

μ_N DENSE SETS AND ITS NATURE

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Abstract: In this article we discuss the nature of contra-continuous functions in the generalized topological space via neutrosophic sets. Also we introduced a new type of set named as μ_N nowhere dense set and by making use of μ_N nowhere dense set we derive μ_N Baire Space and their features are to be discussed. The characters of μ_N rare sets also discussed briefly.

Keywords and Phrases: μ_N contra-continuous, μ_N perfectly continuous, μ_N dense sets, μ_N Rare set, μ_N Baire Spaces.

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1. Introduction

The concept of fuzziness had a great impact in all branches of mathematics which was put forth by Zadeh [13]. Later on, the idea of fuzziness and Topological spaces were put together by C. L. Chang [3] and laid a foundation to the theory of fuzzy topological spaces. By focussing the membership and non membership of the elements, K. T. Atanasov [1] made out intuitionistic fuzzy sets and he extended his research towards and gave out a generalization to intuitionistic L-fuzzy sets with his friend Stoeva [2]. F. Smarandache [7], [8] put his thoughts towards the degree of indeterminacy and bring forth the neutrosophic sets. Subsequently, the neutrosophic topological spaces with the help of neutrosophic sets were found out by A. A. Salama and S. A. Albowi [12]. By making all the works together as inspiration, we [11] made Generalized topological spaces via neutrosophic sets and named it as $\mu_N TS$. The neutrosophic nowhere dense sets in NTS were put forth by

R. Dhavaseelan [10]. Later on, R. Dhavaseelan [9] and his friends worked together and made out neutrosophic Baire spaces. In this paper, we contemplated all the important features of μ_N dense sets and μ_N Baire Space. Also a new type of μ_N continuous function called μ_N contra-continuous were introduced.

2. Preliminaries

The concepts given here helps us to recall our memories regarding the basic concepts of μ_N Topological Space.

Definition 2.1. [12] *Let X be a non-empty fixed set. A Neutrosophic set [NS for short] A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A .*

Remark 2.2. [12] *A neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ can be identified to an ordered triple $A = \{ \langle \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle \}$ in $]^{-0}, 1^+[$ on X .*

Remark 2.3. [12] *For the sake of simplicity, we shall use the symbol $A = \{ \langle \mu(x), \sigma(x), \gamma(x) \rangle \}$ for the neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$.*

Remark 2.4. [12] *Every intuitionistic fuzzy set A is a non empty set in X is obviously on Neutrosophic sets having the form $A = \{ \langle \mu_A(x), 1 - \mu_A(x) + \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$. In order to construct the tools for developing Neutrosophic Set and Neutrosophic topology, here we introduce the neutrosophic sets 0_N and 1_N in X as follows:*

0_N may be defined as follows

$$(0_1)0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$$

$$(0_2)0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$$

$$(0_3)0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$$

$$(0_4)0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$$

1_N may be defined as follows

$$(1_1)1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$$

$$(1_2)1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$$

$$(1_3)1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$$

$$(1_4)1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$$

Definition 2.5. [12] *Let $A = \{ \langle \mu_A, \sigma_A, \gamma_A \rangle \}$ be a NS on X , then the complement of the set A [$C(A)$ for short] may be defined in three ways as follows:*

$$(C_1)C(A) = \{ \langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \gamma_A(x) \rangle : x \in X \}$$

$$(C_2)C(A) = \{ \langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$$

$$(C_3)C(A) = \{ \langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$$

Definition 2.6. [12] Let X be a non-empty set and neutrosophic sets A and B in the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$. Then we may consider two possibilities for definitions for subsets ($A \subseteq B$).

$A \subseteq B$ may be defined as:

$$(A \subseteq B) \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x), \forall x \in X$$

$$(A \subseteq B) \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x), \forall x \in X$$

Proposition 2.7. [12] For any neutrosophic set A , the following conditions holds:

$$0_N \subseteq A, 0_N$$

$$A \subseteq 1_N, 1_N$$

Definition 2.8. [12] Let X be a non empty set and $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$, $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$ are NSs.

Then $A \cap B$ may be defined as:

$$(I_1)A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$$

$$(I_2)A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$$

$A \cup B$ may be defined as:

$$(I_1)A \cup B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$$

$$(I_2)A \cup B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$$

Definition 2.9. [11] A μ_N topology is a non - empty set X is a family of neutrosophic subsets in X satisfying the following axioms:

$$(\mu_{N_1})0_N \in \mu_N$$

$$(\mu_{N_2})G_1 \cup G_2 \in \mu_N \text{ for any } G_1, G_2 \in \mu_N.$$

Remark 2.10. [11] The elements of μ_N are μ_N -open sets and their complement is called μ_N closed sets.

Definition 2.11. [11] Let (X, μ_N) be a μ_N TS and $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle \}$ be a neutrosophic set in X . Then the μ_N -Closure of A is the intersection of all μ_N closed sets containing A .

Definition 2.12. [11] Let (X, μ_N) be a μ_N TS and $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle \}$ be a neutrosophic set in X . Then the μ_N -Interior of A is the union of all μ_N open sets contained in A .

Definition 2.13. [10] A neutrosophic set A in NTS is called neutrosophic dense if there exists no neutrosophic closed sets B in (X, T) such that $A \subset B \subset 1_N$.

Definition 2.14. [9] The neutrosophic topological spaces is said to be neutrosophic

Baire Space if $NInt(\cup_{i=1}^{\infty} G_i) = 0_N$ where G_i 's are neutrosophic nowhere dense set in (X, T) .

3. μ_N Contra-continuous

Definition 3.1. The mapping $f : (X, \gamma) \rightarrow (Y, \delta)$ is called as μ_N Contra-continuous if the inverse image of μ_N - closed set in (Y, δ) is μ_N - open set in (X, γ) .

Theorem 3.2. The mapping $f : (X, \gamma) \rightarrow (Y, \delta)$ is μ_N Contra-continuous if and only if the inverse image of μ_N -open set in (Y, δ) is μ_N -closed set in (X, γ) .

Proof. Requisite Condition: Let $f : (X, \gamma) \rightarrow (Y, \delta)$ be μ_N Contra-continuous and V be a μ_N - open set in (Y, δ) . From this we deduce that $f^{-1}(Y - V) = X - f^{-1}(V)$ is μ_N - open set in (X, γ) because f is μ_N Contra-continuous and hence we conclude that $f^{-1}(V)$ is μ_N -closed set in (X, γ) .

Good enough Condition: Assume that $f^{-1}(V)$ is μ_N - closed set in (X, γ) for each μ_N - open set in (Y, δ) . Let V be a μ_N - open set in (Y, δ) which obviously gives us that $Y - V$ is μ_N -closed set in (Y, δ) . Now, $f^{-1}(Y - V) = X - f^{-1}(V)$ is μ_N -open set in (Y, δ) which provides that $f^{-1}(V)$ is μ_N - closed set in (X, γ) . Hence, f is μ_N Contra-continuous.

Theorem 3.3. For a function $f : (X, \gamma) \rightarrow (Y, \delta)$ the following conditions are equivalent.

(i) f is μ_N Contra-continuous.

(ii) The inverse image of μ_N - open set in (Y, δ) is μ_N - closed set in (X, γ) .

(iii) For each $x \in X$ and each μ_N - closed set V in (Y, δ) with $f(x) \in V$ there exists a μ_N - open set U in (X, γ) such that $x \in U$ and $f(U) \subseteq V$.

Proof. (i) \Rightarrow (ii) The proof is similar to the proof of theorem 3.2.

(i) \Rightarrow (iii) Let B be a μ_N - closed set such that $f(x) \in B$. We have that the inverse image of μ_N - open set in (Y, δ) is μ_N - closed set in (X, γ) which yields us that $x \in f^{-1}(B)$ which is μ_N - open in (X, γ) . Let $A = f^{-1}(B)$ then $x \in A$ and $f(A) \subseteq B$. From this we obtain that for each $x \in X$ and each μ_N - closed set V in (Y, δ) with $f(x) \in V$ there exists a μ_N - open set U in (X, γ) such that $x \in U$ and $f(U) \subseteq V$.

(iii) \Rightarrow (i) Let B be a μ_N - closed set in Y and $x \in f^{-1}(B)$ then we get $f(x) \in B$ and there exists a μ_N -open set U such that $f(U) \subseteq B$. Therefore, $f^{-1}(B)$ is equal to the union of all μ_N - open sets of (X, γ) . Thus, f is μ_N Contra-continuous.

Remark 3.4. The concept of μ_N Contra-continuous and μ_N -continuous are not

depends on each other. This situation can be well explained by the upcoming examples.

Example 3.5. In this example we show that every μ_N -continuous need not be μ_N Contra-continuous. Let $X = \{a, b\}$ and $Y = \{u, v\}$, $0_N = \{< 0, 1, 1 >, < 0, 1, 1 >\}$, $P = \{< 0.7, 0.3, 0.8 >, < 0.5, 0.8, 0.9 >\}$, $Q = \{< 0.4, 0.9, 0.9 >, < 0.3, 0.9, 0.9 >\}$, $R = \{< 0.5, 0.8, 0.7 >, < 0.5, 0.8, 0.8 >\}$, $S = \{< 0.5, 0.8, 0.8 >, < 0.5, 0.8, 0.7 >\}$, $T = \{< 0.3, 0.9, 0.9 >, < 0.4, 0.9, 0.9 >\}$ we define a mapping from $f : (X, \gamma) \rightarrow (Y, \delta)$ by $f(a) = u, f(b) = v$ under the μ_N TS $\gamma = \{P, Q, R, S, 0_N\}$ and $\delta = \{S, T, 0_N\}$. Here $f^{-1}(S) = S$ and $f^{-1}(T) = T$. Here the mapping from $f : (X, \gamma) \rightarrow (Y, \delta)$ is μ_N -continuous but not μ_N Contra-continuous since S and T are not μ_N -closed sets in (X, γ) .

Example 3.6. In this example we made you to understand that every μ_N Contra-continuous need not be μ_N -continuous. Let $f : (X, \gamma) \rightarrow (Y, \delta)$ be a μ_N Contra-continuous mapping which yields us that $f^{-1}(B)$ is μ_N closed in (X, γ) . But there is no way to establish $f^{-1}(B)$ as μ_N open. Hence we cannot establish μ_N -continuous. Thus we conclude that every μ_N Contra-continuous need not be μ_N -continuous. Let $X = \{a, b\}$ and $Y = \{u, v\}$, $0_N = \{< 0, 1, 1 >, < 0, 1, 1 >\}$, $F = \{< 0.5, 0.7, 0.9 >, < 0.3, 0.5, 0.7 >\}$, $G = \{< 0.8, 0.4, 0.7 >, < 0.6, 0.5, 0.7 >\}$, $H = \{< 0.9, 0.3, 0.5 >, < 0.7, 0.5, 0.3 >\}$, $I = \{< 0.7, 0.6, 0.8 >, < 0.7, 0.5, 0.6 >\}$. We define a mapping from $f : (X, \gamma) \rightarrow (Y, \delta)$ by $f(a) = u, f(b) = v$ under the μ_N TS $\gamma = \{G, H, 0_N\}$ and $\delta = \{F, I, 0_N\}$. Here, $f^{-1}(F) = H^c$ which is μ_N closed in (X, γ) . Also, $f^{-1}(I) = G^c$ which is μ_N closed in (X, γ) . Both H^c and G^c are not μ_N open in (X, γ) .

Remark 3.7. Composition of two μ_N Contra-continuous need not be μ_N Contra-continuous. Let $f : (X, \gamma) \rightarrow (Y, \delta)$ and $g : (Y, \delta) \rightarrow (Z, \rho)$ be two μ_N Contra-continuous mappings. Let U be a μ_N open set in (Z, ρ) . Since g is μ_N Contra-continuous, $g^{-1}(U)$ is μ_N closed in (Y, δ) . Since f is also μ_N Contra-continuous, $f^{-1}(g^{-1}(U))$ is μ_N open in (X, γ) . There are no possibilities to get $f^{-1}(g^{-1}(U))$ as μ_N closed sets in (X, γ) .

Example 3.8. In this example we show that Composition of two μ_N Contra-continuous need not be μ_N Contra-continuous. Let $X = \{a, b\}$ and $Y = \{u, v\}$, $Z = \{l, m\}$, $0_N = \{< 0, 1, 1 >, < 0, 1, 1 >\}$, $M_1 = \{< 0.3, 0.2, 0.1 >, < 0.5, 0.4, 0.2 >\}$, $M_2 = \{< 0.2, 0.4, 0.5 >, < 0.6, 0.3, 0.8 >\}$, $M_3 = \{< 0.3, 0.2, 0.1 >, < 0.6, 0.3, 0.2 >\}$, $M_4 = \{< 0.1, 0.8, 0.3 >, < 0.2, 0.6, 0.5 >\}$, $M_5 = \{< 0.5, 0.6, 0.2 >, < 0.8, 0.7, 0.6 >\}$, $M_6 = \{< 0.1, 0.8, 0.3 >, < 0.2, 0.7, 0.6 >\}$ we define the Contra-continuous mappings from $f : (X, \gamma) \rightarrow (Y, \delta)$ by $f(a) = u, f(b) = v$ and also

$g : (Y, \delta) \rightarrow (Z, \rho)$ by $g(u) = l, g(v) = m$ under the μ_N TS $\gamma = \{M_1, M_2, M_4, 0_N\}$, $\delta = \{M_2, M_3, 0_N\}$ and $\rho = \{M_6, 0_N\}$. Here $g^{-1}(M_6) = \{< 0.1, 0.8, 0.3 >, < 0.2, 0.7, 0.6 >\}$ which is μ_N -closed set in (Y, δ) . From this we obtain that $f^{-1}(g^{-1}(M_6)) = \{< 0.1, 0.8, 0.3 >, < 0.2, 0.7, 0.6 >\}$ which is not μ_N -closed set in (X, γ) . Thus $g \circ f$ need not be μ_N Contra-continuous.

Theorem 3.9. *Let $f : (X, \gamma) \rightarrow (Y, \delta)$ and $g : (Y, \delta) \rightarrow (Z, \rho)$ be two functions on μ_N TS. If g is μ_N -continuous and f is μ_N Contra-continuous then $g \circ f$ is μ_N Contra-continuous.*

Proof. Let U be a μ_N open set in (Z, ρ) . Since $g : (Y, \delta) \rightarrow (Z, \rho)$ is μ_N -continuous, $g^{-1}(U)$ is μ_N open in (Y, μ) . Since $f : (X, \gamma) \rightarrow (Y, \delta)$ is μ_N Contra-continuous, $f^{-1}(g^{-1}(U))$ is μ_N closed in (X, γ) . But $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is μ_N closed in (X, γ) . Thus, $g \circ f$ is μ_N Contra-continuous.

Theorem 3.10. *Let $f : (X, \gamma) \rightarrow (Y, \delta)$ and $g : (Y, \delta) \rightarrow (Z, \rho)$ be two functions on μ_N TS. If g is μ_N Contra-continuous and f is μ_N continuous then $g \circ f$ is μ_N Contra-continuous.*

Proof. Let U be μ_N open set in (Z, ρ) . Since $g : (Y, \delta) \rightarrow (Z, \rho)$ is μ_N Contra-continuous, $g^{-1}(U)$ is μ_N closed in (Y, δ) . Also $f : (X, \gamma) \rightarrow (Y, \delta)$ is μ_N continuous, $f^{-1}(g^{-1}(U))$ is μ_N closed in (X, γ) . Hence, $(g \circ f)^{-1}$ is μ_N closed in (X, γ) . Thus $g \circ f$ is μ_N Contra-continuous.

Definition 3.11. *The mapping $f : (X, \gamma) \rightarrow (Y, \delta)$ is called as μ_N Perfectly-continuous if the inverse image of μ_N -open set in (Y, δ) is μ_N -clopen set in (X, γ) .*

Theorem 3.12. *Every μ_N perfectly continuous is μ_N Contra-continuous.*

Proof. Let $f : (X, \gamma) \rightarrow (Y, \delta)$ be μ_N perfectly continuous and G be a μ_N open set of (Y, δ) . Since f is μ_N perfectly continuous, $f^{-1}(G)$ is μ_N -clopen set in (X, γ) . That is $f^{-1}(G)$ is both μ_N open and μ_N closed in (X, γ) . Thus here we got the inverse image of $G \in (Y, \delta)$ is μ_N closed in (X, γ) . Thus, $f : (X, \gamma) \rightarrow (Y, \delta)$ is μ_N Contra-continuous.

Remark 3.13. *The contrary statement of the above theorem need not be true which is established in the forthcoming example.*

Example 3.14. Let $X = \{a, b\}$ and $Y = \{u, v\}, 0_N = \{< 0, 1, 1 >, < 0, 1, 1 >\}$, $A = \{< 0.5, 0.7, 0.9 >, < 0.3, 0.5, 0.7 >\}$, $B = \{< 0.8, 0.4, 0.7 >, < 0.6, 0.5, 0.7 >\}$, $C = \{< 0.9, 0.3, 0.5 >, < 0.7, 0.5, 0.3 >\}$, $D = \{< 0.7, 0.6, 0.8 >, < 0.7, 0.5, 0.6 >\}$, we define a mapping from $f : (X, \gamma) \rightarrow (Y, \delta)$ by $f(a) = a, f(b) = b$ under the μ_N TS $\gamma = \{B, C, 0_N\}$ and $\delta = \{A, D, 0_N\}$. Here, $f^{-1}(A) = C^c$ which is μ_N closed in (X, γ) . $f^{-1}(D) = B^c$ which is also μ_N closed in (X, γ) . But both C^c and B^c are

not μ_N open in (X, γ) . Hence, $f : (X, \gamma) \rightarrow (Y, \delta)$ is μ_N Contra-continuous but not μ_N perfectly continuous.

Theorem 3.15. *If the mapping $f : (X, \gamma) \rightarrow (Y, \delta)$ is μ_N perfectly continuous then f is both μ_N -continuous and μ_N Contra-continuous.*

Proof. Let G be a μ_N open set in (Y, δ) . Since, f is μ_N perfectly continuous, $f^{-1}(G)$ is a μ_N -clopen in (X, γ) that implies us $f^{-1}(G)$ is both μ_N closed and μ_N open in (X, γ) . Thus $f : (X, \gamma) \rightarrow (Y, \delta)$ is both μ_N -continuous and μ_N Contra-continuous.

4. μ_N Dense Set

Definition 4.1. *A neutrosophic set A in μ_N TS (X, μ_N) is called μ_N dense set if there exists no μ_N closed set B in (X, μ_N) such that $A \subset B \subset 1_N$.*

Definition 4.2. *A neutrosophic set A in μ_N TS is called μ_N nowhere dense set if \exists no μ_N open set G in (X, μ_N) such that $G \subset \mu_N Cl(A)$. (i.e) $\mu_N Int(\mu_N Cl(A)) = 0_N$.*

Example 4.3. Let $X = \{a\}$ define neutrosophic sets $0_N = \{< 0, 1, 1 >\}$, $A = \{< 0.3, 0.3, 0.5 >\}$, $B = \{< 0.1, 0.2, 0.3 >\}$, $C = \{< 0.3, 0.2, 0.3 >\}$, $D = \{< 0.3, 0.6, 0.2 >\}$, $E = \{< 0.3, 0.8, 0.5 >\}$, $1_N = \{< 1, 0, 0 >\}$ and so we define a μ_N TS $\mu_N = \{0_N, A, B, C\}$. Here, $\mu_N Int(\mu_N Cl(0_N)) = \{< 0, 1, 1 >\}$, $\mu_N Int(\mu_N Cl(A)) = \{< 0.3, 0.2, 0.3 >\}$, $\mu_N Int(\mu_N Cl(B)) = \{< 0.3, 0.2, 0.3 >\}$, $\mu_N Int(\mu_N Cl(C)) = \{< 0.3, 0.2, 0.3 >\}$, $\mu_N Int(\mu_N Cl(D)) = \{< 0.3, 0.2, 0.3 >\}$, $\mu_N Int(\mu_N Cl(E)) = \{< 0, 1, 1 >\}$, $\mu_N Int(\mu_N Cl(1_N)) = \{< 0.3, 0.2, 0.3 >\}$, $\mu_N Int(\mu_N Cl(0_N)^c) = \{< 0.3, 0.2, 0.3 >\}$, $\mu_N Int(\mu_N Cl(A)^c) = \{< 0, 1, 1 >\}$, $\mu_N Int(\mu_N Cl(B)^c) = \{< 0, 1, 1 >\}$, $\mu_N Int(\mu_N Cl(C)^c) = \{< 0, 1, 1 >\}$, $\mu_N Int(\mu_N Cl(D)^c) = \{< 0.3, 0.2, 0.3 >\}$, $\mu_N Int(\mu_N Cl(E)^c) = \{< 0.3, 0.2, 0.3 >\}$, $\mu_N Int(\mu_N Cl(1_N)^c) = \{< 0, 1, 1 >\}$. Here, $0_N, E, A^c, B^c, C^c, 1_N^c$ are the μ_N nowhere dense sets of (X, μ_N) and $A, B, C, D, 1_N, 0_N^c, E^c$ are not μ_N nowhere dense sets of (X, μ_N) .

Theorem 4.3. *Every μ_N nowhere dense set is μ_N semi closed.*

Proof. Let $A \subseteq X$ be a μ_N nowhere dense set which gives us that $\mu_N Int(\mu_N Cl A) = 0_N$. From this we retrieve that $\mu_N Int(\mu_N Cl A) = 0_N \subseteq A$ which implies us that $\mu_N Int(\mu_N Cl A) \subseteq A$.

Remark 4.5. *Reverse statement of the above proposition need not be true. It is exemplified below.*

Example 4.6. Let $X = \{a, b\}$ and $\mu_N = \{0_N, A, B, C, D\}$, $0_N = \{< 0, 1, 1 > < 0, 1, 1 >\}$, $A = \{< 0.6, 0.4, 0.8 > < 0.8, 0.6, 0.9 >\}$, $B = \{< 0.6, 0.3, 0.8 > < 0.9, 0.2, 0.7 >\}$, $C = \{< 0.5, 0.4, 0.9 > < 0.7, 0.8, 0.9 >\}$, $D = \{< 0.4, 0.6, 0.9 >$

$\langle 0.6, 0.8, 0.9 \rangle$, $E = \{\langle 0.3, 0.7, 0.9 \rangle, \langle 0.5, 0.9, 0.9 \rangle\}$, $1_N = \{\langle 1, 0, 0 \rangle, \langle 1, 0, 0 \rangle\}$. The μ_N semi-closed sets of (X, μ_N) are $\{E, A, B, A^c, B^c, C^c, E^c, (1_N)^c\}$. The μ_N Nowhere Dense sets of (X, μ_N) are $\{E, B^c, 0_N\}$. From this we can conclude that Every μ_N semi closed sets need not be μ_N nowhere Dense.

Proposition 4.7. *If A is a μ_N closed set in (X, μ_N) with $\mu_N \text{Int}(A) = 0_N$ then A is a μ_N nowhere dense set in (X, μ_N) .*

Proof. Let G_1 be a μ_N Closed set in (X, μ_N) which yields that $\mu_N \text{Cl}(G_1) = G_1$. Now let me Take that $\mu_N \text{Int}(\mu_N \text{Cl}(G_1))$. But we already know that G_1 is μ_N Closed which obviously leads us to $\mu_N \text{Int}(\mu_N \text{Cl}(G_1)) = \mu_N \text{Int}(G_1) = 0_N$. Since by going back to G_1 is a μ_N Closed set in (X, μ_N) with $\mu_N \text{Int}(G_1) = 0_N$. Hence, we acquire that G_1 is μ_N nowhere dense set in (X, μ_N) .

Definition 4.8. *A μ_N closed set in (X, μ_N) is said to be is μ_N nowhere dense set then $\mu_N \text{Int}(A) = 0_N$.*

Theorem 4.9. *If A is μ_N nowhere dense set in (X, μ_N) then $\mu_N \text{Int}(A) = 0_N$.*

Proof. Let A be a μ_N nowhere dense set in (X, μ_N) . We are having that $A \subseteq \mu_N \text{Cl}(A)$ from this we acquire that $\mu_N \text{Int}(A) \subseteq \mu_N \text{Int}(\mu_N \text{Cl}(A)) = 0_N$. Because of A is μ_N nowhere dense set in (X, μ_N) . Hence we come to a decision that A is μ_N nowhere dense set in (X, μ_N) then $\mu_N \text{Int}(A) = 0_N$.

Remark 4.10. *The reverse statement of the above theorem is not necessarily be true. This can be exemplified as follows.*

Example 4.11. Let $X = \{a\}$ and $\mu_N = \{0_N, A, C\}$ be a μ_N TS where $0_N = \{\langle 0, 1, 1 \rangle\}$, $A = \{\langle 0.7, 0.8, 0.9 \rangle\}$, $B = \{\langle 0.3, 0.4, 0.6 \rangle\}$, $C = \{\langle 0.9, 0.7, 0.6 \rangle\}$. Here, $\mu_N \text{Int}(B) = 0_N$ but $\mu_N \text{Int}(\mu_N \text{Cl}(B)) = \{\langle 0.9, 0.7, 0.6 \rangle\} \neq 0_N$. Hence B is not a μ_N nowhere dense set in (X, μ_N) .

Remark 4.12. *The complement of μ_N nowhere dense set need not be μ_N nowhere dense set. This can be elaborated via an exemplary.*

Example 4.13. Let $\mu_N = \{0_N, A, B\}$ where $0_N = \{\langle 0, 1, 1 \rangle\}$, $A = \{\langle 0.1, 0.4, 0.6 \rangle\}$, $B = \{\langle 0.2, 0.3, 0.5 \rangle\}$, $C = \{\langle 0.6, 0.6, 0.1 \rangle\}$, $D = \{\langle 0.5, 0.7, 0.2 \rangle\}$. Here, $\{0_N, C, D, A^c, B^c\}$ are μ_N nowhere dense but their complements $\{(0_N)^c, C^c, D^c, A, B\}$ are μ_N Dense but not μ_N nowhere dense.

Theorem 4.14. *If A is a μ_N nowhere dense set in (X, μ_N) then \bar{A} is a μ_N dense set in (X, μ_N) .*

Proof. Let A be a μ_N nowhere dense set in (X, μ_N) . Already we have "If A is a μ_N nowhere dense set in (X, μ_N) then $\mu_N \text{Int}(A) = 0_N$ ". Now let us consider that $\mu_N \text{Cl}(\bar{A}) = (\mu_N \text{Int}(A)) = (0_N) = 1_N$. Hence, \bar{A} is a μ_N dense set in (X, μ_N) .

Remark 4.15. *The contrary statement of the above theorem need not be true. That is, If A is a μ_N dense set in (X, μ_N) then \bar{A} need not be a μ_N nowhere dense set in (X, μ_N) .*

Example 4.16. Let $X = \{a\}$ define neutrosophic sets $0_N = \{< 0, 1, 1 >\}$, $A_1 = \{< 0.7, 0.8, 0.9 >\}$, $A_2 = \{< 0.3, 0.4, 0.6 >\}$, $A_3 = \{< 0.9, 0.7, 0.6 >\}$, $1_N = \{< 1, 0, 0 >\}$ and we define a μ_N TS $\mu_N = \{0_N, A_1, A_3\}$. Here, $A_2, A_3, 1_N$ are μ_N dense sets in (X, μ_N) and $(A_3)^c, 0_N$ are μ_N nowhere dense sets in (X, μ_N) . Here, A_2 is μ_N dense set but its complement is not μ_N nowhere dense set in (X, μ_N) .

Theorem 4.17. *If A is a μ_N Dense, μ_N open in (X, μ_N) provided $B \subseteq \bar{A}$, then B is a μ_N nowhere dense in (X, μ_N) .*

Proof. Let us take that $B \subseteq \bar{A}$

$$\begin{aligned} &\Rightarrow \mu_N Cl(B) \subseteq \mu_N Cl(\bar{A}) = \bar{A} \\ &\Rightarrow \mu_N Cl(B) \subseteq \bar{A} \\ &\Rightarrow \mu_N Int(\mu_N Cl(B)) \subseteq \mu_N Int(\bar{A}) \\ &\Rightarrow \mu_N Int(\mu_N Cl(B)) \subseteq \overline{\mu_N Cl(A)} \\ &\Rightarrow \mu_N Int(\mu_N Cl(B)) \subseteq \bar{1}_N \\ &\Rightarrow \mu_N Int(\mu_N Cl(B)) \subseteq 0_N \\ &\Rightarrow \mu_N Int(\mu_N Cl(B)) = 0_N \end{aligned}$$

Hence B is μ_N nowhere dense in (X, μ_N) .

Theorem 4.18. *Let A be a μ_N closed set in (X, μ_N) then A is μ_N nowhere dense in (X, μ_N) iff $\mu_N Int(A) = 0_N$.*

Proof. Let A be a μ_N closed set in (X, μ_N) with $\mu_N Int(A) = 0_N$. By using already existing statement, "If A is μ_N closed set in (X, μ_N) with $\mu_N Int(A) = 0_N$ then A is μ_N nowhere dense in (X, μ_N) ". Thus we conclude that A is μ_N nowhere dense in (X, μ_N) . Conversely assume that A be a μ_N nowhere dense in (X, μ_N) . Then, $\mu_N Int(\mu_N Cl(A)) = 0_N$ which obviously yields us that $\mu_N Int(A) = 0_N$. Since A is μ_N closed set in (X, μ_N) , so $\mu_N Cl(A) = A$. Thus, we acquire that $\mu_N Int(A) = 0_N$.

Definition 4.19. *A neutrosophic set U in μ_N TS is called to be as μ_N Rare set if $\mu_N Int(U) = 0_N$.*

Theorem 4.20. *Every μ_N nowhere dense set in (X, μ_N) is μ_N rare set in (X, μ_N) .*

Proof. Let A be a μ_N nowhere dense set which brings us $\mu_N Int(\mu_N Cl(A)) = 0_N$. Already we have $A \subseteq \mu_N Cl(A)$ by applying this we get $\mu_N Int A \subseteq \mu_N Int(\mu_N Cl(A))$. From this we deduce that $\mu_N Int(A) = 0_N$ which concludes that A is μ_N rare set in (X, μ_N) . Hence we come up with the decision that every μ_N nowhere dense set

in (X, μ_N) is μ_N rare set in (X, μ_N) .

Remark 4.21. *The reverse concept of the above theorem need not be true which is explained in detail with the help of an example in order to help for the readers understanding.*

Example 4.22. Let $X = \{a\}$ define neutrosophic sets $0_N = \{< 0, 1, 1 >\}$, $U_1 = \{< 0.3, 0.3, 0.5 >\}$, $U_2 = \{< 0.1, 0.2, 0.3 >\}$, $U_3 = \{< 0.3, 0.2, 0.3 >\}$, $U_4 = \{< 0.3, 0.6, 0.2 >\}$, $U_5 = \{< 0.3, 0.8, 0.5 >\}$, $1_N = \{< 1, 0, 0 >\}$ and we define a μ_N TS $\mu_N = \{0_N, U_1, U_2, U_3\}$. Here, $\{0_N, U_5, (U_1)^c, (U_2)^c, (U_3)^c, (U_4)^c\}$ are μ_N nowhere dense sets in (X, μ_N) and $\{0_N, U_4, U_5, (U_1)^c, (U_2)^c, (U_3)^c, U_4^c\}$ are μ_N rare sets in (X, μ_N) . From the above two collections we can observe that U_4 is μ_N rare set in (X, μ_N) but it is not μ_N nowhere dense set in (X, μ_N) .

Remark 4.23. *If A is μ_N closed and μ_N rare set in (X, μ_N) then A is μ_N nowhere dense set in (X, μ_N) . Proof is obvious.*

Theorem 4.24. *If A is a μ_N nowhere dense set in (X, μ_N) then $\mu_N Cl A$ is a μ_N nowhere dense set in (X, μ_N) .*

Proof. Let us take $\mu_N Cl(A) = B$. Now consider $\mu_N Int(\mu_N Cl B) = \mu_N Int(\mu_N Cl(\mu_N Cl(A)))$ which brings us that $\mu_N Int(\mu_N Cl B) = \mu_N Int(\mu_N Cl(A))$ which obviously leads us into $\mu_N Int(\mu_N Cl B) = 0_N$. Hence, B is μ_N nowhere dense set in (X, μ_N) . Thus we conclude that $\mu_N Cl A$ is a μ_N nowhere dense set in (X, μ_N) .

Theorem 4.25. *If A is a μ_N nowhere dense set in (X, μ_N) then $\overline{(\mu_N Cl A)}$ is a μ_N dense set in (X, μ_N) .*

Proof. We have "If A is a μ_N nowhere dense set in (X, μ_N) then $\mu_N Cl A$ is a μ_N nowhere dense set in (X, μ_N) ". From the above statement we acquire that $\mu_N Cl(A)$ is a μ_N nowhere dense set in (X, μ_N) . Also we have, If A is a μ_N nowhere dense set in (X, μ_N) then \overline{A} is a μ_N dense set in (X, μ_N) . From this we deduce that $\overline{(\mu_N Cl A)}$ is a μ_N dense set in (X, μ_N) .

Theorem 4.26. *If A is a μ_N dense set in (X, μ_N) and B is a μ_N nowhere dense set in (X, μ_N) then $A \cap B$ is a μ_N nowhere dense set in (X, μ_N) .*

Proof. Let B be a μ_N nowhere dense set in (X, μ_N) and A be a μ_N dense set in (X, μ_N) which implies that $\mu_N Cl A = 1_N$. Now, we are yet to prove that $A \cap B$ is μ_N nowhere dense set in (X, μ_N) . Now let us consider,

$$\begin{aligned} \mu_N Int(\mu_N Cl A \cap B) &\subseteq \mu_N Int(\mu_N Cl A) \cap \mu_N Int(\mu_N Cl B) \\ &\subseteq \mu_N Int(1_N) \cap \mu_N Int(\mu_N Cl B) \\ &\subseteq \mu_N Int(\mu_N Cl B) = 0_N \end{aligned}$$

Hence, $\mu_N Int(\mu_N Cl(A \cap B)) = 0_N$ which yields us that $A \cap B$ is μ_N nowhere dense

set in (X, μ_N) .

In general Neutrosophic Topological spaces, "If A is a μ_N dense set in (X, μ_N) and B is a μ_N nowhere dense set in (X, μ_N) if and only if $A \cap B$ is a μ_N nowhere dense set in (X, μ_N) ". But in μ_N Topological Spaces it is not necessary to be that if $A \cap B$ is a μ_N nowhere dense set in (X, μ_N) then A is a μ_N dense set in (X, μ_N) and B is a μ_N nowhere dense set in (X, μ_N) .

Theorem 4.27. *If A is a μ_N dense set in (X, μ_N) and also μ_N open set in (X, μ_N) then \bar{A} is a μ_N nowhere dense set in (X, μ_N) .*

Proof. Let A be a μ_N dense set in (X, μ_N) such that $\mu_N Cl A = 1_N$. Now consider $\mu_N Int(\mu_N Cl \bar{A}) = \overline{(\mu_N Cl(\mu_N Int(A)))} = \overline{(\mu_N Cl(A))} = 0_N$. Thus, we get $\mu_N Int(\mu_N Cl(\bar{A})) = 0_N$. Hence, we conclude that \bar{A} is μ_N nowhere dense set in (X, μ_N) .

Definition 4.28. *The μ_N TS is called as μ_N open hereditarily irresolvable space if $\mu_N Int(\mu_N Cl A) \neq 0_N$, then $\mu_N Int(A) \neq 0_N$ for any non-zero neutrosophic sets in (X, μ_N) .*

Theorem 4.29. *If (X, μ_N) is a μ_N be a μ_N open hereditarily irresolvable space, any non zero neutrosophic set A with $\mu_N Int(A) = 0_N$ is a μ_N nowhere dense set in (X, μ_N) .*

Proof. Let A be a non zero neutrosophic set in μ_N open hereditarily irresolvable space (X, μ_N) with $\mu_N Int(A) = 0_N$ from this we retrieve that $\mu_N Int(\mu_N Cl A) \neq 0_N$ which is contrary to $\mu_N Int(A) = 0_N$. Hence, we must obtain that $\mu_N Int(\mu_N Cl A) = 0_N$. Hence, A is μ_N nowhere dense set in (X, μ_N) .

5. μ_N Baire Space

Definition 5.1. *Let (X, μ_N) be a μ_N Topological space A neutrosophic set in (X, μ_N) is called μ_N first category if $A = \cup_{i=1}^{\infty} G_i$ where G_i 's are μ_N nowhere dense set in (X, μ_N) . The remaining neutrosophic sets in (X, μ_N) is said to be μ_N second category.*

Proposition 5.2. *If A is a μ_N first category set in (X, μ_N) , then $\bar{A} = \cap_{i=1}^{\infty} G_i$ where $\mu_N Cl(G_i) = 1_N$.*

Proof. Let A be a μ_N first category set in (X, μ_N) which yields us that $A = \cup_{i=1}^{\infty} A_i$ where A_i 's are μ_N nowhere dense set in (X, μ_N) .

Now, $\bar{A} = \overline{(\cup_{i=1}^{\infty} A_i)} = \cap_{i=1}^{\infty} \overline{(A_i)}$, we have, If A is a μ_N nowhere dense set in (X, μ_N) , then \bar{A} is μ_N dense set in (X, μ_N) . Let us put $G_i = \overline{(A_i)}$. Thus, we get $\bar{A} = \cap_{i=1}^{\infty} G_i$ where $\mu_N Cl(G_i) = 1_N$.

Remark 5.3. *The reverse process of the above theorem need not be true. It can be*

explained below with the help of an example.

Let $X = \{a\}$ define neutrosophic sets $0_N = \{< 0, 1, 1 >\}$, $A_1 = \{< 0.7, 0.8, 0.9 >\}$, $A_2 = \{< 0.3, 0.4, 0.6 >\}$, $A_3 = \{< 0.9, 0.7, 0.6 >\}$, $1_N = \{< 1, 0, 0 >\}$ and we define a μ_N TS $\mu_N = \{0_N, A_1, A_3\}$. Here, $A_2, A_3, 1_N$ are μ_N dense sets in (X, μ_N) . In this case $\bar{A} = \{< 0.3, 0.7, 0.6 >\}$ from this we get $A = \{< 0.6, 0.3, 0.3 >\}$ which is not μ_N first category set in (X, μ_N) .

Definition 5.4. Let A be a μ_N first category set in (X, μ_N) then \bar{A} is called μ_N Residual set in (X, μ_N) .

Definition 5.5. The μ_N Topological spaces is said to be μ_N Baire Space if $\mu_N \text{Int}(\cup_{i=1}^{\infty} G_i) = 0_N$ where G_i 's are μ_N nowhere dense set in (X, μ_N) .

Example 5.6. Let $X = \{a\}$, we define a neutrosophic sets in μ_N TS such that $0_N = \{< 0, 1, 1 >\}$, $A = \{< 0.1, 0.4, 0.6 >\}$, $B = \{< 0.2, 0.3, 0.5 >\}$, $C = \{< 0.6, 0.6, 0.1 >\}$, $D = \{< 0.5, 0.7, 0.2 >\}$. Let $\mu_N = \{0_N, A, B\}$ be a μ_N TS. Here, $0_N, C, D, A^c, B^c$ are a μ_N nowhere dense set in (X, μ_N) . C and A^c are of μ_N first category set in (X, μ_N) . The left out μ_N nowhere dense sets in (X, μ_N) are of μ_N Second category set in (X, μ_N) . Here, $\mu_N \text{Int}(0_N \cup C \cup D \cup A^c \cup B^c) = 0_N$. Hence, (X, μ_N) is a μ_N Baire Space.

Theorem 5.7. If $\mu_N \text{Cl}(\cap_{i=1}^{\infty} (U_i)) = 1_N$, where U_i 's are μ_N dense set in (X, μ_N) and μ_N open sets in (X, μ_N) , Then (X, μ_N) is a μ_N Baire Space.

Proof. We have $\mu_N \text{Cl}(\cap_{i=1}^{\infty} U_i) = 1_N$, Now taking compliment on both sides we retrieve that $(\mu_N \text{Cl}(\cap_{i=1}^{\infty} U_i)) = 0_N$ from this we easily acquire that $\mu_N \text{Int}(\cup_{i=1}^{\infty} (\overline{U_i})) = 0_N$. We put $V_i = \overline{U_i}$. Then we get $\mu_N \text{Int}(\cup_{i=1}^{\infty} V_i) = 0_N$. Here $U_i \in \mu_N$ gives us that $(\overline{U_i})$ is μ_N closed which gives that V_i is μ_N closed. So, $\mu_N \text{Int}(V_i) = \mu_N \text{Int}(\overline{U_i}) = (\mu_N \text{Cl}(U_i)) = 0_N$. Already we have, "If A is a μ_N closed set in (X, μ_N) with $\mu_N \text{Int}(A) = 0_N$ then A is μ_N nowhere dense set in (X, μ_N) ". From this we say that V_i is μ_N nowhere dense set in (X, μ_N) . Thus we acquire that $\mu_N \text{Int}(\cup_{i=1}^{\infty} V_i) = 0_N, V_i$'s are μ_N nowhere dense set in (X, μ_N) that leads us into (X, μ_N) is a μ_N Baire Space.

In the above theorem, U must be μ_N dense set and μ_N open set in (X, μ_N) . If anyone of the condition is not present then the theorem fails to occur. Counter examples are provided to illustrate the scenario.

Example 5.8. Let $X = \{a\}$ and $\mu_N = \{0_N, A, C, E\}$ be a μ_N TS where $0_N = \{< 0, 1, 1 >\}$, $A = \{< 0.3, 0.4, 0.5 >\}$, $B = \{< 0.3, 0, 0.1 >\}$, $C = \{< 0.4, 0.6, 0.8 >\}$, $D = \{< 0.4, 0, 0.1 >\}$, $E = \{< 0.4, 0.4, 0.5 >\}$, $1_N = \{< 1, 0, 0 >\}$. Here B and D are the only μ_N dense sets in (X, μ_N) but not μ_N open in (X, μ_N) . In this case we cannot get any μ_N no where dense sets

and so we cannot retrieve μ_N Baire Space.

Example 5.9. Let $X = \{a, b\}$ and $\mu_N = \{0_N, A, B, C\}$ be a μ_N TS where $0_N = \{\langle 0, 1, 1 \rangle, \langle 0, 1, 1 \rangle\}$, $A = \{\langle 0.7, 0.3, 0.8 \rangle, \langle 0.5, 0.8, 0.9 \rangle\}$, $B = \{\langle 0.8, 0.2, 0.7 \rangle, \langle 0.7, 0.2, 0.4 \rangle\}$, $C = \{\langle 0.5, 0.8, 0.9 \rangle, \langle 0.3, 0.8, 0.8 \rangle\}$, $D = \{\langle 0.5, 0.8, 0.8 \rangle, \langle 0, 5, 0.8, 0.8 \rangle\}$, $E = \{\langle 0.7, 0.2, 0.8 \rangle, \langle 0.8, 0.2, 0.7 \rangle\}$. Here A, B, C are μ_N open in (X, μ_N) but not μ_N dense sets in (X, μ_N) . In this case also we have not get any μ_N no where dense sets and so we cannot deduce μ_N Baire Space.

Proposition 5.10. Let (X, μ_N) be a μ_N TS. Then the following are equivalent.

- (i) (X, μ_N) is μ_N Baire Space.
- (ii) $\mu_N \text{Int}(A) = 0_N$, for all μ_N first category set in (X, μ_N) .
- (iii) $\mu_N \text{Cl}(A) = 1_N$, for every μ_N Residual set in (X, μ_N) .

Proof. (i) \Rightarrow (ii) Assume (X, μ_N) is a μ_N Baire Space. Let A be a μ_N first category set in (X, μ_N) from this we obtain that $A = \cup_{i=1}^{\infty} A_i$ where A_i 's are μ_N nowhere dense sets in (X, μ_N) . Now $\mu_N \text{Int}(A) = \mu_N \text{Int}(\cup_{i=1}^{\infty} A_i) = 0_N$. Since, (X, μ_N) is μ_N Baire Space which obviously $\mu_N \text{Int}(A) = 0_N$.

(ii) \Rightarrow (iii) Assume $\mu_N \text{Int}(A) = 0_N$, for all μ_N first category set in (X, μ_N) . Let us assume B be a μ_N Residual set in (X, μ_N) then \overline{B} will be a μ_N first category set in (X, μ_N) . By our assumption we get that $\mu_N \text{Int}(\overline{B}) = 0_N$ which yields us that $\overline{(\mu_N \text{Cl}(\overline{B}))} = 0_N$. Now taking compliment on both sides we obtain that $\mu_N \text{Cl}(B) = (0_N) \Rightarrow \mu_N \text{Cl}(B) = 1_N$ where B is a μ_N residual set in (X, μ_N) .

(iii) \Rightarrow (i) Assume that $\mu_N \text{Cl}(A) = 1_N$, μ_N residual set in (X, μ_N) . Let A be a μ_N first category set in (X, μ_N) which implies us that $A = \cup_{i=1}^{\infty} A_i$, A_i 's are μ_N nowhere dense set in (X, μ_N) . We have, If A is μ_N first category set in (X, μ_N) then \overline{A} is μ_N residual set in (X, μ_N) . Since, \overline{A} is μ_N residual set in (X, μ_N) by (iii) we get $\mu_N \text{Cl}(\overline{A}) = 1_N$ which gives us that $\overline{(\mu_N \text{Int}(A))} = 1_N$. By taking complement we deduce that $\mu_N \text{Int}(A) = 0_N$. From this we acquire that $\mu_N \text{Int}(\cup_{i=1}^{\infty} A_i) = 0_N$, A_i 's are μ_N nowhere dense set in (X, μ_N) . Thus, (X, μ_N) is μ_N Baire Space.

Proposition 5.11. Every μ_N first category set in (X, μ_N) is μ_N Rare set.

Proof. Let A be a μ_N first category set in a μ_N Baire Space (X, μ_N) from this we obtain that $A = \cup_{i=1}^{\infty} A_i$ where A_i 's are μ_N nowhere dense sets in (X, μ_N) . Now $\mu_N \text{Int}(A) = \mu_N \text{Int}(\cup_{i=1}^{\infty} A_i) = 0_N$. Since, (X, μ_N) is μ_N Baire Space which obviously $\mu_N \text{Int}(A) = 0_N$. From this we say that A is μ_N Rare set.

Remark 5.12. The reverse process of the above statement need not be true. This

can be illustrated with the help of the upcoming example.

Example 5.13. Let $X = \{a\}$, we define a neutrosophic sets in μ_N TS such that $0_N = \{< 0, 1, 1 >\}$, $A = \{< 0.1, 0.4, 0.6 >\}$, $B = \{< 0.2, 0.3, 0.5 >\}$, $C = \{< 0.6, 0.6, 0.1 >\}$, $D = \{< 0.5, 0.7, 0.2 >\}$. Let $\mu_N = \{0_N, A, B\}$ be a μ_N TS. Here, $0_N, C, D, A^c, B^c$ are a μ_N nowhere dense set in (X, μ_N) . C and A^c are of μ_N first category set in (X, μ_N) . The μ_N Rare sets of (X, μ_N) are $A^c, B^c, C, D, 0_N$. From this we can observe that $B^c, D, 0_N$ are μ_N Rare sets of (X, μ_N) but not μ_N first category set in (X, μ_N) .

6. Conclusion

In this article, we have emanated some characterizations of μ_N Topological Spaces. Here we discussed about μ_N Dense sets, μ_N Nowhere Dense sets, μ_N rare set. Some comparisons were done. Some New type of μ_N continuous functions such as μ_N Perfectly Continuous, μ_N Contra-Continuous functions were discovered and their properties were listed out. In future, we wish to put our thoughts towards μ_N -Connected, μ_N -Compact and so on.

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