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# ON FUZZY GENERALIZED PRE REGULAR WEAKLY CONTINUITY

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**Abstract:** In this paper Fuzzy generalized pre regular weakly continuity has been introduced. Also, Fuzzy generalized pre regular weakly-irresolute functions, Fuzzy generalized pre regular weakly open mappings, Fuzzy generalized pre regular weakly closed mappings has been introduced and investigated.

**Keywords and Phrases:** Fuzzy generalized pre regular weakly continuity, Fuzzy generalized pre regular weakly-irresolute functions, Fuzzy generalized pre regular weakly open mappings, Fuzzy generalized pre regular weakly closed mappings.

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## 1. Introduction

The concept of fuzziness has been implemented in almost all branches of mathematics. With the advent of fuzzy theory in 1965 research has taken place at a rapid pace in this field and almost fuzzification of every branch of mathematics took place. Fuzzy topology introduced by C. L Chang [2] in 1968 is the generalization of the classical topology and with its introduction various fuzzy sets were introduced and studied. Fuzzy function, Fuzzy continuous function and fuzzy uniformly continuous functions were introduced and studied by Zhang Guang-Quan in 1989 in the paper [9] titled “Fuzzy continuous function and its properties”. In 1992 fuzzy-continuous functions were introduced and studied by Jae Rong Choi et.al [3]. In 2017 Fuzzy weakly e-closed functions were introduced and studied by

Veerappan Chandrasekar et.al [7]. And based on other fuzzy closed sets various fuzzy functions were introduced and studied in a timely manner.
In the same framework in this paper we have introduced fuzzy generalized pre regular weakly continuous functions and fuzzy generalized pre regular weakly-irresolute functions and find out their relation with other fuzzy functions and also investigated their some other properties. In this paper we also give a brief introduction of fuzzy generalized pre regular weakly open mappings, fuzzy generalized pre regular weakly closed mappings and fuzzy generalized pre regular weakly homeomorphism and find out some important results regarding these.

## 2. Preliminaries

**Definition 2.1.** [4] *Suppose $(Y, \tau)$ is a fuzzy topological space. Then a fuzzy subset $\lambda$of $(Y, \tau)$ is called fuzzy generalized pre regular weakly closed (briefly Fgprw-closed) if $pcl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ & $\mu$ is a fuzzy regular semi open set in $(Y, \tau)$.*

**Definition 2.2.** [4] *Suppose $(Y, \tau)$ is a fuzzy topological space and $\lambda \leq Y$ be a fuzzy subset. Then we call $\lambda$ fuzzy gprw-open (briefly Fgprw-open) iff $(1 - \lambda)$ is fuzzy gprw closed in $(Y, \tau)$.*

**Definition 2.3.** [8] *Let X be a space of objects, with a generic element of X denoted by x. Then a fuzzy set A in X is a set of ordered pairs $\{(x, f(x))\}$ where $f_A(x)$ is called the membership function which associates each point in X a real number in the interval [0,1].*

**Definition 2.4.** [2] *A collection $\tau$ of fuzzy subsets of a fuzzy space X is called fuzzy topology on X if fuzzy subsets with membership value 0 and 1 belong to $\tau$ and $\tau$ is closed with respect to arbitrary union and finite intersection. The elements of $\tau$ are called fuzzy open sets and their complements are called fuzzy closed sets. The space X with topology $\tau$ is called fuzzy topological space denoted by (X,$\tau$).*

**Definition 2.5.** [2] *For a fuzzy set $\alpha$ of $X$, the closure cl $\alpha$ and the interior int $\alpha$ of $\alpha$ are defined respectively, as*
$cl\alpha = \wedge\{\mu : \mu \geq \alpha, 1 - \mu \in \tau\}$*and*
$int\alpha = \vee\{\mu : \mu \leq \alpha, \mu \in \tau\}$

**Definition 2.6.** [6] *Let $(X, \tau)$ be a topological space. A fuzzy set $\lambda \in X$ is called a fuzzy w-closed set in $(X, \tau)$ if $cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and$\mu$ is fuzzy semi-open in $(X, \tau)$.*

**Definition 2.7.** [6] *A mapping $f : (X, T) \to (Y, S)$ is called fuzzy w-continuous if $f^{-1}(\lambda)$ is fuzzy w-closed in (X,$\tau$) for every fuzzy closed set $\lambda$ in$(Y, S)$.*

**Definition 2.8.** [5] *A fuzzy set in $X$ is called a fuzzy point iff it takes the value 0 for all $y \in X$ except one, say $x \in X$. If its value at $x$ is $\lambda(0 < \lambda \leq 1)$ we denote this fuzzy point by $x_\lambda$ , where the point $x$ is called its support.*

**Definition 2.9.** [5] *A fuzzy point $x_\lambda$ is said to be quasi-coincident with A, denoted by $x_\lambda q A$, iff $\lambda > A'(x)$, or $\lambda + A(x) > 1$.*

**Definition 2.10.** [5] *A fuzzy set $A$ in $(X, \tau)$ is called a fuzzy q-neighborhood (fuzzy q-nbhd)of $x_\lambda$ iff there exists $B \in \tau$ such that $x_\lambda q B < A$. The family consisting of all the q-neighborhoods of $x_\lambda$ is called the system of q-neighborhoods of $x_\lambda$.*

**Definition 2.11.** [1] *A mapping $f$ is called a fuzzy continuous mapping if $f^{-1}(\lambda) \in \tau X$ for each $\lambda \in \tau Y$ or, equivalently $f^{-1}(\mu)$ is a fuzzy closed set of $X$ for each fuzzy closed set $\mu$ of $Y$.*

## 3. Fuzzy Generalized Pre Regular Weakly Continuity

**Definition 3.1.** *For a Fuzzy generalized pre regular weakly closed set $\mu$ of $Y$, Fuzzy generalized pre regular weakly closure (symbolically cl_) and Fuzzy generalized pre regular weakly interior (symbolically Int_) of $\mu$ are respectively defined as,*

$$cl_{\_} = \wedge\{\lambda : \mu \leq \lambda, \lambda isFgprw - closed\}$$
$$Int_{\_} = \vee\{\lambda : \lambda \leq \mu, \lambda isFgprw - open\}.$$

**Definition 3.2.** *In fuzzy topological space $(U, \tau_1)$, a fuzzy set $A$ is called fuzzy generalized pre regular weakly quasi neighborhood (Fgprw-q-nbhd) if there exists a Fgprw-open set $B$ such that for an fuzzy point $u_\lambda$ we have $u_\lambda q B < U$.*

**Definition 3.3.** *A function $h : H \to K$ is called fuzzy generalized pre regular weakly continuous (briefly Fgprw-continuous) if inverse image of every fuzzy closed set in fuzzy topological space $K$ is fuzzy generalized pre regular weakly closed (Fgprw-closed) in fuzzy topological space $H$.*

**Definition 3.4.** *A function $h : (H, \tau_1) \to (K, \tau_2)$ is said to be fuzzy generalized pre regular weakly-irresolute (briefly Fgprw-irresolute) if $h^{-1}(\{\psi\})$ is fuzzy gprw-closed for every fuzzy gprw-closed $\{\psi\}$ in $(K, \tau_2)$.*

**Definition 3.5.** *A mapping $l : (G, \tau_1) \to (H, \tau_2)$ is called Fuzzy generalized pre regular weakly open (Fgprw-open) mapping if for each $\lambda \in \tau_1, l(\lambda)$ is fuzzy gprw-open in $H$.*

**Definition 3.6.** *A mapping $l : (G, \tau_1) \to (H, \tau_2)$ is called Fuzzy generalized pre regular weakly closed (Fgprw-closed) mapping if for each $\{\lambda : 1 - \lambda \in \tau_1\}, l(\lambda)$ is fuzzy gprw-closed in $H$.*

**Definition 3.7.** *A bijection $l : (G, \tau_1) \to (H, \tau_2)$ is said to be Fuzzy generalized pre regular weakly homeomorphism (Fgprw-homeomorphism) if $l$ is a Fgprw-continuous and Fgprw-open mapping.*

**Definition 3.8.** *A function $l : (G, \tau_1) \to (H, \tau_2)$ is said to be Fuzzy strongly generalized pre regular weakly continuous (Fsgprw-continuous) if $l^{-1}(\lambda) < G$ is a fuzzy closed set for every fuzzy gprw-closed set in $(H, \tau_2)$.*

**Theorem 3.9.** *Suppose $l : (M, \tau_1) \to (N, \tau_2)$ is a function,*

*(I) The following statements are equivalent.*

*(a) $l$ is Fgprw-continuous.*

*(b) Inverse image of every open fuzzy set in $N$ is Fgprw-open in $M$.*

*(II) If $l : (M, \tau_1) \to (N, \tau_2)$ is Fgprw-continuous then $l(cl_(\psi)) \leq cl(l(\psi))$ for a fuzzy set $\psi$ in $M$.*

*(III) The following statements are equivalent.*

*(a) For every fuzzy point $m_\lambda$ in $M$ and every fuzzy q-nbhd of $l(m_\lambda)$, There exists a Fgprw q-nbhd $R$ of $m_\lambda$ such that $l(R) \leq S$.*

*(b) For every fuzzy set $\psi$ in $M$, $l(cl\_(\psi)) \leq cl(l(\psi))$ .*

*(c) For every fuzzy set $\phi$ in $N$, $cl\_(l^{-1}(\phi)) \leq l^{-1}(cl(\phi))$.*

**Proof.**

(I) The equivalence of the given statements is obvious from the definitions.

(II) Suppose $\psi$ is a fuzzy set in $M$. Then $\psi \leq l^{-1}(l(\psi)) \leq l^{-1}(cl(l(\psi)))$ Now, as $l$ is Fgprw-continuous, so $cl\_(\psi) \leq l^{-1}(cl(l(\psi)))$. Implying $l(cl\_(\psi)) \leq cl(l(\psi))$.

(III) $(a) \Rightarrow (b)$ Suppose $n_\alpha \in l(cl\_(\psi))$ and $S$ is a fuzzy q-nbhd of $n_\alpha$. So their exists an $m_\alpha$ such that $l(m_\alpha) = n_\alpha$ and $m_\alpha \in cl\_(\psi)$, Now by $(a)$ their exists a Fgprw q-nbhd $R$ of $m_\alpha$ such that $l(R) \leq S$. Since $m_\alpha \in cl\_(\psi)$, $Rq\psi$ and hence $S_q l(\psi)$. Implying $n_\alpha = l(m_\alpha) \in cl(l(\psi))$.
$(b) \Rightarrow (a)$ Let $m_\alpha \in M$ and $S$ be a fuzzy q-nbhd of $l(m_\alpha)$ . Put $\psi = l^{-1}(1 - S)$. Then $m_\alpha \notin \psi$. Now $l(cl\_(\psi)) \leq cl(l(\psi)) \leq 1 - S$ implies $cl\_(\psi) \leq l^{-1}(1 - S) = \psi$, which in turn implies that $cl\_(\psi) = \psi$. So $m_\alpha \notin \psi$ implies $m_\alpha \notin cl(\psi)$ and hence there exists a Fgprw q-nbhd $R$ of $m_\alpha$ such that $R\overline{q}\psi$ , implying that $l(R) \leq l(1 - \psi) \leq S$.
$(b) \Rightarrow (c)$ Straightforward.

**Theorem 3.10.** *If $l : (M, \tau_1) \to (N, \tau_2)$ is a fuzzy continuous function, then it is Fgprw-continuous also.*

**Proof.** $l : (M, \tau_1) \to (N, \tau_2)$ is a fuzzy continuous function implies that for a fuzzy open set $\psi$ in $N$, $l^{-1}(\psi)$ is fuzzy open in $M$. Now by [4] $l^{-1}(\psi)$ is fuzzy gprw open, implying $l : (M, \tau_1) \to (N, \tau_2)$ is Fgprw-continuous.

**Theorem 3.11.** *All Fgprw-irresolute functions are Fgprw-continuous.*

**Proof.** Suppose $l : (M, \tau_1) \to (N, \tau_2)$ be a Fgprw-irresolute function. So for any fuzzy gprw-closed set $P$ in $(N, \tau_2)$, $l^{-1}(P)$ is fuzzy gprw-closed in $(M, \tau_1)$ . Now by [4] every fuzzy closed set is fuzzy gprw closed, implying $l^{-1}(P)$ is fuzzy closed in $(M, \tau_1)$ for a fuzzy gprw- closed set $P$ in $(N, \tau_2)$. Hence $l$ is a Fgprw-continuous function.

**Theorem 3.12.** *If $l : (M, \tau_1) \to (N, \tau_2)$ is a Fgprw-continuous function, then we have $l(cl_{-}(\psi)) < cl(l(\psi))$.*

**Proof.** Consider a fuzzy subset $\psi < M$, implying that $l(\psi) < N$. Now as $l$ is Fgprw-continuous, so $l^{-1}(cl(l(\psi)))$ is fuzzy gprw-closed set in $M$. Now $l(\psi) < cl(l(\psi))$ implying that $\psi < l^{-1}(cl(l(\psi)))$. So $cl_{-}(\psi) < cl_{-}(l^{-1}(cl(l(\psi)))$ . Now as $l^{-1}(cl(l(\psi)))$ is fuzzy gprw-closed, implying that $cl_{-}(\psi) < (l^{-1}(cl(l(\psi)))$ or $l(cl_{-}(\psi)) < cl(l(\psi))$ .

**Theorem 3.13.** *Suppose $U$, $V$ & $W$ are fuzzy topological spaces and $l : U \to V$, $m : V \to W$ are two mappings. Then the composition map mol is Fgprw-continuous if the following are true.*

*(I) $m$ is Fgprw-continuous and $l$ is Fgprw-irresolute.*

*(II) $m$ is a fuzzy continuous map and $l$ is Fgprw-continuous.*

**Proof.**

(I) let $\lambda$ be a fuzzy closed set in $W$. Now $(mol)^{-1}(\lambda) = l^{-1}(m^{-1}(\lambda))$, and as $m$ is Fgprw-continuous implies $A = m^{-1}(\lambda)$ is fuzzy gprw-closed in $V$. Also $l$ is Fgprw-irresolute, so $l^{-1}(A) = l^{-1}(m^{-1}(\lambda))$ is fuzzy gprw-closed in $U$, implying that $mol$ is a Fgprw-continuous mapping.

(II) Let $\lambda < W$ be a fuzzy closed set. As $m$ is a fuzzy continuous mapping, so $A = m^{-1}(\lambda)$ is fuzzy closed set in $V$. And as $l$ is Fgprw-continuous implies $l^{-1}(A) = l^{-1}(m^{-1}(\lambda)) = (mol)^{-1}(\lambda)$ is a fuzzy closed set in $U$, implying $mol$ is a Fgprw-continuous mapping.

**Theorem 3.14.** *Suppose $U$,$V$ & $W$ are fuzzy topological spaces and $l : U \to V$, $m : V \to W$ are Fgprw-irresolute functions then $mol$ is also Fgprw-irresolute.*

**Proof.** Let $\lambda < W$ be a fuzzy gprw-closed set. Now $m$ is Fgprw-irresolute map, implies $A = m^{-1}(\lambda)$ is fuzzy gprw closed in $V$. Also $l$ is Fgprw-irresolute map so $l^{-1}(A) = l^{-1}(m^{-1}(\lambda)) = (mol)^{-1}(\lambda)$ is fuzzy gprw-closed in $U$, implying $mol$ is also Fgprw-irresolute.

**Theorem 3.15.** *If $l : (M, \tau_1) \to (N, \tau_2)$ is a fuzzy w-continuous function, then it is Fgprw-continuous also.*

**Proof.** Let $l : (M, \tau_1) \to (N, \tau_2)$ is a fuzzy w-continuous function. So for a fuzzy open set $\psi < N$, $l^{-1}(\psi) < M$ is a fuzzy weakly closed set. Now by [4] $l^{-1}(\psi)$ is fuzzy gprw closed also, implying that $l$ is Fgprw-continuous.

**Theorem 3.16.** *For a bijection $l : (G, \tau_1) \to (H, \tau_2)$ the given statements are equivalent.*

*a)* $l^{-1} : (H, \tau_2) \to (G, \tau_1)$ *is Fgprw-continuous.*

*b)* $l$ *is Fgprw-open mapping.*

*c)* $l$ *is Fgprw-closed mapping.*

**Proof.**
$(a) \Rightarrow (b)$ By hypothesis and assumption $l$ is Fgprw-homeomorphism.

$(b) \Rightarrow (c)$ Since $l$ is Fgprw-homeomorphism, it is a Fgprw-open map so by Theorem 3.15 it is also a Fgprw-closed mapping.

$(c) \Rightarrow (a)$ Let $\lambda$ be a fuzzy open set in $G$. So $1 - \lambda$ is a fuzzy closed set and $l$ being a fuzzy gprw-closed set implies $l(1 - \lambda)$ is a fuzzy gprw-closed set in $Y$. But $l(1 - \lambda) = 1 - l(\lambda)$ implying $l(\lambda)$ is fuzzy gprw-open set in $H$. Hence $l$ is a Fgprw-open mapping.

**Theorem 3.17.** *A function $l : (G, \tau_1) \to (H, \tau_2)$ is Fsgprw-continuous iff $l^{-1}(\lambda)$ is fuzzy open in $G$ for arbitrary fuzzy gprw-open set $\lambda$ in $(H, \tau_2)$.*

**Proof.** let $l : (G, \tau_1) \to (H, \tau_2)$ be a Fsgprw-mapping and $\lambda < H$ be an arbitrary fuzzy gprw-open set. So $1 - \lambda$ is fuzzy gprw-closed and $l^{-1}(1 - \lambda) = 1 - l^{-1}(\lambda)$ is fuzzy closed in $G$. Hence $l^{-1}(\lambda)$ is fuzzy open in $G$. Conversely suppose that $l^{-1}(\lambda)$ is fuzzy open in $G$ for arbitrary fuzzy gprw-open set $\lambda$ in $H$. So for fuzzy gprw-closed set $\mu$ in $H$, $1 - \mu$ is fuzzy gprw-open and so $l^{-1}(1 - \mu) = 1 - l^{-1}(\mu)$ is fuzzy open in $G$ implying $l^{-1}(\mu)$ is fuzzy closed in $G$. Hence $l$ is Fsgprw-continuous.

**Theorem 3.18.** *Every Fsgprw-continuous map is Fgprw-irresolute.*

**Proof.** Let $l : (G, \tau_1) \to (H, \tau_2)$ is a Fsgprw-continuous mapping and let $\lambda$ is a fuzzy gprw-closed set in $H$, Then $l^{-1}(\lambda)$ is fuzzy closed in $G$. Now by [4] every fuzzy closed set is fuzzy gprw-closed. Implying $l^{-1}(\lambda)$ is fuzzy gprw- closed, so $l$ is Fgprw-irresolute.

**Theorem 3.19.** *: Let $l : (G, \tau_1) \to (H, \tau_2)$ and $k : (H, \tau_2) \to (M, \tau_3)$ be two functions, then the following are true.*

a) *Suppose $l$ and $k$ are Fsgprw-continuous mappings, then the composition map $kol : (G, \tau_1) \to (M, \tau_3)$ is also Fsgprw-continuous.*

b) *Suppose $l$ is fuzzy continuous and $k$ is Fsgprw-continuous, then $kol : (G, \tau_1) \to (M, \tau_3)$ is Fsgprw-continuous.*

c) *Suppose $l$ is Fgprw-continuous and $k$ is Fsgprw-continuous, then the composition map $kol : (G, \tau_1) \to (M, \tau_3)$ is Fgprw-irresolute.*

d) *Suppose $l$ is Fsgprw-continuous and $k$ is Fgprw-continuous, then the composition map $kol : (G, \tau_1) \to (M, \tau_3)$ is Fuzzy continuous.*

**Proof.**

a) Suppose $\lambda$ is a fuzzy gprw-open set in $(M, \tau_3)$. So as mapping $k$ is Fsgprw-continuous means $k^{-1}(\lambda)$ is fuzzy open in (H,$\tau_2$). Now by [4] every fuzzy open set is fuzzy gprw-open, so $k^{-1}(\lambda)$ is fuzzy gprw-open in (H,$\tau_2$). Also $l$ is Fsgprw-continuous implies $l^{-1}(k^{-1}(\lambda)) = (kol)^{-1}$ is fuzzy open in (G,$\tau_1$). Hence $kol$ is Fsgprw-continuous.

b) Suppose $\lambda < M$ is a fuzzy gprw-open set. Now $k$ is Fsgprw-continuous, so $k^{-1}(\lambda)$ is fuzzy open in (H,$\tau_2$). Now $l$ is fuzzy continuous implies $(kol)^{-1}(\lambda) = l^{-1}(k^{-1}(\lambda))$ is fuzzy open in $(G, \tau_1)$. Hence $kol$ is Fsgprw-continuous.

c) Suppose $\lambda < M$ be a fuzzy gprw-open set. Now $k$ is Fsgprw-continuous means $k^{-1}(\lambda)$ is fuzzy open in (H,$\tau_2$). Also $l$ is fuzzy gprw-continuous, so $(kol)^{-1}(\lambda) = l^{-1}(k^{-1}(\lambda))$ is fuzzy gprw-open set in $(G, \tau_1)$ , implying $kol$ : $(G, \tau_1) \to (M, \tau_3)$ is Fgprw-irresolute.

d) Suppose $\lambda < M$ is a fuzzy open set. Now as $k$ is a Fgprw-continuous mapping, so $k^{-1}(\lambda)$ is fuzzy gprw-open in (H,$\tau_2$). Also $l$ is Fsgprw-continuous implies $(kol)^{-1}(\lambda) = l^{-1}(k^{-1}(\lambda))$ is fuzzy open in $(G, \tau_1)$. Therefore $kol : (G, \tau_1) \to (M, \tau_3)$ is fuzzy continuous.

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