = 1,$$

$$[a;q]_n = (1-a)(1-aq)...(1-aq^{n-1}), \quad n \in N,$$

.

$$[a_1, a_2, ..., a_k; q]_n = [a_1;q]_n[a_2;q]_n...[a_k;q]_n,$$

$$[a;q]_\infty = \prod_{r=0}^{\infty}(1-aq^r)$$

and

$$[a_1, a_2, ..., a_r; q]_\infty = (a_1;q)_\infty(a_2;q)_\infty...(a_r;q)_\infty.$$

A basic hypergeometric series is defined by

$${}_r\Phi_s\left[\begin{array}{l} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s \end{array}\right] = \sum_{n=0}^{\infty}\frac{[a_1, a_2, ..., a_r; q]_n z^n}{[q, b_1, b_2, ..., b_s; q]_n}, \quad |z| < 1, \qquad (1.1)$$

where as a very well poised basic hypergeometric series defined by,

$${}_{r+3}W_{r+2}\left[a; b_1, b_2, ..., b_r; q; z\right]$$

$$= {}_{r+3}\Phi_{r+2}\left[\begin{array}{c} a, q\sqrt{a}, -q\sqrt{a}, b_1, b_2, ..., b_r; q; z \\ \sqrt{a}, -\sqrt{a}, aq/b_1, aq/b_2, ..., aq/b_r \end{array}\right]. \tag{1.2}$$

A pair of sequences $\{\alpha(a,k;q), \beta(a,k;q)\}$ is called WP-Bailey pair if

$$\beta_n(a,k;q) = \sum_{r=0}^{n} \frac{[k/a;q]_{n-r}[k;q]_{n+r}}{[q;q]_{n-r}[aq;q]_{n+r}} \alpha(a,k;q)$$

$$= \frac{[k/a,k;q]_n}{[q,aq;q]_n} \sum_{r=0}^{n} \frac{[kq^n, q^{-n};q]_r}{[aq^{1-n}/k, aq^{n+1};q]_r} \alpha_r(a,k;q). \tag{1.3}$$

In order to obtain WP-Bailey pairs, we shall make use of following summation formulae

$${}_6\Phi_5\left[\begin{array}{c} a, q\sqrt{a}, -q\sqrt{a}, b, kq^n, q^{-n}; q; aq/bk \\ \sqrt{a}, -\sqrt{a}, aq/b, aq^{1-n}/k, aq^{1+n} \end{array}\right] = \frac{[aq, kb/a;q]_n}{[k/a, aq/b;q]_n b^n}, \tag{1.4}$$

which can be deduced from [Gasper & Rahman 3; App II (II.21)].

$${}_8\Phi_7\left[\begin{array}{c} a, q\sqrt{a}, -q\sqrt{a}, b, c, a^2q/bck, kq^n, q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bck/a, aq^{1-n}/k, aq^{1+n} \end{array}\right]$$

$$= \frac{[aq, aq/bc, kb/a, kc/a;q]_n}{[aq/b, aq/c, k/a, kbc/a;q]_n}, \tag{1.5}$$

which can be deduced from [Gasper & Rahman 3; App. II (II.22)].

$${}_6\Phi_5\left[\begin{array}{c} a, q\sqrt{a}, -q\sqrt{a}, a/k, kq^n, q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, kq, aq^{1-n}/k, aq^{1+n} \end{array}\right] = \delta_{n,0}, \tag{1.6}$$

which can be deduced from (1.4) by taking b=a/k.
Following are certain theorems for constructing the WP-Bailey pairs from a known pair.

Theorem 1 (Andrews) If $\{\alpha_n(a,k;q), \beta_n(a,k;q)\}$ is a WP-Bailey pair then so is the $\{\alpha_n'(a,k;q), \beta_n'(a,k;q)\}$ given by

$$\alpha_n'(a,k;q) = \frac{[\rho_1, \rho_2;q]_n}{[aq/\rho_1, aq/\rho_2;q]_n}\left(\frac{k}{m}\right)^n \alpha_n(a,m;q) \tag{1.7}$$

$$\beta_n'(a,k;q) = \frac{[mq/\rho_1, mq/\rho_2;q]_n}{[aq/\rho_1, aq/\rho_2;q]_n} \sum_{r=0}^{n}\left(\frac{1-mq^{2r}}{1-m}\right)\frac{[\rho_1,\rho_2;q]_r}{[mq/\rho_1, mq/\rho_2;q]_r} \times$$

$$\times \frac{[k/m;q]_{n-r}[k;q]_{n+r}}{[q;q]_{n-r}[mq;q]_{n+r}} \left(\frac{k}{m}\right)^r \beta_r(a,m;q), \tag{1.8}$$

where $m = k\rho_1\rho_2/aq$.

Theorem 2 (Andrews) If $\{\alpha_n(a,k;q), \beta_n(a,k;q)\}$ is a WP-Bailey pair then so is the $\{\alpha'_n(a,k;q), \beta'_n(a,k;q)\}$ given by

$$\alpha'_n(a,k;q) = \frac{[m;q]_{2n}}{[k;q]_{2n}} \left(\frac{k}{m}\right)^n \alpha_n(a,m;q) \tag{1.9}$$

and

$$\beta'_n(a,k;q) = \sum_{r=0}^{n} \frac{[k/m;q]_{n-r}}{[q;q]_{n-r}} \left(\frac{k}{m}\right)^r \beta_r(a,m;q), \tag{1.10}$$

where $m = a^2q/k$.

Theorem 3 (Warnaar) If $\{\alpha_n(a,k;q), \beta_n(a,k;q)\}$ is a WP-Bailey pair then so is the $\{\alpha'_n(a,k;q), \beta'_n(a,k;q)\}$ given by

$$\alpha'_n(a,k;q) = \left(\frac{1-\sigma k^{1/2}}{1-\sigma k^{1/2}q^n}\right)\left(\frac{1+\sigma m^{1/2}q^n}{1+\sigma m^{1/2}}\right) \frac{[m;q]_{2n}}{[k;q]_{2n}} \left(\frac{k}{m}\right)^n \alpha_n(a,m;q) \tag{1.11}$$

and

$$\beta'_n(a,k;q) = \left(\frac{1-\sigma k^{1/2}}{1-\sigma k^{1/2}q^n}\right) \sum_{r=0}^{n} \left(\frac{1+\sigma m^{1/2}q^r}{1+\sigma m^{1/2}}\right) \frac{[k/m;q]_{n-r}}{[q;q]_{n-r}} \left(\frac{k}{m}\right)^r \beta_r(a,m;q), \tag{1.12}$$

where $m = a^2/k$ and $\sigma \in (-1,1)$.

Theorem 4 (Warnaar) If $\{\alpha_n(a,k;q), \beta_n(a,k;q)\}$ is a WP-Bailey pair then so is the $\{\alpha'_n(a,k;q), \beta'_n(a,k;q)\}$ given by

$$\alpha'_n(a^2,k;q^2) = \alpha_n(a,m;q) \tag{1.13}$$

and

$$\beta'_n(a^2,k;q^2) = \frac{[-mq;q]_{2n}}{[-aq;q]_{2n}} \sum_{r=0}^{n} \left(\frac{1-mq^{2r}}{1-m}\right) \frac{[k/m^2;q^2]_{n-r}}{[q^2;q^2]_{n-r}} \times$$

$$\times \frac{[k;q^2]_{n+r}}{[m^2q^2;q^2]_{n+r}} \left(\frac{m}{a}\right)^{n-r} \beta_r(a,m;q), \tag{1.14}$$

where m=k/aq.

Theorem 5 (Warnaar) If $\{\alpha_n(a,k;q), \beta_n(a,k;q)\}$ is a WP-Bailey pair then so is the $\{\alpha'_n(a,k;q), \beta'_n(a,k;q)\}$ given by

$$\alpha'_n(a^2,k;q^2) = q^{-n}\left(\frac{1+aq^{2n}}{1+a}\right) \alpha_n(a,m;q) \tag{1.15}$$

$$\beta'_n(a^2,k;q^2) = q^{-n}\frac{[-mq;q]_{2n}}{[-a;q]_{2n}}\sum_{r=0}^{n}\left(\frac{1-mq^{2r}}{1-m}\right)\frac{[k/m^2;q^2]_{n-r}}{[q^2;q^2]_{n-r}}\times$$

$$\times\frac{[k;q^2]_{n+r}}{[m^2q^2;q^2]_{n+r}}\left(\frac{m}{a}\right)^{n-r}\beta_r(a,m;q), \qquad (1.16),$$

where $m = k/a$.

Theorem 6 (Warnaar) If $\{\alpha_n(a,k;q), \beta_n(a,k;q)\}$ is a WP-Bailey pair then so is the $\{\alpha'_n(a,k;q), \beta'_n(a,k;q)\}$ given by

$$\alpha'_{2n}(a,k;q) = \alpha_n(a,m;q^2), \qquad \alpha_{2n+1}(a,k;q) = 0 \qquad (1.17)$$

$$\beta'_n(a,k;q) = \frac{[mq;q^2]_n}{[aq;q^2]_n}\sum_{r=0}^{[n/2]}\left(\frac{1-mq^{2r}}{1-m}\right)\frac{[k/m;q]_{n-2r}}{[q;q]_{n-2r}}\times$$

$$\times\frac{[k;q]_{n+2r}}{[mq;q]_{n+2r}}\left(-\frac{k}{a}\right)^{n-2r}\beta_r(a,m;q^2), \qquad (1.18),$$

where $m = k^2/a$.

2. WP-Bailey Pairs

In this section we shall establish certain WP-Bailey pairs.

(i) Taking $\alpha_r(a,k;q) = \dfrac{[a, q\sqrt{a}, -q\sqrt{a}, b; q]_r}{[q, \sqrt{a}, -\sqrt{a}, aq/b; q]_r}\left(\dfrac{1}{b}\right)^r$ in (1.3) and using (1.4) we get,

$$\beta_n(a,k;q) = \frac{[k, kb/a; q]_n}{[q, aq/b; q]_n b^n}.$$

Thus we find that

$$\alpha_r(a,k;q) = \frac{[a, q\sqrt{a}, -q\sqrt{a}, b; q]_n}{[q, \sqrt{a}, -\sqrt{a}, aq/b; q]_n}\left(\frac{1}{b}\right)^n \qquad (2.1)$$

and

$$\beta_n(a,k;q) = \frac{[k, kb/a; q]_n}{[q, aq/b; q]_n b^n}. \qquad (2.2)$$

is a WP-Bailey pair.

(ii) Taking $\alpha_r(a,k;q) = \dfrac{[a, q\sqrt{a}, -q\sqrt{a}, b, c, a^2q/bck; q]_r}{[q, \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bck/a; q]_r}\left(\dfrac{k}{a}\right)^r$ in (1.3) and using (1.5) we get,

$$\beta_n(a,k;q) = \frac{[k, aq/bc, kb/a, kc/a; q]_n}{[q, aq/b, aq/c, kbc/a; q]_n b^n}.$$

Thus

$$\alpha_r(a,k;q) = \frac{[a, q\sqrt{a}, -q\sqrt{a}, b, c, a^2q/bck; q]_n}{[q, \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bck/a; q]_n} \left(\frac{k}{a}\right)^n \tag{2.3}$$

and

$$\beta_n(a,k;q) = \frac{[k, aq/bc, kb/a, kc/a; q]_n}{[q, aq/b, aq/c, kbc/a; q]_n b^n}. \tag{2.4}$$

is a WP-Bailey pair. This pair was first obtained by Singh U.B. [4]
(iii) If we take b=a/k in (2.1) and (2.2) we get another WP-Bailey pair,

$$\alpha_n(a,k;q) = \frac{[a, q\sqrt{a}, -q\sqrt{a}, a/k; q]_n}{[q, \sqrt{a}, -\sqrt{a}, kq; q]_n} \left(\frac{k}{a}\right)^n \tag{2.5}$$

and

$$\beta_n(a,k;q) = \delta_{n,0}. \tag{2.6}$$

(iv) If we take $\alpha_r(a,k;q) = \delta_{r,0}$ in (1.3) then

$$\beta_n(a,k;q) = \frac{[k, k/a; q]_n}{[q, aq; q]_n}.$$

Thus,

$$\alpha_n(a,k;q) = \delta_{n,0}. \tag{2.7}$$

and

$$\beta_n(a,k;q) = \frac{[k, k/a; q]_n}{[q, aq; q]_n}. \tag{2.8}$$

from a WP Bailey pair.

3. Main Results

In this section we shall construct new WP Bailey pairs.
(A) (i) Using the WP-Bailey pair given in (2.1) and (2.2) in theorem 1; (1.7)-(1.8), we get new WP-Bailey pair,

$$\alpha'_n(a,k;q) = \frac{[a, q\sqrt{a}, -q\sqrt{a}, b, \rho_1, \rho_2; q]_n}{[q, \sqrt{a}, -\sqrt{a}, aq/b, aq/\rho_1, aq/\rho_2; q]_n} \left(\frac{k}{mb}\right)^n \tag{3.1}$$

$$\beta'_n(a,k;q) = \frac{[k, k/m, mq/\rho_1, mq/\rho_2; q]_n}{[q, mq, aq/\rho_1, aq/\rho_2; q]_n} \times$$

$$\times \sum_{n=0}^{\infty} \left(\frac{1-mq^{2r}}{1-m}\right) \frac{[m, mb/a, \rho_1, \rho_2, kq^n, q^{-n}; q]_r q^r}{[q, aq/b, mq/\rho_1, mq/\rho_2, mq^{1-n}/k, mq^{1+n}; q]_r b^r}, \tag{3.2}$$

where $m = \dfrac{k\rho_1\rho_2}{aq}$.

(ii) Using the WP-Bailey pair given in (2.1) and (2.2) in theorem 2; (1.9)-(1.10), we get new WP-Bailey pair

$$\alpha'_n(a,k;q) = \frac{[m;q]_{2n}}{[k;q]_{2n}}\left(\frac{k}{m}\right)^n \frac{[a, q\sqrt{a}, -q\sqrt{a}, b; q]_n}{[q, \sqrt{a}, -\sqrt{a}, aq/b; q]_n b^n} \tag{3.3}$$

$$\beta'_n(a,k;q) = \sum_{r=0}^{n} \frac{[k/m;q]_{n-r}}{[q;q]_{n-r}}\left(\frac{k}{m}\right)^r \frac{[m, mb/a; q]_r}{[q, aq/b; q]_r b^r}$$

$$= \frac{[k/m;q]_n}{[q;q]_n} \sum_{r=0}^{n} \frac{[m, mb/a, q^{-n}; q]_r}{[q, aq/b, mq^{1-n}/k; q]_r}\left(\frac{q}{b}\right)^r, \tag{3.4}$$

where $m = a^2q/k$.

(iii) Using the WP-Bailey pair given in (2.1) and (2.2) in theorem 3; (1.11)-(1.12) we get another new WP-Bailey pair,

$$\alpha'_n(a,k;q) = \left(\frac{1-\sigma k^{1/2}}{1-\sigma k^{1/2}q^n}\right)\left(\frac{1-\sigma m^{1/2}q^n}{1-\sigma m^{1/2}}\right)\frac{[m;q]_{2n}}{[k;q]_{2n}}\left(\frac{k}{mb}\right)^n$$

$$\times \frac{[a, q\sqrt{a}, -q\sqrt{a}, b; q]_n}{[q, \sqrt{a}, -\sqrt{a}, aq/b; q]_n}, \tag{3.5}$$

$$\beta'_n(a,k;q) = \left(\frac{1-\sigma k^{1/2}}{1-\sigma k^{1/2}q^n}\right)\frac{[k/m;q]_n}{[q;q]_n}\sum_{r=0}^{n}\left(\frac{1+\sigma m^{1/2}q^r}{1+\sigma m^{1/2}}\right)\times$$

$$\times \frac{[q^{-n};q]_r}{[mq^{1-n}/k;q]_r}\frac{[m, mb/a; q]_r}{[q, aq/b; q]_r}\left(\frac{q}{b}\right)^r, \tag{3.6}$$

where $m = a^2/k$ and $\sigma \in (-1,1)$.

(iv) Using the WP-Bailey pair given in (2.1) and (2.2) in theorem 4; (1.13)-(1.14) we get the new WP-Bailey pair.

$$\alpha'_n(a^2,k;q^2) = \frac{[a, q\sqrt{a}, -q\sqrt{a}, b; q]_n}{[q, \sqrt{a}, -\sqrt{a}, aq/b; q]_n b^n}, \tag{3.7}$$

$$\beta'_n(a^2,k;q^2) = \frac{[-mq;q]_{2n}}{[-aq;q]_{2n}}\frac{[k/m^2;q^2]_n[k;q^2]_n}{[q^2;q^2]_n[m^2q^2;q^2]_n}\left(\frac{m}{a}\right)^n\sum_{r=0}^{n}\left(\frac{1-mq^{2r}}{1-m}\right)\times$$

$$\times \frac{[m, mb/a; q]_r[kq^{2n}, q^{-2n}; q^2]_r}{[q, aq/b; q]_r[m^2q^{2-2n}/k, m^2q^{2+2n}; q^2]_r}, \tag{3.8}$$

where m=k/aq.
(v) Using the WP-Bailey pair given in (2.1) and (2.2) in theorem 5; (1.15)-(1.16) we get the new WP-Bailey pair.

$$\alpha'_n(a^2,k;q^2) = q^{-n}\left(\frac{1+aq^{2n}}{1+a}\right)\frac{[a,q\sqrt{a},-q\sqrt{a},b;q]_n}{[q,\sqrt{a},-\sqrt{a},aq/b;q]_n b^n} \tag{3.9}$$

$$\beta'_n(a^2,k;q^2) = q^{-n}\frac{[-mq;q]_{2n}}{[-aq;q]_{2n}}\frac{[k/m^2;q^2]_n[k;q^2]_n}{[q^2;q^2]_n[m^2q^2;q^2]_n}\left(\frac{m}{a}\right)^n\sum_{r=0}^{n}\left(\frac{1-mq^{2r}}{1-m}\right)\times$$
$$\times\frac{[m,mb/a;q]_r[kq^{2n},q^{-2n};q^2]_r}{[q,aq/b;q]_r[m^2q^{2-2n}/k,m^2q^{2+2n};q^2]_r}\left(\frac{amq^2}{bk}\right), \tag{3.10}$$

where m=k/a.
(vi) Using the WP-Bailey pair given in (2.1) and (2.2) in theorem 6; (1.17)-(1.18) we get the new WP-Bailey pair.

$$\alpha'_{2n}(a,k;q) = \frac{[a,q^2\sqrt{a},-q^2\sqrt{a},b;q^2]_n}{[q^2,\sqrt{a},-\sqrt{a},aq^2/b;q^2]_n b^n}, \qquad \alpha'_{2n+1}(a,k;q) = 0 \tag{3.11}$$

$$\beta'_n(a,k;q) = \frac{[mq;q^2]_n}{[aq;q^2]_n}\frac{[k,k/m;q]_n}{[q,mq;q]_n}\left(-\frac{k}{a}\right)^n\sum_{r=0}^{n/2}\left(\frac{1-mq^{2r}}{1-m}\right)\times$$
$$\times\frac{[q^{-n};q]_{2r}[kq^n;q]_{2r}[m,mb/a;q^2]_r}{[mq^{1-n}/k;q]_{2r}[mq^{1+n};q]_{2r}[q^2,aq^2/b;q^2]_r}\left(\frac{amq}{bk^2}\right)^r \tag{3.12},$$

where $m = k^2/a$.
(B) (vii) Using the WP-Bailey pair given in (2.3) and (2.4) in theorem 1; (1.7)-(1.8), we get the new Bailey pair,

$$\alpha'_n(a,k;q) = \frac{[\rho_1,\rho_2;q]_n}{[aq/\rho_1,aq/\rho_2;q]_n}\frac{[a,q\sqrt{a},-q\sqrt{a},b,c,a^2q/bcm;q]_n}{[q,\sqrt{a},-\sqrt{a},aq/b,aq/c,bcm/a;q]_n}\left(\frac{k}{a}\right)^n \tag{3.13}$$

$$\beta'_n(a,k;q) = \frac{[k,k/m,mq/\rho_1,mq/\rho_2;q]_n}{[q,mq,aq/\rho_1,aq/\rho_2;q]_n}\sum_{r=0}^{n}\left(\frac{1-mq^{2r}}{1-m}\right)\frac{[\rho_1,\rho_2;q]_r}{[mq/\rho_1,mq/\rho_2;q]_r}\times$$
$$\times\frac{[q^{-n},kq^n;q]_r q^r[m,aq/bc,mb/a,mc/a;q]_r}{[mq^{1-n}/k,mq^{1+n};q]_r[q,aq/b,aq/c,mbc/a;q]_r}, \tag{3.14}$$

where $m = k\rho_1\rho_2/aq$.
(viii) Using the WP-Bailey pair given in (2.3) and (2.4) in theorem 2; (1.9)-(1.10), we get the new Bailey pair,

$$\alpha'_n(a,k;q) = \frac{[m;q]_{2n}}{[k;q]_{2n}}\left(\frac{k}{a}\right)^n\frac{[a,q\sqrt{a},-q\sqrt{a},b,c,a^2q/bcm;q]_n}{[q,\sqrt{a},-\sqrt{a},aq/b,aq/c,bcm/a;q]_n} \tag{3.15}$$

$$\beta'_n(a,k;q) = \frac{[k/m;q]_n}{[q;q]_n} \sum_{r=0}^{n} \frac{[q^{-n};q]_r q^r [m, aq/bc, mb/a, mc/a;q]_r}{[mq^{1-n}/k;q]_r [q, aq/b, aq/c, mbc/a;q]_r}, \tag{3.16}$$

where $m = a^2q/k$.

(ix) Using the WP-Bailey pair given in (2.3) and (2.4) in theorem 3; (1.11)-(1.12), we get the new Bailey pair,

$$\alpha'_n(a,k;q) = \left(\frac{1-\sigma k^{1/2}}{1-\sigma k^{1/2}q^n}\right)\left(\frac{1+\sigma m^{1/2}q^n}{1+\sigma m^{1/2}}\right)\frac{[m;q]_{2n}}{[k;q]_{2n}}\times$$

$$\times \frac{[a, q\sqrt{a}, -q\sqrt{a}, b, c, a^2q/bcm;q]_n}{[q, \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bcm/a;q]_n}\left(\frac{k}{a}\right)^n \tag{3.17}$$

$$\beta'_n(a,k;q) = \left(\frac{1-\sigma k^{1/2}}{1-\sigma k^{1/2}q^n}\right)\frac{[k/m;q]_n}{[q;q]_n}\sum_{r=0}^{n}\left(\frac{1+\sigma m^{1/2}q^r}{1+\sigma m^{1/2}}\right)\times$$

$$\times \frac{[q^{-n};q]_r q^r [m, aq/bc, mb/a, mc/a;q]_r}{[mq^{1-n}/k;q]_r [q, aq/b, aq/c, mbc/a;q]_r}, \tag{3.18}$$

where $m = a^2q/k$ and $\sigma \in (-1,1)$.

(x) Using the WP-Bailey pair given in (2.3) and (2.4) in theorem 4; (1.13)-(1.14), we get the new Bailey pair,

$$\alpha'_n(a^2,k;q^2) = \frac{[a, q\sqrt{a}, -q\sqrt{a}, b, c, a^2q/bcm;q]_n}{[q, \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bcm/a;q]_n}\left(\frac{m}{a}\right)^n \tag{3.19}$$

$$\beta'_n(a^2,k;q^2) = \frac{[-mq;q]_{2n}}{[-aq;q]_{2n}}\frac{[k/m^2;q^2]_n[k;q^2]_n}{[q^2;q^2]_n[m^2q^2;q^2]_n}\left(\frac{m}{a}\right)^n\sum_{r=0}^{n}\left(\frac{1-mq^{2r}}{1-m}\right)\times$$

$$\times \frac{[q^{-2n}, kq^{2n};q^2]_r [m, aq/bc, mb/a, mc/a;q]_r}{[m^2q^{2-2n}/k, m^2q^{2+2n};q^2]_r [q, aq/b, aq/c, mbc/a;q]_r}, \tag{3.20}$$

where m=k/aq.

(xi) Using the WP-Bailey pair given in (2.3) and (2.4) in theorem 5; (1.15)-(1.16), we get the new Bailey pair,

$$\alpha'_n(a^2,k;q^2) = q^{-n}\left(\frac{1+aq^{2n}}{1+a}\right)\frac{[a, q\sqrt{a}, -q\sqrt{a}, b, c, a^2q/bcm;q]_n}{[q, \sqrt{a}, -\sqrt{a}, aq/b, aq/c, bcm/a;q]_n}\left(\frac{m}{a}\right)^n \tag{3.21}$$

$$\beta'_n(a^2,k;q^2) = q^{-n}\frac{[-mq;q]_{2n}}{[-a;q]_{2n}}\frac{[k/m^2;q^2]_n[k;q^2]_n}{[q^2;q^2]_n[m^2q^2;q^2]_n}\left(\frac{m}{a}\right)^n\sum_{r=0}^{n}\left(\frac{1-mq^{2r}}{1-m}\right)\times$$

$$\times \frac{[q^{-2n}, kq^{2n}; q^2]_r [m, aq/bc, kb/a, mc/a; q]_n}{[m^2q^{2-2n}/k, m^2q^{2+2n}; q^2]_r [q, aq/b, aq/c, mbc/a; q]_n} \left(\frac{maq^2}{k}\right)^r, \tag{3.22}$$

where $m = k/a$.

(xii) Using the WP-Bailey pair given in (2.3) and (2.4) in theorem 6; (1.17)-(1.18), we get the new Bailey pair,

$$\alpha'_{2n}(a, k; q) = \frac{[a, q^2\sqrt{a}, -q^2\sqrt{a}, b, c, a^2q^2/bcm; q^2]_n}{[q^2, \sqrt{a}, -\sqrt{a}, aq^2/b, aq^2/c, bcm/a; q^2]_n} \left(\frac{m}{a}\right)^n, \qquad \alpha_{2n+1}(a, k; q) = 0 \tag{3.23}$$

$$\beta'_n(a, k; q) = \frac{[mq; q^2]_n}{[aq; q^2]_n} \frac{[k/m, k; q]_n}{[q, mq; q]_n} \left(-\frac{k}{a}\right)^n \sum_{r=0}^{n/2} \left(\frac{1 - mq^{2r}}{1 - m}\right) \times$$

$$\times \frac{[q^{-n}; q]_{2r} [kq^n; q]_{2r}}{[mq^{1-n}/k; q]_{2r} [mq^{1+n}; q]_{2r}} \left(\frac{mq}{k}\right)^{2r} \left(\frac{a}{k}\right)^{2r} \times$$

$$\times \frac{[m, aq^2/bc, mb/a, mc/a; q^2]_n}{[q^2, aq^2/b, aq^2/c, mbc/a; q^2]_n}, \tag{3.24}$$

where $m = k^2/a$.

(C) (xiii) Using the WP-Bailey pair given in (2.7) and (2.8) in theorem 1; (1.7)-(1.8), we get the new Bailey pair,

$$\alpha'_n(a, k; q) = \delta_{n,0} \tag{3.25}$$

$$\beta'_n(a, k; q) = \frac{\left[\frac{mq}{\rho_1}, \frac{mq}{\rho_2}, \frac{k}{m}, k; q\right]_n}{\left[\frac{aq}{\rho_1}, \frac{aq}{\rho_2}, q, mq; q\right]_n} \sum_{r=0}^{n} \left(\frac{1 - mq^{2r}}{1 - m}\right) \times$$

$$\times \frac{[\rho_1, \rho_2, q^{-n}, kq^n; q]_r q^r [m, m/a; q]_r}{\left[\frac{mq}{\rho_1}, \frac{mq}{\rho_2}, \frac{mq^{1-n}}{k}, mq^{1+n}; q\right]_r [q, aq; q]_r}, \tag{3.26}$$

where $m = \dfrac{k\rho_1\rho_2}{aq}$.

(xiv) Using the WP-Bailey pair given in (2.7) and (2.8) in theorem 2; (1.9)-(1.10), we get the new Bailey pair,

$$\alpha'_n(a, k; q) = \delta_{n,0} \tag{3.27}$$

$$\beta'_n(a, k; q) = \frac{[k/m; q]_n}{[q; q]_n} \sum_{r=0}^{n} \frac{[q^{-n}; q]_r [m, m/a; q]_r}{[mq^{1-n}/k; q]_r [q, aq; q]_r}, \tag{3.28}$$

where $m = a^2q/k$.

(xv) Using the WP-Bailey pair given in (2.7) and (2.8) in theorem 3; (1.11)-(1.12), we get the new Bailey pair,

$$\alpha'_n(a,k;q) = \delta_{n,0} \tag{3.29}$$

$$\beta'_n = \left(\frac{1-\sigma\sqrt{k}}{1-\sigma q^n\sqrt{k}}\right)\frac{[k/m;q]_n}{[q;q]_n}\sum_{r=0}^{n}\left(\frac{1+\sigma q^r\sqrt{m}}{1+\sigma\sqrt{m}}\right)\times$$
$$\times\frac{[q^{-n};q]_r q^r[m,m/a;q]_r}{[mq^{1-n}/k;q]_r[q,aq;q]_r}, \tag{3.30}$$

where $m = a^2/k$ and $\sigma \in (-1,1)$.

(xvi) Using the WP-Bailey pair given in (2.7) and (2.8) in theorem 4; (1.13)-(1.14), we get the new Bailey pair,

$$\alpha'_n(a^2,k;q^2) = \delta_{n,0} \tag{3.31}$$

$$\beta'_n(a^2,k;q^2) = \frac{[-mq;q]_{2n}[k,k/m^2;q^2]_n}{[-aq;q]_{2n}[q^2,m^2q^2;q^2]_n}\left(\frac{m}{a}\right)^n\sum_{r=0}^{n}\left(\frac{1-mq^{2r}}{1-m}\right)\times$$
$$\times\frac{[kq^{2n},q^{-2n};q^2]_r}{[m^2q^{2-2n}/k,m^2q^{2+2n};q^2]_r}\left(\frac{amq^2}{k}\right)^r\frac{[m,m/a;q]_r}{[q,aq;q]_r}, \tag{3.32}$$

where m=k/aq.

(xvii) Using the WP-Bailey pair given in (2.7) and (2.8) in theorem 5; (1.15)-(1.16), we get the new Bailey pair,

$$\alpha'_n(a^2,k;q^2) = \delta_{n,0} \tag{3.33}$$

$$\beta'_n(a^2,k;q^2) = q^{-n}\frac{[-mq;q]_{2n}[k,k/m^2;q^2]_n}{[-a;q]_{2n}[q^2,m^2q^2;q^2]_n}\left(\frac{m}{a}\right)^n\sum_{r=0}^{n}\left(\frac{1-mq^{2r}}{1-m}\right)\times$$
$$\times\frac{[kq^{2n},q^{-2n};q^2]_r}{[m^2q^{2-2n}/k,m^2q^{2+2n};q^2]_r}\left(\frac{amq^2}{k}\right)^r\frac{[m,m/a;q]_r}{[q,aq;q]_r}, \tag{3.34}$$

where m=k/a.

(xviii) Using the WP-Bailey pair given in (2.7) and (2.8) in theorem 6; (1.17)-(1.18), we get the new Bailey pair,

$$\alpha'_{2n}(a,k;q) = \delta_{n,0}, \qquad \alpha_{2n+1} = 0 \tag{3.35}$$

$$\beta'_n(a,k;q) = \frac{[mq;q^2]_n[k/m;q]_n[k;q]_n}{[aq;q^2]_n[q;q]_n[mq;q]_n}\left(-\frac{k}{a}\right)^n\times$$

$$\times \sum_{n=0}^{n/2} \left(\frac{1-mq^{2r}}{1-m}\right) \frac{[q^{-n};q]_{2r}[kq^n;q]_{2r}[m,m/a;q^2]_r}{[mq^{1-n}/k;q]_{2r}[mq^{1+n};q]_{2r}[q^2,aq^2;q^2]_r} \left(\frac{maq}{k^2}\right)^{2r} \tag{3.36}$$

where $m = k^2/a$.

(D) (xix) Using the WP-Bailey pair given in (2.5) and (2.6) in theorem 1; (1.7)-(1.8), we get the new Bailey pair,

$$\alpha'_n(a,k;q) = \frac{[\rho_1,\rho_2;q]_n}{[aq/\rho_1,aq/\rho_2;q]_n} \left(\frac{k}{a}\right)^n \frac{[a,q\sqrt{a},-q\sqrt{a},a/k;q]_n}{[q,\sqrt{a},-\sqrt{a},kq;q]_n} \tag{3.37}$$

$$\beta'_n(a,k;q) = \frac{[mq/\rho_1,mq/\rho_2;q]_n[k,k/m;q]_n}{[aq/\rho_1,aq/\rho_2;q]_n[q,mq;q]_n}, \tag{3.38}$$

where $m = k\rho_1\rho_2/aq$.

(xx) Using the WP-Bailey pair given in (2.5) and (2.6) in theorem 2; (1.9)-(1.10), we get the new Bailey pair,

$$\alpha'_n(a,k;q) = \frac{[m;q]_{2n}}{[k;q]_{2n}} \left(\frac{k}{a}\right)^n \frac{[a,q\sqrt{a},-q\sqrt{a},a/m;q]_n}{[q,\sqrt{a},-\sqrt{a},mq;q]_n} \tag{3.39}$$

$$\beta'_n(a,k;q) = \frac{[k/m;q]_n}{[q;q]_n}, \tag{3.40}$$

where $m = a^2q/k$.

(xxi) Using the WP-Bailey pair given in (2.5) and (2.6) in theorem 3; (1.11)-(1.12), we get the new Bailey pair,

$$\alpha'_n(a,k;q) = \left(\frac{1-\sigma k^{1/2}}{1-\sigma k^{1/2}q^n}\right)\left(\frac{1+\sigma m^{1/2}q^n}{1+\sigma m^{1/2}}\right)\frac{[m;q]_{2n}}{[k;q]_{2n}} \times$$

$$\times \left(\frac{k}{a}\right)^n \frac{[a,q\sqrt{a},-q\sqrt{a},a/m;q]_n}{[q,\sqrt{a},-\sqrt{a},mq;q]_n} \tag{3.41}$$

$$\beta'_n(a,k;q) = \frac{[k/m;q]_n}{[q;q]_n}, \tag{3.42}$$

where $m = a^2/k$ and $\sigma \in (-1,1)$.

(xxii) Using the WP-Bailey pair given in (2.5) and (2.6) in theorem 4; (1.13)-(1.14), we get the new Bailey pair,

$$\alpha'_n(a^2,k;q^2) = \frac{[a,q\sqrt{a},-q\sqrt{a},a/m;q]_n}{[q,\sqrt{a},-\sqrt{a},mq;q]_n}\left(\frac{m}{a}\right)^n \tag{3.43}$$

$$\beta'_n(a^2,k;q^2)=\frac{[-mq;q]_{2n}[k/m^2;q^2]_n[k;q^2]_n}{[-aq;q]_{2n}[q^2;q^2]_n[m^2q^2;q^2]_n}\left(\frac{m}{a}\right)^n, \tag{3.44}$$

where m=k/aq.

(xxiii) Using the WP-Bailey pair given in (2.5) and (2.6) in theorem 5; (1.15)-(1.16), we get the new Bailey pair,

$$\alpha'_n(a^2,k;q^2)=q^{-n}\left(\frac{1+aq^{2n}}{1+a}\right)\frac{[a,q\sqrt{a},-q\sqrt{a},a/m;q]_n}{[q,\sqrt{a},-\sqrt{a},mq;q]_n}\left(\frac{m}{a}\right)^n \tag{3.45}$$

$$\beta'_n(a^2,k;q^2)=\frac{[-mq;q]_{2n}[k/m^2;q^2]_n[k;q^2]_n}{[-a;q]_{2n}[q^2;q^2]_n[m^2q^2;q^2]_n}\left(\frac{m}{a}\right)^n \qquad (3.46),$$

where $m=k/a$.

(xxiii) Using the WP-Bailey pair given in (2.5) and (2.6) in theorem 6; (1.17)-(1.18), we get the new Bailey pair,

$$\alpha'_{2n}(a,k;q)=\frac{[a,q^2\sqrt{a},-q^2\sqrt{a},a/m;q^2]_n}{[q^2,\sqrt{a},-\sqrt{a},mq^2;q^2]_n}\left(\frac{m}{a}\right)^n \qquad \alpha'_{2n+1}=0, \tag{3.47}$$

$$\beta'_n(a,k;q)=\frac{[mq;q^2]_n[k/m;q]_n[k;q]_n}{[aq;q^2]_n[q;q]_n[mq;q]_n}\left(-\frac{k}{a}\right)^n, \tag{3.48}$$

where $m=k^2/a$.

4. Transformation and summation formulae for q-series

In this section we shall establish transformation and summation formulae for basic hypergeometric series by making use of (1.3) and new WP Bailey pairs established in previous section.

(i) Using the WP-Bailey pair of (3.1)-(3.2) in (1.3) we get the transformation formula,

$$\begin{aligned}&{}_8\Phi_7\left[\begin{matrix}m,q\sqrt{m},-q\sqrt{m},mb/a,\rho_1,\rho_2,kq^n,q^{-n};q;q/b\\ \sqrt{m},-\sqrt{m},aq/b,aq/\rho_1,aq/\rho_2,mq^{1-n}/k,mq^{1+n}\end{matrix}\right]\\ &=\frac{[mq,aq/\rho_1,aq/\rho_2,k/a;q]_n}{[aq,k/m,mq/\rho_1,mq/\rho_2;q]_n}\times\\ &{}_8\Phi_7\left[\begin{matrix}a,q\sqrt{a},-q\sqrt{a},b,\rho_1,\rho_2,kq^n,q^{-n};q;aq/mb\\ \sqrt{a},-\sqrt{a},aq/b,aq/\rho_1,aq/\rho_2,aq^{1-n}/k,aq^{1+n}\end{matrix}\right]\end{aligned} \tag{4.1}$$

where $m=\dfrac{k\rho_1\rho_2}{aq}$.

(ii) Using the WP-Bailey pair of (3.3)-(3.4) in (1.3) we get the following transformation formula,

$$ {}_3\Phi_2\left[\begin{matrix}m,mb/a,q^{-n};q;q/b\\ aq/b,mq^{1-n}/k\end{matrix}\right]=\frac{[k,k/a;q]_n}{[aq;q]_n[k/m;q]_n}\times$$

$$\times {}_{10}\Phi_9\left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, b, \sqrt{m}, -\sqrt{m}, \sqrt{mq}, -\sqrt{mq}, kq^n, q^{-n}; q; aq/mb \\ \sqrt{a}, -\sqrt{a}, aq/b, \sqrt{k}, -\sqrt{k}, \sqrt{kq}, -\sqrt{kq}, aq^{1-n}/k, aq^{1+n} \end{array}\right], \quad (4.2)$$

where $m = a^2q/k$.

(iii) Using the WP-Bailey pair of (3.5)-(3.6) in (1.3) we get the following transformation formula,

$$\left(\frac{1-\sigma\sqrt{k}}{1-\sigma q^n\sqrt{k}}\right)\frac{[k/m;q]_n[aq;q]_n}{[k,k/a;q]_n}\,{}_4\Phi_3\left[\begin{array}{l} m, mb/a, -\sigma q\sqrt{m}q^{-n}; q; q/b \\ aq/b, \sigma\sqrt{m}, mq^{1-n}/k \end{array}\right]$$

$$= {}_{10}\Phi_9\left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, b, \sigma\sqrt{k}, -\sigma q\sqrt{m}, \sqrt{m}, -\sqrt{m}, \sqrt{mq}, -\sqrt{mq}; q; aq/mb \\ \sqrt{a}, -\sqrt{a}, aq/b, \sigma q\sqrt{k}, -\sigma\sqrt{m}, \sqrt{k}, -\sqrt{k}, \sqrt{kq}, -\sqrt{kq} \end{array}\right], \quad (4.3)$$

where $m = a^2/k$ and $\sigma \in (-1, 1)$.

(iv) Using the WP-Bailey pair of (3.7)-(3.8) in (1.3) we get the following transformation formula,

$$\frac{[-mq;q]_{2n}[k/m^2;q^2]_n}{[-aq;q]_{2n}[m^2q^2;q^2]_n}\left(\frac{m}{a}\right)^n\frac{[a^2q^2;q^2]_n}{[k/a^2;q^2]_n}\times$$

$${}_8\Phi_7\left[\begin{array}{l} m, q\sqrt{m}, -q\sqrt{m}, \frac{mb}{a}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; \frac{amq^2}{bk} \\ \sqrt{m}, -\sqrt{m}, \frac{aq}{b}, \frac{m}{\sqrt{k}}q^{1-n}, -\frac{m}{\sqrt{k}}q^{1-n}, mq^{1+n}, -mq^{1+n} \end{array}\right]$$

$$= {}_8\Phi_7\left[\begin{array}{l} a, , q\sqrt{a}, -q\sqrt{a}, b, q\sqrt{k}, -q\sqrt{k}, q^{-n}, -q^{-n}; q; \frac{aq}{bk} \\ \sqrt{a}, -\sqrt{a}, \frac{aq}{b}, \frac{a}{\sqrt{k}}q^{1-n}, -\frac{a}{\sqrt{k}}q^{1-n}, aq^{1+n}, -aq^{1+n} \end{array}\right], \quad (4.4)$$

where m=k/aq.

(v) Using the WP-Bailey pair of (3.9)-(3.10) in (1.3) we get the following transformation formula,

$$\frac{[-mq;q]_{2n}[k/m^2,k;q^2]_n}{[-a;q]_{2n}[m^2q^2;q^2]_n}\left(\frac{m}{aq}\right)^n\times$$

$${}_8\Phi_7\left[\begin{array}{l} m, q\sqrt{m}, -q\sqrt{m}, \frac{mb}{a}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; \frac{amq^2}{bk} \\ \sqrt{m}, -\sqrt{m}, \frac{aq}{b}, \frac{m}{\sqrt{k}}q^{1-n}, -\frac{m}{\sqrt{k}}q^{1-n}, mq^{1+n}, -mq^{1+n} \end{array}\right]$$

$$= \frac{[k/a^2,k;q^2]_n}{[a^2q^2;q^2]_n}\times$$

$$= {}_{10}\Phi_9\left[\begin{matrix} a,, q\sqrt{a}, -q\sqrt{a}, b, iq\sqrt{a}, -iq\sqrt{a}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; \frac{a^2q}{k} \\ \sqrt{a}, -\sqrt{a}, \frac{aq}{b}, i\sqrt{a}, -i\sqrt{a}, \frac{a}{\sqrt{k}}q^{1-n}, -\frac{a}{\sqrt{k}}q^{1-n}, aq^{1+n}, -aq^{1+n} \end{matrix}\right], \tag{4.5}$$

where $m = \frac{k}{a}, \left|\frac{a^2q}{k}\right| < 1, \left|\frac{q^2}{b}\right| < 1.$

(vi) Using the WP-Bailey pair of (3.11)-(3.12) in (1.3) we get the following transformation formula,

$$\frac{[mq;q^2]_n[k,k/m;q]_n}{[aq;q^2]_n[mq;q]_n}\left(-\frac{k}{a}\right)^n\sum_{r=0}^{[n/2]}\left(\frac{1-mq^{2r}}{1-m}\right)\times$$

$$\times\frac{[q^{-n},kq^n;q]_{2r}[m,mb/a;q^2]_r}{[mq^{1-n}/k,mq^{1+n};q]_{2r}[q^2,aq^2/b;q^2]_r}\left(\frac{q}{b}\right)^r$$

$$=\frac{[k,k/a;q]_n}{[aq;q]_n}\sum_{r=0}^{[n/2]}\frac{[q^{-n},kq^n;q]_{2r}}{[aq^{1-n}/k,aq^{1+n};q]_{2r}}\left(\frac{aq}{k}\right)^{2r}\times$$

$$\times\frac{[a,q^2\sqrt{a},-q^2\sqrt{a},b;q^2]_r}{[q^2,\sqrt{a},-\sqrt{a},aq^2/b;q^2]_r b^r}. \tag{4.6}$$

(vii) Using the WP-Bailey pair of (3.13)-(3.14) in (1.3) we get the following transformation formula,

$$\frac{[k/m,mq/\rho_1,mq/\rho_2;q]_n[aq;q]_n}{[mq,aq/\rho_1,aq/\rho_2;q]_n[k/a;q]_n}\times$$

$${}_{10}\Phi_9\left[\begin{matrix} m, q\sqrt{m}, -q\sqrt{m}, \rho_1, \rho_2, \frac{aq}{bc}, \frac{mb}{a}, \frac{mc}{a}, kq^n, q^{-n}; q; q \\ \sqrt{m}, -\sqrt{m}, \frac{mq}{\rho_1}, \frac{mq}{\rho_2}, \frac{bcm}{a}, \frac{aq}{b}, \frac{aq}{c}, \frac{mq^{1-n}}{k}, mq^{1+n} \end{matrix}\right]$$

$$= {}_{10}\Phi_9\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, \rho_1, \rho_2, \frac{a^2q}{bcm}, b, c, kq^n, q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, \frac{aq}{\rho_1}, \frac{aq}{\rho_2}, \frac{bcm}{a}, \frac{aq}{b}, \frac{aq}{c}, \frac{aq^{1-n}}{k}, aq^{1+n} \end{matrix}\right], \tag{4.7}$$

where $m = \frac{k\rho_1\rho_2}{aq}$.

(viii) Using the WP-Bailey pair of (3.15)-(3.16) in (1.3) we get the following transformation formula,

$$ {}_5\Phi_4\left[\begin{matrix} m, \frac{aq}{bc}, \frac{mb}{a}, \frac{mc}{a}, q^{-n}; q; q \\ \frac{mbc}{a}, \frac{aq}{b}, \frac{aq}{c}, \frac{mq^{1-n}}{k} \end{matrix}\right] = \frac{[k, k/a; q]_n}{[aq, k/m; q]_n} \times $$

$$ {}_{12}\Phi_{11}\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, \frac{a^2q}{bcm}, \sqrt{m}, -\sqrt{m}, \sqrt{mq}, -\sqrt{mq}, kq^n, q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, \sqrt{k}, -\sqrt{k}, \sqrt{kq}, -\sqrt{kq}, \frac{aq}{b}, \frac{aq}{c}, \frac{bcm}{a}, \frac{aq^{1-n}}{k}, aq^{1+n} \end{matrix}\right], \quad (4.8) $$

where $m = \dfrac{a^2q}{k}$.

(ix) Using the WP-Bailey pair of (3.17)-(3.18) in (1.3) we get the following transformation formula,

$$ \left(\frac{1-\sigma\sqrt{k}}{1-\sigma q^n\sqrt{k}}\right)\frac{[k/m; q]_n}{[q; q]_n}\, {}_6\Phi_5\left[\begin{matrix} m, \frac{aq}{bc}, \frac{mb}{a}, \frac{mc}{a}, -\sigma q\sqrt{m}, q^{-n}; q; q \\ \frac{mbc}{a}, \frac{aq}{b}, \frac{aq}{c}, -\sigma\sqrt{m}, \frac{mq^{1-n}}{k} \end{matrix}\right] $$

$$ = \frac{[k, k/a; q]_n}{[q, aq; q]_n} \times $$

$$ {}_{14}\Phi_{13}\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, \frac{a^2q}{bcm}, \sigma\sqrt{k}, -\sigma q\sqrt{m}, \sqrt{m}, -\sqrt{m}, \sqrt{mq}, -\sqrt{mq}, kq^n, q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, \sqrt{k}, -\sqrt{k}, \sqrt{kq}, -\sqrt{kq}, \frac{aq}{b}, \frac{aq}{c}, \frac{bcm}{a}, \sigma\sqrt{m}, -\sigma q\sqrt{k}, \frac{aq^{1-n}}{k}, aq^{1+n} \end{matrix}\right], \quad (4.9) $$

where $m = \dfrac{a^2}{k}$ and $\sigma \in (-1, 1)$.

(x) Using the WP-Bailey pair of (3.19)-(3.20) in (1.3) we get the following transformation formula,

$$ \frac{[-mq; q]_{2n}[k/m^2; q^2]_n[k; q^2]_n}{[-aq; q]_{2n}[q^2; q^2]_n[m^2q^2; q^2]_n}\left(\frac{m}{a}\right)^n \times $$

$$ {}_{10}\Phi_9\left[\begin{matrix} m, q\sqrt{m}, -q\sqrt{m}, \frac{mb}{a}, \frac{mc}{a}, \frac{aq}{bc}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; q \\ \sqrt{m}, -\sqrt{m}, \frac{bcm}{a}, \frac{aq}{b}, \frac{aq}{c}, \frac{mq^{1-n}}{\sqrt{k}}, -\frac{mq^{1-n}}{\sqrt{k}}, mq^{1+n}, -mq^{1+n} \end{matrix}\right] $$

$$ = \frac{[k, k/a^2; q^2]_n}{[q^2, a^2q^2; q^2]_n} \times $$

$$\times {}_{10}\Phi_9\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, \dfrac{a^2q}{bcm}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, \dfrac{bcm}{a}, \dfrac{aq}{b}, \dfrac{aq}{c}, \dfrac{aq^{1-n}}{\sqrt{k}}, -\dfrac{aq^{1-n}}{\sqrt{k}}, aq^{1+n}, -aq^{1+n} \end{matrix}\right] \tag{4.10}$$

where m=k/aq.

(xi) Using the WP-Bailey pair of (3.21)-(3.22) in (1.3) we get the following transformation formula,

$$\frac{[-mq;q]_{2n}[k/m^2;q^2]_n[k;q^2]_n}{[-a;q]_{2n}[q^2;q^2]_n[m^2q^2;q^2]_n}\left(\frac{m}{aq}\right)^n \times$$

$${}_{10}\Phi_9\left[\begin{matrix} m, q\sqrt{m}, -q\sqrt{m}, \dfrac{mb}{a}, \dfrac{mc}{a}, \dfrac{aq}{bc}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; q \\ \sqrt{m}, -\sqrt{m}, \dfrac{bcm}{a}, \dfrac{aq}{b}, \dfrac{aq}{c}, \dfrac{mq^{1-n}}{\sqrt{k}}, -\dfrac{mq^{1-n}}{\sqrt{k}}, mq^{1+n}, -mq^{1+n} \end{matrix}\right]$$

$$= \frac{[k, k/a^2; q^2]_n}{[q^2, a^2q^2; q^2]_n}\times$$

$$\times {}_{12}\Phi_{11}\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, \dfrac{a^2q}{bcm}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}, iq\sqrt{a}, -iq\sqrt{a}; q; q \\ \sqrt{a}, -\sqrt{a}, \dfrac{bcm}{a}, \dfrac{aq}{b}, \dfrac{aq}{c}, \dfrac{aq^{1-n}}{\sqrt{k}}, -\dfrac{aq^{1-n}}{\sqrt{k}}, aq^{1+n}, -aq^{1+n}, -i\sqrt{a}, i\sqrt{a} \end{matrix}\right] \tag{4.11}$$

where m=k/a.

(xii) Using the WP-Bailey pair of (3.23)-(3.24) in (1.3) we get the following transformation formula,

$$\frac{[mq;q^2]_n[k,k/m;q]_n}{[aq;q^2]_n[q,mq;q]_n}\left(-\frac{k}{a}\right)\sum_{n=0}^{[n/2]}\left(\frac{1-mq^{2r}}{1-m}\right)\times$$

$$\times\frac{[m, aq^2/bc, mb/a, mc/a; q^2]_r[kq^n;q]_{2r}[q^{-n};q]_{2r}q^{2r}}{[q^2, aq^2/b, aq^2/c, mbc/a; q^2]_r[mq^{1+n};q]_{2r}[mq^{1-n}/k;q]_{2r}}$$

$$= \frac{[k, k/a; q]_n}{[q, aq; q]_n}\sum_{r=0}^{[n/2]}\frac{[q^{-n}, kq^n; q]_{2r}}{[aq^{1-n}/k, aq^{1+n}; q]_{2r}}\times$$

$$\times\frac{[a, q^2\sqrt{a}, -q^2\sqrt{a}, b, c, a^2q^2/bcm; q^2]_r q^{2r}}{[q^2, \sqrt{a}, -\sqrt{a}, aq^2/b, aq^2/c, bcm/a; q^2]_r}, \tag{4.12}$$

where $m = k^2/a$.

(xiii) Using the WP-Bailey pair of (3.25)-(3.26) in (1.3) we get the following transformation formula,

$$
{}_8\Phi_7\left[\begin{array}{l} m, q\sqrt{m}, -q\sqrt{m}, m/a, \rho_1, \rho_2, kq^n, q^{-n}; q; q \\ \sqrt{m}, -\sqrt{m}, aq, mq/\rho_1, mq/\rho_2, mq^{1-n}/k, mq^{1+n} \end{array}\right]
$$

$$
= \frac{[mq, aq/\rho_1, aq/\rho_2, k/a; q]_n}{[aq, k/m, mq/\rho_1, mq/\rho_2; q]_n}, \tag{4.13}
$$

where $m = \dfrac{k\rho_1\rho_2}{aq}$.

(xiv) Using the WP-Bailey pair of (3.27)-(3.28) in (1.3) we get the Saalschiitzian summation formula,

$$
{}_3\Phi_2\left[\begin{array}{l} m, m/a, q^{-n}; q; q \\ aq, mq^{1-n}/k \end{array}\right] = \frac{[k, k/a; q]_n}{[k/m, aq; q]_n}, \tag{4.14}
$$

where $m = \dfrac{a^2q}{k}$.

(xv) Using the WP-Bailey pair of (3.29)-(3.30) in (1.3) we get,

$$
{}_4\Phi_3\left[\begin{array}{l} m, m/a, -\sigma q\sqrt{m}, q^{-n}; q; q \\ aq, -\sigma\sqrt{m}, mq^{1-n}/k \end{array}\right] = \frac{[k, k/a, \sigma q\sqrt{k}; q]_n}{[aq, k/m, \sigma\sqrt{k}; q]_n}, \tag{4.15}
$$

where $m = a^2/k$ and $\sigma \in (-1, 1)$.

(xvi) Using the WP-Bailey pair of (3.31)-(3.32) in (1.3) we get,

$$
{}_8\Phi_7\left[\begin{array}{l} m, \frac{m}{a}, q\sqrt{m}, -q\sqrt{m}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; q \\ aq, \sqrt{m}, -\sqrt{m}, \frac{m}{\sqrt{k}}q^{1-n}, -\frac{m}{\sqrt{k}}q^{1-n}, mq^{1+n}, -mq^{1+n} \end{array}\right]
$$

$$
= \frac{[-aq; q]_{2n}[m^2q^2; q^2]_n[k/a^2; q^2]_n}{[-mq; q]_{2n}[k/m^2; q^2]_n[a^2q^2; q^2]_n}\left(\frac{a}{m}\right)^n, \tag{4.16}
$$

where m=k/aq.

(xvii) Using the WP-Bailey pair of (3.33)-(3.34) in (1.3) we get,

$$
{}_8\Phi_7\left[\begin{array}{l} m, \frac{m}{a}, q\sqrt{m}, -q\sqrt{m}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; q^2 \\ aq, \sqrt{m}, -\sqrt{m}, \frac{m}{\sqrt{k}}q^{1-n}, -\frac{m}{\sqrt{k}}q^{1-n}, mq^{1+n}, -mq^{1+n} \end{array}\right]
$$

$$= \frac{[-a;q]_{2n}[m^2q^2;q^2]_n[k/a^2;q^2]_n}{[-mq;q]_{2n}[k/m^2;q^2]_n[a^2q^2;q^2]_n}\left(\frac{aq}{m}\right)^n, \tag{4.17}$$

where m=k/a.

(xviii) Using the WP-Bailey pair of (3.35)-(3.36) in (1.3) we get,

$$\sum_{r=0}^{[n/2]}\left(\frac{1-mq^{2r}}{1-m}\right)\frac{[m,m/a;q^2]_r[q^{-n};q]_{2r}[kq^n;q]_{2r}q^{2r}}{[q^2,aq^2;q^2]_r[mq^{1-n}/k;q]_{2r}[mq^{1+n};q]_{2r}}$$

$$= \frac{[aq;q^2]_n[mq;q]_n[k/a;q]_n(-a/k)^n}{[mq;q^2]_n[k/m;q]_n[aq;q]_n}, \tag{4.18}$$

where $m = k^2/a$.

(xix) Using the WP-Bailey pair of (3.37)-(3.38) in (1.3) we get,

$${}_8\Phi_7\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, a/m, \rho_1, \rho_2, kq^n, q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, mq, aq/\rho_1, aq/\rho_2, aq^{1-n}/k, aq^{1+n} \end{matrix}\right]$$

$$= \frac{[aq, mq/\rho_1, mq/\rho_2, k/m; q]_n}{[mq, k/a, aq/\rho_1, aq/\rho_2; q]_n}, \tag{4.19}$$

where $m = \dfrac{k\rho_1\rho_2}{aq}$.

(xx) Using the WP-Bailey pair of (3.39)-(3.40) in (1.3) we get,

$${}_{10}\Phi_9\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, \dfrac{a}{m}, \sqrt{m}, -\sqrt{m}, \sqrt{mq}, -\sqrt{mq}, kq^n, q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, \sqrt{k}, -\sqrt{k}, \sqrt{kq}, -\sqrt{kq}, mq, \dfrac{aq^{1-n}}{k}, aq^{1+n} \end{matrix}\right]$$

$$= \frac{[aq, k/m; q]_n}{[k, k/a; q]_n}, \tag{4.20}$$

where $m = \dfrac{a^2q}{k}$.

(xxi) Using the WP-Bailey pair of (3.41)-(3.42) in (1.3) we get,

$${}_{12}\Phi_{11}\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, \dfrac{a}{m}, \sigma\sqrt{k}, -\sigma q\sqrt{m}, \sqrt{m}, -\sqrt{m}, \sqrt{mq}, -\sqrt{mq}, kq^n, q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, \sqrt{k}, -\sqrt{k}, \sqrt{kq}, -\sqrt{kq}, mq, -\sigma\sqrt{m}, \sigma q\sqrt{k}, \dfrac{aq^{1-n}}{k}, aq^{1+n} \end{matrix}\right]$$

$$= \frac{[aq, k/m; q]_n}{[k, k/a; q]_n}, \tag{4.21}$$

where $m = \frac{a^2}{k}$ and $\sigma \in (-1, 1)$.

(xxii) Using the WP-Bailey pair of (3.43)-(3.44) in (1.3) we get,

$$ {}_8\Phi_7 \left[\begin{matrix} a, \frac{a}{m}, q\sqrt{a}, -q\sqrt{a}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; q \\ mq, \sqrt{a}, -\sqrt{a}, \frac{a}{\sqrt{k}}q^{1-n}, -\frac{a}{\sqrt{k}}q^{1-n}, aq^{1+n}, -aq^{1+n} \end{matrix} \right] $$

$$ = \frac{[-mq; q]_{2n}[a^2q^2; q^2]_n[k/m^2; q^2]_n}{[-aq; q]_{2n}[k/a^2; q^2]_n[m^2q^2; q^2]_n} \left(\frac{m}{a}\right)^n, \tag{4.22} $$

where m=k/aq.

(xxiii) Using the WP-Bailey pair of (3.45)-(3.46) in (1.3) we get,

$$ = {}_{10}\Phi_9 \left[\begin{matrix} a, , q\sqrt{a}, -q\sqrt{a}, a/m, iq\sqrt{a}, -iq\sqrt{a}, q^n\sqrt{k}, -q^n\sqrt{k}, q^{-n}, -q^{-n}; q; q \\ \sqrt{a}, -\sqrt{a}, mq, i\sqrt{a}, -i\sqrt{a}, \frac{a}{\sqrt{k}}q^{1-n}, -\frac{a}{\sqrt{k}}q^{1-n}, aq^{1+n}, -aq^{1+n} \end{matrix} \right], \tag{4.5} $$

$$ = \frac{[-mq; q]_{2n}[a^2q^2; q^2]_n[k/m^2; q^2]_n}{[-a; q]_{2n}[k/a^2; q^2]_n[m^2q^2; q^2]_n} \left(\frac{m}{a}\right)^n, \tag{4.23} $$

where m=k/a.

(xxiv) Using the WP-Bailey pair of (3.47)-(3.48) in (1.3) we get,

$$ {}_8\Phi_7 \left[\begin{matrix} a, \frac{a}{m}, q^2\sqrt{a}, -q^2\sqrt{a}, kq^{n+1}, kq^n, q^{-n}, q^{1-n}; q^2; q^2 \\ mq^2, \sqrt{a}, -\sqrt{a}, \frac{aq^{1-n}}{k}q^{1-n}, \frac{aq^{2-n}}{kq^{1-n}}, aq^{1+n}, aq^{2+n} \end{matrix} \right] $$

$$ = \frac{[mq; q^2]_n[aq, k/m; q]_n}{[aq; q]_n[mq, k/a; q]_n} \left(-\frac{k}{a}\right)^n, \tag{4.24} $$

where $m = k^2/a$.

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