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A note on Palindrome numbers

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History of Palindromes : Palindromes as a form of wordplay have been created for many centuries. For example, the ancient Greeks are known to have often inscribed the following onto their fountains:

Nipson anomemata me monan opsin.

The third century BCE Greek poet Sotades was said to have written such vulgar satires that King Ptolemy II had him sewn up in a sack and thrown into the sea, for insulting the king in one of his verses. But his coarse, vile verses must have been clever, for Sotades is reputed to have invented **palindromes**, which are sometimes called Sotadics in his honor.


Sotades

The word palindrome is derived from the Greek palíndromos, meaning running backagain (palín = AGAIN + drom–, drameîn = RUN). A palindrome is a word or phrase which reads the same in both directions. The word "palindrome" was coined from the Greek roots by the English writer Ben Jonson in the 17th century.

The Romans were also admirers of palindromes, and produced such sentences as:

In girum imus nocte et consumimur igni.

It means **we enter the circle after dark and are consumed by fire** and is said to describe the movement of moths.

The following 2D palindrome square also dates back to Roman times. It is inscribed on a stone tablet outside Rome in Italy and is the earliest known 2D palindrome.

S	A	T	O	R
A	R	E	P	O
T	E	N	E	T
O	P	E	R	A
R	O	T	A	S

Sator Arepo tenet opera rotas means the sower Arepo works with the help of a wheel.

Palindrome Word Squares

2D Palindromes are word squares in which every row and column reads as a word in both directions:

Palindrome sentence:

Was it a car or a cat I saw?
He lived as a devil, eh?

Love is This and This is Love
by : J.A. Linden

Darling, my love
Is great, so great;
Recalling Heaven's calm above.
Fate is sweet this---
All after Fall!
Fall? After all.
This, sweet, is fate--
Above calm Heaven's recalling.
Great, so great is Love, my darling!

Panama Palindromes:

A man, a plan, a canal – Panama!

It first appeared in 1948, but James Puder believes that it must have been discovered before this. Read his Word Ways article: Who first found The Panama Palindrome? But it is not the only one. In fact there is a whole family of them, some of them very long indeed. In 1983, Jim Saxe added a cat to the list, creating:

A man, a plan, a cat, a canal – Panama!

Here is another one of the best Panama Palindromes.

A yak (a kind of ox) and a yam (a potato-like tuber) can be inserted, lengthening it to: Here is another one of the best Panama Palindromes.

A man, a plan, a cam, a yak, a yam, a canal – Panama!

Guy Jacobson found two more items that could be added to this extended Panama palindrome, making the 17-word:

A man, a plan, a cat, a ham, a yak, a yam, a hat, a canal – Panama!

The Magic Square Palindrome

5x5 word square

M	U	S	E	S
U	N	E	V	E
S	E	Y	E	S
E	V	E	N	U
S	E	S	U	M

Muse sun, Eve's e(y)es even use sum

5x5 word square

E	R	O	S	E
R	O	D	E	S
O	D	I	D	O
S	E	D	O	R
E	S	O	R	E

Eros, erode sod. (1) dosed ore sore.
[7 words / 25 letter]

5x5 word square

T	R	A	P	A
R	A	T	S	P
A	T	A	T	A
P	S	T	A	R
A	P	A	R	T

Trap : a rat spat a tap's tar apart.
[8 words / 25 letter]

What is your definition of number?

The question is a challenging one because defining the abstract idea of number is extremely difficult. More than 2,500 years ago, the great number enthusiast Pythagoras described number as "the first principle, a thing which is undefined, incomprehensible, and having in itself all numbers." Even today, we still struggle with the notion of what numbers mean.

Waring's problem for cubes:

Every positive integer can be written as the sum of nine (or fewer) positive cubes. This upper limit of nine cubes cannot be reduced because, for example, 23 cannot be written as the sum of fewer than nine positive cubes:

$23 = 2^3 + 2^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3$.

Sum of cubes in arithmetic progression

There are examples of cubes in arithmetic progression whose sum is a cube,

$3^3 + 4^3 + 5^3 = 6^3$

$11^3 + 12^3 + 13^3 + 14^3 = 20^3$

$31^3 + 33^3 + 35^3 + 37^3 + 39^3 + 41^3 = 66^3$

Palindrome numbers:

- A number (natural number) is a palindrome number, if it produces the same number when it is written in the reverse order.
- Example: 151, 125521, 1001, 7007 etc
- The smallest three digit palindrome is 101.
- All single digit numbers are palindrome.

Palindrome numbers like palindrome words or lines obey the same property of being the same whether they are read from left to right or vice-versa. Examples include:

484

123321

Palindromes could be formed from a number that is not a palindrome by adding the original number to the number formed by reversing the digits. For example:

$$38 + 83 = 121$$

which is a palindrome. Sometimes, however, more than one reversal is necessary. Thus,

$$49 + 94 = 143$$

which is not a palindrome. But

$$143 + 341 = 484$$

is a palindrome.

Palindrome Primes

- All even palindrome numbers of order more than three are divisible by 11 and hence are composite numbers. What is about palindrome numbers with odd digits? Of the first ten 3 digit palindromes, five of them are primes (101, 131, 151, 181, and 191). Is this a sign that there are a lot of palindrome primes?
- Among 90 palindrome numbers of order three, 15 of them are primes. Among the 900 palindrome numbers of order five, 93 are primes. Among the 9000, palindromes of order seven, 668 are primes. The ratio drops dramatically after that, so in general palindrome primes are in fact, fairly rare compared to the number of primes to the composites.

The largest known palindrome prime was discovered in 2001, has 39027 digits, and is equal to $10^{39026} + 4538354 * 10^{19510} + 1$.

A beautiful Palindrome Prime 313:

This is a three-digit palindrome whose digits sum to a lucky number, 3+1+3 =7. . . . Two of these digits, 3 and 7, are also prime nos and the other isn't composite! Twice the lucky number less 1 is an unlucky number: 2*7-1=13. The sum of the digits of 14 is 5, another prime. The unlucky number, 13, with its digits reversed is a prime, 31, terminating a five-digit palindrome: 1 3 1 3 1. (A prime, such as 13, which is reversible to another prime, such as 31, is called an emirp.) The first digit of 313, when squared, factors the larger palindrome evenly: 13131/9 = 1459. The digits of this second factor, 1459, which is also a prime, when summed and summed again (1+4+5+9=19, and 9+1=10), give us the base of our number system, 10. To clinch it all, 313 = 12^2 + 13^2: 144 + 169 = 313, the sum of two consecutive squares. Can you discover anything else about 313 ?"

'373' the king amidst the three digit palprimes

The sum of five consecutive primes
373 = 67 + 71 + 73 + 79 + 83

The sum of the squares of five consecutive primes
373 = 32 + 52 + 72 + 112 + 132

373 is a sum of positive powers of its digits. 3^1 + 7^3 + 3^3

Factorial 199 or 199! has exactly 373 digits

373 can be expressed in three ways as prime1 + prime2 + 1 - (Carlos Rivera)

199 + 173 + 1 | 193 + 179 + 1 | 191 + 181 + 1

Starting with composite number 38 and applying the procedure

Repeated Factorisation of Concatenated Primefactors

we arrive at 38 = 2 x 19 and 219 = 3 x 73 which is 373 after two steps.

Here is a beautiful Magic Square filled only with

Can you guess the missing number in the middle

282	737	646
919	555	191
464	?	828

373 is highly decomposable and transformable into other primes.
Every prefix is prime 373 - 37 - 3
Every suffix is prime 373 - 73 - 3
Every permutation of its digits is prime 373 - 337 - 733

373 is palindrome in other bases as well.
37310 = 4549 = 5658 = 113114

373 is the average of its two 'neighbour primes'
373 = (367 + 379) / 2

Decimal palindrome numbers

All numbers in base 10 with one digit are palindrome. The number of palindrome numbers with two digits is 9 :

{11, 22, 33, 44, 55, 66, 77, 88, 99}.

There are 90 palindrome numbers with three digits (Using the Rule of product: 9 choices for the first digit - which determines the third digit as well - multiplied by 10 choices for the second digit) :

{101, 111, 121, 131, 141, 151, 161, 171, 181, 191, …, 909, 919, 929, 939, 949, 959, 969, 979, 989, 999}

and also 90 palindrome numbers with four digits: (Again, 9 choices for the first digit multiplied by ten choices for the second digit. The other two digits are determined by the choice of the first two)

{1001, 1111, 1221, 1331, 1441, 1551, 1661, 1771, 1881, 1991, …, 9009, 9119, 9229, 9339, 9449, 9559, 9669, 9779, 9889, 9999},

So there are 199 palindrome numbers below 104. Below 105 there are 1099 palindrome numbers and for other exponents of 10n we have: 1999, 10999, 19999, 109999, 199999, 1099999, …. For some types of palindrome numbers these values are listed below in a table. Here 0 is included.

Perfect powers

....

There are many palindrome perfect powers n^k, where n is a natural number and k is 2, 3 or 4.

- Palindrome squares: 0, 1, 4, 9, 121, 484, 676, 10201, 12321, 14641, 40804, 44944, ...
- Palindrome cubes: 1, 8, 343, 1331, 1030301, 1367631, 1003003001, …
- Fourth power of n is palindrome : n = 0, 1, 11, 101, 1001, 10001, 100001, 1000001, ..
- No palindromes of form n^5 (or higher exponent) have been found. The only known non-palindrome number, whose cube is a palindrome, is 2201. It is not yet known if the only such occurrence is 22013 = 10662526601.

G. J. Simmons conjectures there are no palindromes of form n^k for k > 4.

M. Markovič conjectures that if n^4 is a positive palindrome, then n begins and ends with digit 1, all other digits of n being 0. The conjecture is verified for n up to 100000001.

HYPOTHESIS

All palindromes with an even number of digits are divisible by 11.

The Palindrome Conjecture

- A famous unsolved number problem called the "palindrome conjecture" says that you can start with any number greater than 10, reverse it, and add the two numbers. After a certain number of times repeating this process, you will come up with a palindrome:

1) 97+79=176
2) 176+671=847
3) 847+748=1595
4) 1595+5951=7546
5) 7546+6457=14003
6) 14003+30041=44044 which is a palindrome.

An activity for palindrome numbers is to explore the Palindromic Number Conjecture. This conjecture states that any number can be changed into a palindrome number by reversing the digits and adding to the original number a finite amount of times. For example, the number 23 can be turned into a palindrome in just one step:

23 + 32 => 55

Some numbers take two steps, such as 49 :

49 + 94 => 143 + 431 => 484

Try this experiment with the number 196. What did you find? Mathematicians have tried the reverse and add process for 196 at least nine million times without finding a palindrome. So is 196 an exception? Mathematicians have not been able to prove it either way, that is why it is still a conjecture instead of a theorem.

Other "base" numbers that (as far as anyone knows) do not go to palindromes are 879, 1997, 7059, 10553, 10563, 32419, 36973, and 43844.

The current record for the longest time to converge belongs to the number 100,120,849,299,260 which after 201 iterations, converges to the palindrome number:
16616723795884852455598564101455698426624444977944442662489655410146589555425848859732761661 which incidentally is the largest palindrome number calculated by this method.

In the paper "Palindrome prime pyramids" [HC2000] we discuss a type of pyramid first proposed by G. L. Honaker, Jr. For example, starting with the prime 2 we have

2	2
929	929
39293	39293
7392937	3392933
373929373	733929337

Note that each row is a palindrome prime (a palprime) with the previous row as the central digit. These are the tallest that can be built starting with 2 and with step size one (adding one digit at a time to each side.)

```
                              2
                            7 2 7
                          3 7 2 7 3
                      3 3 3 7 2 7 3 3 3
                    9 3 3 3 7 2 7 3 3 3 9
                3 0 9 3 3 3 7 2 7 3 3 3 9 0 3
            1 8 3 0 9 3 3 3 7 2 7 3 3 3 9 0 3 8 1
          9 2 1 8 3 0 9 3 3 3 7 2 7 3 3 3 9 0 3 8 1 2 9
        3 9 2 1 8 3 0 9 3 3 3 7 2 7 3 3 3 9 0 3 8 1 2 9 3
  1 3 3 3 9 2 1 8 3 0 9 3 3 3 7 2 7 3 3 3 9 0 3 8 1 2 9 3 3 3 1
1 8 1 3 3 3 9 2 1 8 3 0 9 3 3 3 7 2 7 3 3 3 9 0 3 8 1 2 9 3 3 3 1 8 1
```

there are many interesting and big such pyramids.

List of few Palindromes and their construction:

39893 39993 40004 40104
40204 40304 40404 40504 40604 40704 40804 40904
41014 41114 41214 41314 41414 41514 41614 41714 41814 41914
42024 42124 42224 42324 42424 42524 42624 42724 42824 42924
43034 43134 43234 43334 43434 43534 43634 43734 43834 etc

Counting the Palindromes

All the digits are palindromes (1,2,3,...,9).

There are also **9 palindromes** with two digits (11,22,33, ...,99).

You can find to every two-digit number one, and only one number with three digits and with four digits.

For example: For the number 34 there are 343 and 3443.

You can conclude that there are **90 palindromes** with three and also **90 palindromes** with four digits.

You can find to every three-digit number one, and only one number with five digits and with six digits.

For example: To the number 562 there are 56265 and 562265.

You can conclude that there are **900 palindromes** with five and **900 palindromes** with six digits.

You have 9+9+90+90+900+900 = 1998 palindromes up to one million. That's 0,1998 %. About every 500th number is a palindrome.

A research problem with prime palindrome:

a. Patrick De Geest, observes that:

   10499 + 10501 + 10513 = 31513

   is the first case of a prime palindrome (31513) obtained adding three consecutive primes, the central one of them being a palindrome prime also (10501), and asks if this will ever happen again?

b. By my own I ask if exists any palindrome prime obtained adding the first k primes, that is to say:

   What is the first k, such that:

   Nk = 2 + 3 + 5 + 7 + 11 +…..+ pk = Prime Palindrome?

[January 8, 2013]

The newest palindromic prime record is again from Darren Bedwell

*10314727 - 8 * 10157363 - 1 (314727 digits) Written out, it's 157363 nines, a one, and 157363 more nines and is therefore a PWP or a Palindromic Wing Prime.*

Palindrome Triangular

n	n(n+1)/2
11	66
1111	617716
111111	6172882716

These are 3 of the 40 palindrome triangular numbers with n < 10,000,000. Unfortunately, the next number in the above series (11111111) is not palindrome,

although it does contain all 10 digits.
Palindromic Squares of Palindromes

$$10001^2 = 100020001$$
$$11011^2 = 121242121$$
$$11111^2 = 123454321$$
$$11211^2 = 125686521$$

$$11^1 = 11$$
$$11^2 = 121$$
$$11^3 = 1331$$
$$11^4 = 14641$$

Cube root not palindrome

$$2201^3 = 10662526601$$

The only known palindrome cube whose root is not palindrome !

Palindrome powers pattern

Base	Square	Cube	Forth Power
11	121	1331	14641
101	10201	1030301	104060401
1001	1002001	1003003001	1004006004001

And so on...

8 x 8 + 13 = 77
88 x 8 + 13 = 717
888 x 8 + 13 = 7117
8888 x 8 + 13 = 71117
88888 x 8 + 13 = 711117
And so on...

Interesting Palindrome Triangular Numbers

539593131395935
8208268228628028

Multiples of 9 (with a 9 at the end) **918273645546372819**
Products of consecutive numbers 77x78 = 6006 77x78x79 = 474474

Fascinating Palindromes

Start Palindrome	Divided by	Gives this Palindrome	Divided by	Gives this Palindrome
121	11	11	11	1
1234321	11	112211	11	10201
12345654321	11	1122332211	11	102030201
123456787654321	11	11223344332211	11	1020304030201

This pattern constructed from material on Peter Collins
He calls these palindromes 'Fascinating'!

The 57th positive number palindrome is 484 (57 = Heinz's number)
The 23456788th positive number palindrome is 12345678987654321

Palindrome prime number patterns
Depression Primes

727	**757**
72227	**75557**
722222227	**75555555557**

The above numbers are called depression primes. The next ones in the 'two' series contain 27 and 63 two's! Note the 'seven' two's in the one above. The next ones in the 'five' series contain 19, 21, 57, 73 & 81 fives.

Interesting 9-Digit Palindrome Primes

188888881	111181111	323232323
199999991	111191111	727272727
355555553	777767777	919191919
Plateau Primes	8 like digits	Smoothly Undulating
123494321		765404567
345676543	354767453	987101789
345686543	759686957	987646789
Peak Primes	5 consecutive digits	Valley Primes

Palindrome Primes : There are a total of 5172 nine digit primes that read the same forward or backward. Many of them have extra properties.

Plateau Primes : There are 3 primes where all the interior digits are alike and are higher than the terminal digits. There are two primes, 322222223 & 722222227 in which the interior digits are smaller than the end ones. These are called Depression Primes.

Undulating Primes : So called when adjacent digits are alternately greater or less then their neighbors. If there are only two distinct digits, they are called smoothly undulating. Of the total of 1006 undulating nine digit palindrome primes, seven are smoothly undulating.

Peak & Valley Primes : If the digits of the prime, reading left to right, steadily increase to a maximum value, and then steadily decrease, they are called peak primes. Valley primes are just the opposite. There are a total of 10 peak and 20 valley primes.

345676543 is unique because of the five consecutive digits.

As an example of palindromic primes, here is a pyramid (list) of palindromic primes supplied by G. L. Honaker, Jr.

2
30203
133020331
1713302033171
12171330203317121
151217133020331712151
1815121713302033171215181
16181512171330203317121518161
331618151217133020331712151816133
9333161815121713302033171215181613339
11933316181512171330203317121518161333911

Ten 27-digit palindrome primes in arithmetic progression

Found April 23, 1999

About 20 PC's were used, with the search team consisting of: Harvey Dubner, Manfred Toplic, Tony Forbes, Jonathan Johnson, Brian Schroeder and Carlos Rivera.

742950290870000078092059247
742950290871010178092059247
742950290872020278092059247
742950290873030378092059247
742950290874040478092059247
742950290875050578092059247
742950290876060678092059247
742950290877070778092059247
742950290878080878092059247
742950290879090978092059247

The Ubiquitous 196

Take any positive integer of two digits or more, reverse the digits, and add to the original number.

If the resulting number is not a palindrome, repeat the procedure with the sum until the resulting number is a palindrome.

For example, start with 87 or 88 or 89. Applying this process, we obtain :

87	88	89
87 + 78 = 165	88 is a palindrome	89 + 98 = 187
165 + 561 = 726		187 + 781 = 968
726 + 627 = 1353		968 + 869 = 1837
1353 + 3531 = 4884		until finally after 24 steps
4884 is a palindrome		becomes 8813200023188

Using the above algorithm, do all numbers become palindromes eventually? The answer to this problem is not known.

The venerable David Wells (Curious and Interesting Numbers, pp.211, 212) says that "196 is the only number less than 10,000 that by this process has not yet produced a palindrome."

Many palindrome web sites imply the same thing, but a little reflection reveals that is not correct. What is correct is that 196 is the smallest number that may not produce a palindrome.

Of the 900 3-digit numbers 90 are palindrome and 735 require from 1 to 5 reversals and additions.

Of the remaining 75 numbers, most form chains of numbers that eventually result in a palindrome. One such chain (part of the 89 chain in the example above) is 187, 286, 385, 583, 682, 781,869, 880, 968.

Others (of the 75) form a chain that so far has not resulted in a palindrome. This chain starts with the 196 mentioned above, The first few numbers of this chain are 196, 887, 1675, 7436, 13783. Each number in this chain must also be included with the 196 as possibly not converging to a palindrome.

Mathematicians are unable to prove that these numbers will eventually form a palindrome.

Consequently, a tremendous amount of time and effort has been expended in the search to resolve this issue.

By Sept. 11, 2003, Wade VanLandingham (Florida, U.S.A.) had tested the number 196 to 278,837,830 iterations, resulting in a number of 117,905,317 digits. It was still not a palindrome! Wade has tested a number of programs written by different people to perform this search. The one he is currently using was written by Eric Goldstein of the Netherlands, and is the fastest to date.

The first few numbers not known to produce palindromes, sometimes known as Lychrel numbers (VanLandingham), are 196, 295, 394, 493, 592, 689, 691, 788, 790, 879, 887, ...

Products of Successive Numbers which are palindrome

16x17 = 272
77x78 = 6006
77x78x79 = 474474
538x539 = 289982
1621x1622 = 2629262
2457x2458 = 6039306

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