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NEW INTUITIONISTIC FUZZY SCORE FUNCTION AND ITS APPLICATION

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Abstract: In this paper we introduce a new score function and accuracy function for Intuitionistic fuzzy sets, which connects membership function, non-membership function and hesitancy. Ranking of intuitionistic fuzzy numbers based on the new score function is discussed. It is also applied in Medical Diagnosis.

Keywords and Phrases: Intuitionistic Fuzzy Sets, Intuitionistic Fuzzy Numbers, Score Function, Accuracy Function, Medical Diagnosis.

2020 Mathematics Subject Classification: 03E72.

1. Introduction

K. T. Atanassov in 1986 [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS), characterized by a membership function and a non-membership function. It is more suitable for dealing with uncertainty. The concept of IFS has been widely studied and applied in various areas such as decision making problem, medical diagnosis, pattern recognition etc. Intuitionistic Fuzzy numbers (IFN) are special kind of IFS which are fit more suitably to describe uncertainty. In practical decision making situations ranking or comparing several Intuitionistic fuzzy values is important.

In 1994 Chen and Tan [4] developed a score function and utilized it in multiple attribute decision making problems based on vague sets. To evaluate the accuracy level of vague values, Hong and Choi [8] developed an accuracy function. Based on score function and accuracy function Z. Xu [13] developed a method for the comparison between two Intuitionistic Fuzzy Values. Kharal [8] proposed three types of score functions to compute the net predisposition of positive and negative outcomes. The first score function is defined as the degree of membership minus the product of the non-membership and hesitation degrees. The second score function is similar but subtracts the arithmetic mean of the non-membership and hesitation degrees. The third score function is defined as the arithmetic mean of the membership and non-membership degrees minus the hesitation degree. Many researches have done on the applications of score functions. At present the concept of score functions has been found to be useful in diverse fields including similarity measures, aggregation operators, ranking procedures, Choquet integrals, preference relations, programming models, multiple attribute decision making and group decision making.

In this paper we propose a new Intuitionistic Fuzzy Score function and accuracy function to compare IFNs. To show the effectiveness of the score function we have given some examples and applied it in medical diagnosis.

2. Preliminaries

Definition 2.1. [1] *Let X be a given set. An Intuitionistic Fuzzy Set A in X is given*
$A = (x, \mu_A(x), \nu_A(x))|x\varepsilon X$, *where* $\mu_A, \nu_A : X \to [0, 1]$, *and* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.
$\mu_A(x)$ *is the degree of membership of the element x in A and $\nu_A(x)$ is the degree of nonmembership of x in A. For each $x\varepsilon X$, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the degree of hesitation.*

Definition 2.2. [3] *An Intuitionistic Fuzzy Set $\tilde{A}$ is called an Intuitionistic Fuzzy Number if it satisfies the following conditions,*
1.$\tilde{A}_1$ is normal, (i.e) there exists at least two points x_0, $x_1 \in X$ such that $\mu_A(x_0) = 1$ and $\nu_A(x_0) = 1$.
2. $\tilde{A}_1$ is convex, (i.e) its membership function is fuzzy convex and its non-membership function is concave.
3. Its membership function is upper semi continuous and its non-membership function is lower semi continuous and the set $\tilde{A}_1$ is bounded.

Definition 2.3. [4, 5] *For any IFN $\alpha = (\mu_\alpha, \nu_\alpha)$, the Score of α can be evaluated by the score function denoted by S as follows: $S(\alpha) = \mu_\alpha - \nu_\alpha$ where $S(\alpha)\epsilon[-1, 1]$.*

Definition 2.4. [7] *For any IFN* $\alpha = (\mu_\alpha, \nu_\alpha)$*, the accuracy function is defined as* $H(\alpha) = \mu_\alpha + \nu_\alpha$ *where* $H(\alpha)\epsilon[0,1]$.

Definition 2.5. [5] *Let* $\alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1})$ *and* $\alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ *be two IFNs. Then* $S(\alpha_1) = \mu_{\alpha_1} - \nu_{\alpha_1}$ *and* $S(\alpha_2) = \mu_{\alpha_2} - \nu_{\alpha_2}$ *also* $H(\alpha_1) = \mu_{\alpha_1} + \nu_{\alpha_1}$ *and* $H(\alpha_2) = \mu_{\alpha_2} + \nu_{\alpha_2}$ *be the scores and accuracy functions of the IFNs* α_1 *and* α_2 *respectively. Then*
(I) If $S(\alpha_1) < S(\alpha_2)$ *then* $\alpha_1 < \alpha_2$.
(II) If $S(\alpha_1) = S(\alpha_2)$ *then,*
(i)If $H(\alpha_1) = H(\alpha_2)$ *then* $\alpha_1 = \alpha_2$.
(ii) $H(\alpha_1) < H(\alpha_2)$ *then* $\alpha_1 < \alpha_2$.

3. New Score and Accuracy functions

Definition 3.1. *The new Score function for IFN* $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *is given by* $S_1(\alpha) = \mu_\alpha - max(\nu_\alpha, \pi_\alpha)$*, where* $S_1(\alpha) \in [-1, 1]$.

Remark 3.1.2.
(a) If $\alpha = (1, 0, 0)$ *then* $S_1(\alpha)$*=1*
(b) If $\alpha = (0, 1, 0)$ *or* $\alpha = (0, 0, 1)$ *then clearly,* $S_1(\alpha)$*=-1.*

Definition 3.2. *The new Accuracy function for IFN* $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *is given by* $H_1(\alpha) = \mu_\alpha + max(\nu_\alpha, \pi_\alpha)$*, where* $H_1(\alpha) \in [0, 1]$.

4. Numerical Examples

4.1. Consider the IFNs $\alpha_1 = (0.65, 0.15, 0.2)$, $\alpha_2 = (0.75, 0.25, 0)$. Apply definition 3.1 to α_1 and α_2 we get $S_1(\alpha_1)$=0.65 - max(0.15,.2)= 0.4. $S_1(\alpha_2)$=0.75 - max(0.25,0)= 0.5.
Since $S_1(\alpha_2) > S_1(\alpha_1)$ we get $\alpha_2 > \alpha_1$.
4.2. Consider the IFNs $\alpha_1 = (0.65, 0.25, 0.1)$, $\alpha_2 = (0.5, 0.15, 0.35)$, $\alpha_3 =$ (0.55, 0.2, 0.25). By definition 3.1 we get $S_1(\alpha_1)$=0.65 - max(0.25,.1)= 0.4, $S_1(\alpha_2)$=0.5 - max(0.15,.35)= 0.15, $S_1(\alpha_3)$=0.55 - max(0.2,.25)= 0.3.
Order of the scores is $S_1(\alpha_1) > S_1(\alpha_3) > S_1(\alpha_2)$.
Therefore the ordering of α_1, α_2 and α_3 is given by $\alpha_1 > \alpha_3 > \alpha_2$.
4.3. Consider the IFNs $\alpha_1 = (0.75, 0.15, 0.1)$, $\alpha_2 = (0.65, 0.15, 0.2)$, $\alpha_3 =$ (0.55, 0.25, 0.2), $\alpha_4 = (0.45, 0.25, 0.3)$, $\alpha_5 = (0.8, 0.2, 0)$.
By definition 3.1 we get
$S_1(\alpha_1)$=0.75- max(0.15,.1)= 0.6, $S_1(\alpha_2)$=0.65-max(0.15,.2)= 0.45, $S_1(\alpha_3)$=0.55-max(0.25,.2)= 0.3, $S_1(\alpha_4)$=0.45-max(0.25,.3)= 0.15, $S_1(\alpha_5)$=0.8-max(0.2-0)= 0.6.
Here $S_1(\alpha_1) = S_1(\alpha_5) > S_1(\alpha_2) > S_1(\alpha_3) > S_1(\alpha_4)$.
So apply the new accuracy function given in definition 3.2, we get $H_1(\alpha_1)$=0.75+

max(0.15,.1)= 0.9 and $H_1(\alpha_5)$=0.8+ max(0.2,0)= 1.
So the ordering of α_1, α_2, α_3, α_4 and α_5 is given by $\alpha_5 > \alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$.

5. Illustration

Here we consider a medical diagnosis example to demonstrate the application of the proposed Score function.

Let there be four patients$P = (P_1, P_2, P_3, P_4)$ and the set of symptoms $S = (S_1 Temperature, S_2 Headache, S_3 Stomachpain, S_4 Cough, S_5 Chestpain)$. Let the set of diseases $D = (D_1 Viralfever$, $D_2 Malaria$, $D_3 Typhoid$, $D_4 Stomachproblem$, $D_5 Chestproblem)$. Table 1 represents the patient-symptom relation and Table 2 represents symptom-disease relation.

The relation between patients and symptoms are presented in Table 1. The relation between symptoms and diseases is presented in Table 2.

Table 1: The relation between patient and symptoms

Relation1	Temperature	Head Ache	Stomach Pain	Cough	Chest Pain
P_1	$(0.8, 0.1, 0.1)$	$(0.5, 0.2, 0.3)$	$(0.2, 0.8, 0.0)$	$(0.6, 0.2, 0.2)$	$(0.1, 0.6, 0.3)$
P_2	$(0.0, 0.8, 0.2)$	$(0.4, 0.4, 0.2)$	$(0.6, 0.1, 0.3)$	$(0.1, 0.7, 0.2)$	$(0.1, 0.8, 0.1)$
P_3	$(0.8, 0.1, 0.1)$	$(0.8, 0.1, 0.1)$	$(0.0, 0.6, 0.4)$	$(0.2, 0.7, 0.1)$	$(0.0, 0.5, 0.5)$
P_4	$(0.3, 0.6, 0.1)$	$(0.5, 0.4, 0.1)$	$(0.3, 0.4, 0.3)$	$(0.7, 0.2, 0.1)$	$(0.3, 0.4, 0.3)$

Now we calculate the score of all diseases from the set of symptoms by the proposed score function. For example, $Score_{Viralfever}(Temperature) = 0.4 - max(0, 0.6) = -.2$. Similarly, $Score_{Viralfever} = (-.2, -.2, -.6, .1, -.6)$ Also calculate score of each patient from the set of symptoms by the proposed Score function. For example, $Score_{p_1}(Temperature) = .8 - max(.1, .1) = .7$ Similarly, $Score_{p_1} = (.7, .2, -.6, .4, -.5)$.

Now we calculate the Score distance [15] between each patient and each disease as given below.

Table 2: The relation among Symptoms and Diseases

Relation2	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	$(0.4, 0.0, 0.6)$	$(0.7, 0.0, 0.3)$	$(0.3, 0.3, 0.4)$	$(0.1, 0.7, 0.2)$	$(0.1, 0.8, 0.1)$
Head Ache	$(0.3, 0.5, 0.2)$	$(0.2, 0.6, 0.2)$	$(0.6, 0.1, 0.3)$	$(0.2, 0.4, 0.4)$	$(0.0, 0.8, 0.2)$
Stomach Pain	$(0.1, 0.7, 0.2)$	$(0.0, 0.9, 0.1)$	$(0.2, 0.7, 0.1)$	$(0.8, 0.0, 0.2)$	$(0.2, 0.8, 0.0)$
Cough	$(0.4, 0.3, 0.3)$	$(0.7, 0.0, 0.3)$	$(0.2, 0.6, 0.2)$	$(0.2, 0.7, 0.1)$	$(0.2, 0.8, 0.0)$
Chest Pain	$(0.1, 0.7, 0.2)$	$(0.1, 0.8, 0.1)$	$(0.1, 0.9, 0.0)$	$(0.2, 0.7, 0.1)$	$(0.8, 0.1, 0.1)$

$d(Score_{P_1}, Score_{Viralfever}) = \sum_{k=1}^{n} \frac{1}{2k}|Score_{P_1} - Score_{Viralfever}| = \frac{1}{10}(|0.7-(-0.2)| + |0.2-(-0.2)| + |-0.6-(-0.6)| + |0.4-.1| + |-0.5-(-0.6)|) = \frac{1}{10}(0.9+0.4+0+0.3+0.1)$ $=0.17$ Similarly, we can obtain the other results as in table 3.

Table 3: Score-distance description between each patient and each disease

Relation5	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
P_1	0.17	**0.14**	0.24	0.38	0.46
P_2	0.25	0.38	0.18	**0.1**	0.32
P_3	0.25	0.28	**0.2**	0.22	0.42
P_4	**0.18**	0.27	0.29	0.23	0.39

From table 3, P_1 is diagnosed with Malaria, P_2 is diagnosed with Stomach Problem, P_3 is diagnosed with Typhoid and P_4 is diagnosed with Viral fever.
Note: If the score distance between a patient a particular disease is the shortest, the patient is likely to have that disease.

6. Conclusion

In this chapter we introduced a new score function and accuracy function for IFNs. It connects the membership function, non-membership function and the hesitancy part. To show the effectiveness of the proposed results we have given comparative numerical examples and applied in the field of medical diagnosis.

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