J. of Ramanujan Society of Mathematics and Mathematical Sciences
Vol. 8, No. 2 (2021), pp. 95-108

ISSN (Online): 2582-5461

ISSN (Print): 2319-1023

A TRANSFORMATION INVOLVING BASIC MULTIVARIABLE I-FUNCTION OF PRATHIMA

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(**Received:** *Jun. 10, 2021* **Accepted:** *Jun. 24, 2021* **Published:** *Jun. 30, 2021)*

Abstract: In this document an expansion formula for q- analogue of multivariable I-function have been given by the applications of the q-Leibniz law for the q-derivatives of multiplication of two functions. Expansion formulas concerning the basic I-function of two variables, q - analogue H function of two variables, basic analogue Meijer G-function of two variables, basic I-function of one variable, basic analogue H-function of one variables, basic analogue Meijer G-function of one variables were given as special cases of the main formula.

Keywords and Phrases: Fractional q-integral, q-derivative operators, analogue basic I-function of several variables, q- analogue I-function of two variables, basic analogue I-function of one variable, q-Leibniz rule.

2020 Mathematics Subject Classification: 33C99, 33C60, 44A20.

1. Introduction

The origin of fractional calculus is as old as calculus, because this idea came to existence by L'Hospital, when he asked to Leibniz in 1695 [16] that "what if the order of derivative is a fraction number instead of an integer ?". After that it has been came to notice that fractional calculus has a vast application in various fields of mathematics ([14], [16], [17], [23]). The q - calculus has also evolved in twentieth century by Slater [27], Exton [8], Gasper [11] and a thesis [9]. The q - analogue of ordinary fractional calculus is termed as fractional q-calculus.

Al-Salam given the concept of fractional q-calculus, like q analogue of Cauchy's formula ([1], [5], [6]). Agarwal [3] studied certain fractional q-integral operators and q-derivatives. Further, Isogawa et al. [13] studied some basic properties of fractional q-derivatives. Rajkovic et al. [20] generalized the notion of the left fractional q-integral operators and fractional q-derivatives. Garg et al. [10] introduced q-analogues of hyper-Bessel type Kober fractional derivatives. Further, Saxena et al. [26] and Yadav et al. ([35]) have obtained values of many q- functions with the help of fractional q- operators. Recently, Purohit and Yadav [19] have defined q-extensions of the Saigo's fractional integral operators [22]. By inducing these ideas several mathematicians have used these operators for different funtions to evaluate fractional order derivative [9]-[11], [24] and [25], [30]-[35]. In this paper, we study the Kober fractional q-integral operator [10] and Riemann-Liouville fractional q-integral of the multiple basic analogue of multivariable I-function defined by Prathima et al. [18].

In this paper, we have established three theorems involving the fractional q-integral and q-derivative operator, which generalizes the classical definitions.

In the theory of q-series, for real or complex a and $|q| < 1$, the q-shifted factorial is defined as :

$$(a;q)_n = \prod_{i=1}^{n-1}(1-aq^i) = \frac{(a;q)_\infty}{(aq^n;q)_\infty}, \qquad n \in \mathbb{N}. \tag{1.1}$$

so that $(a;q)_0 = 1$, or equivalently

$$(a,q)_n = \frac{\Gamma_q(a+n)(1-q)^n}{\Gamma_q(a)} \qquad (a \neq 0, -1, -2, \cdots). \tag{1.2}$$

The q-gamma function [8] is given by

$$\Gamma_q(\alpha) = \frac{(q;q)_\infty (1-q)^{\alpha-1}}{(q^\alpha;q)_\infty} = \frac{[1-q]_{\alpha-1}}{(1-q)^{\alpha-1}} = \frac{(q;q)_{\alpha-1}}{(1-q)^{\alpha-1}} \quad (\alpha \neq 0,-1,-2,\cdots). \tag{1.3}$$

The basic analogue of well known Riemann-Liouville fractional integral operator of a function $f(x)$ due to Al-Salam [6], is given by

$$I_q^\mu\{f(x)\} = \frac{1}{\Gamma_q(\mu)} \int_0^x (x-tq)_{\mu-1} f(t) d_q t \qquad (Re(\alpha) > 0, |q| < 1). \tag{1.4}$$

also

$$[x-y]_v = x^v \prod_{n=0}^{\infty} \left[\frac{1-(y/x)q^n}{1-(y/x)q^{n+v}}\right]. \tag{1.5}$$

Also the basic integral, see Gasper and Rahman [11] are given by

$$\int_0^x f(t) d_q t = x(1-q) \sum_{k=0}^{\infty} q^k f(xq^k). \tag{1.6}$$

The equation (1.4) in connection with (1.6) gives the following operator in series form.

$$I_q^\mu f(x) = \frac{x^\mu (1-q)}{\Gamma_q(\alpha)} \sum_{k=0}^{\infty} q^k [1-q^{k+1}]_{\mu-1} f(xq^k). \tag{1.7}$$

Now, for $f(x) = x^{\lambda-1}$, we obtain (see [30]).

$$I_q^\mu \left(x^{\lambda-1}\right) = \frac{\Gamma_q(\lambda)}{\Gamma_q(\lambda+\mu)} x^{\lambda+\mu-1}. \tag{1.8}$$

2. Main Results

Now, we will deduce fractional q-integral formulae for the q- analogue of multivariable I-function defined by Prathima [18].

$$G(q^a) = \left[\prod_{n=0}^{\infty} (1-q^{a+n})\right]^{-1} = \frac{1}{(q^a;q)_\infty}. \tag{2.1}$$

We have

Recently I-function of several variables has been introduced and studied by Prathima et al. [18], it's an extension of the H-function of several variables defined by Srivastava and Panda [28, 29]. In this paper, we define a new function, the q - analogue

of multivariable I-function defined by Prathima et al. [18].
We note

$$I(z_1,\cdots,z_r;q)=I^{0,n:m_1,n_1\cdots;m_r,n_r}_{p,q':p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} z_1 & (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p}: \\ \vdots\quad ;q & \\ z_r & (b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q'}: \end{array}\right.$$

$$\left.\begin{array}{c} (c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \\ \\ (d_j^{(1)},\delta_j^{(1)};D_1)_{1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_r)_{1,q_r} \end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_s}\pi^r\phi(t_1,\cdots t_r;q)\prod_{i=1}^{r}\theta_i(t_i;q)z_i^{t_i}dt_1\cdots dt_r, \tag{2.2}$$

where $\phi(t_1,\cdots,t_r;q),\theta_i(t_i;q),i=1,\cdots,r$ are given by:

$$\phi(t_1,\cdots,t_r;q)=\frac{\prod_{j=1}^{n}G(q^{1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}A_jt_j})}{\prod_{j=n+1}^{p}G(q^{a_j-\sum_{i=1}^{r}\alpha_j^{(i)}A_jt_j})\prod_{j=1}^{q'}G(q^{1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}B_jt_j})}, \tag{2.3}$$

$$\theta_i(t_i;q)=\frac{\prod_{j=1}^{n_i}G(q^{1-c_j^{(i)}+\gamma_j^{(i)}C_j^{(i)}t_j})\prod_{j=1}^{m_i}G(q^{d_j^{(i)}-\delta_j^{(i)}D_j^{(i)}t_j})}{\prod_{j=n_i+1}^{p_i}G(q^{c_j^{(i)}-\gamma_j^{(i)}C_j^{(i)}t_j})\prod_{j=m_i+1}^{q_i}G(q^{1-d_j^{(i)}+\delta_j^{(i)}D_j^{(i)}t_i})G(1-q^{t_i})sin\pi t_i}, \tag{2.4}$$

where z_i are not zero and an empty product is interpreted as unity. Also n,p,q,m_i, $n_i,p_i,q_i(i=1,\cdots,r)$ are all positive integers such that $0\le n\le p,0\le q',0\le m_i\le q_i;0\le n_i\le p_i(i=1,\cdots,r)$. The letters $A_j,B_j,C_j^{(i)},D_j^{(i)}$ and $\alpha_j^{(i)},\beta_j^{(i)},\gamma_j^{(i)},\delta_j^{(i)}(i=1,\cdots,r)$ and are all positive numbers and the letters $a_j,b_j,c_j^{(i)},d_j^{(i)}$ are complex numbers. The contour L_i is in the s-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of , $G(q^{d_j^{(i)}-\delta_j^{(i)}D_j^{(i)}t_j})(j=1,\cdots,M_i)$ are to the right and all the poles of $G(q^{1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}A_jt_j})(j=1,\cdots,N)$, $G(q^{1-c_j^{(i)}+\gamma C_j^{(i)}t_j})$,$(j=1,\cdots,N_i)$ lie to the left of L_i. For large values of $|s_i|$, $Re(s_ilog(z_i)-log\sin\pi s_i)<0,i=1,\cdots,r$. The integrand has simple poles.
We will use following notations throughout in this work:

$$X=m_1,n_1;\cdots;m_r,n_r;V=p_1,q_1;\cdots;p_r,q_r. \tag{2.5}$$

$$A=(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p}. \tag{2.6}$$

$$B=(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q'}. \tag{2.7}$$

$$C = (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \cdots ; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r}. \tag{2.8}$$

$$D = (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \cdots ; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r}. \tag{2.9}$$

3. Main Theorems

Now, we will deduce two fractional basic integral formulae for the basic analogue of multivariable I-function.

Theorem 3.1. *Let* $Re(\mu) > 0, |q| < 1$, $\rho_i > 0 (i = 1, \cdots, r)$ *being any positive integers, the Riemann Liouville fractional q-integral of a product of two basic functions exists an under*

$$I_q^{\mu} \left\{ x^{\lambda-1} I \left(\begin{array}{c|c} z_1 x^{\rho_1} & A : C \\ \vdots \quad ; q & \vdots \\ z_r x^{\rho_r} & B : D \end{array} \right) \right\} = (1-q)^{\mu} x^{\lambda+\mu-1}$$

$$I_{p+1,q'+1;V}^{0,n+1;X} \left(\begin{array}{c|c} z_1 x^{\rho_1} & (1-\lambda; \rho_1, \cdots, \rho_r; 1), A : C \\ \vdots \quad ; q & \vdots \\ z_r x^{\rho_r} & B, (1-\lambda-\mu; \rho_1, \cdots, \rho_r; 1) : D \end{array} \right), \tag{3.1}$$

where $Re(t_i \log(z_i) - \log \sin \pi t_i) < 0, (i = 1, \cdots, r)$.

Proof. To deduce above theorem, we take the left hand side of equation (3.1) (say L) and make use of the definition (1.4) and (2.2), we obtain

$$L = I_q^{\mu} \left\{ x^{\lambda-1} \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \phi(t_1, \cdots, t_r; q) \prod_{i=1}^{r} \theta_i(t_r; q) z_i^{t_i} x^{\sum_{i=1}^{r} \rho_i t_i} d_q t_1 \cdots d_q t_r \right\}. \tag{3.2}$$

interchanging the order of integrations, justified under the above given conditions, we obtain

$$L = \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \phi(t_1, \cdots, t_r; q) \prod_{i=1}^{r} \theta_i(t_i; q) z_k^{t_k} I_q^{\mu} \left\{ x^{\sum_{i=1}^{r} \rho_i t_i + \lambda - 1} \right\} d_q t_1 \cdots d_q t_r. \tag{3.3}$$

We use the formula (1.8) and obtain

$$L = \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \phi(t_1, \cdots, t_r; q) \prod_{i=1}^{r} \theta_i(t_i; q) z_k^{t_k} \frac{(q^{\sum_{i=1}^{r} \rho_i t_i + \lambda + \mu}; q)_\infty}{(q^{\sum_{i=1}^{r} \rho_i t_i + \lambda}; q)_\infty} d_q t_1 \cdots d_q t_r. \tag{3.4}$$

Now, explicating the q-Mellin-Barnes double contour integral in terms of the basic analogue of I-function of several variables, we obtain the required equation (3.1).

If we put $-\mu$ in place of μ and apply in the following operator as :

$$I_q^{-\mu} f(x) = D_{x,q}^{\mu} f(x) = I_q^{\mu}\{f(x)\} = \frac{1}{\Gamma_q(-\mu)} \int_0^x (x - yq)_{-\mu-1} f(t) d_q y, \tag{3.5}$$

where $Re(\mu) < 0$ and we get the result as follows.

Theorem 3.2. *For $Re(\mu) > 0, |q| < 1, \rho_i (i = 1, \cdots, r)$ being any positive integers, the Riemann Liouville fractional q-integral of a product of two functions are as*

$$D_{x,q}^{\mu} \left\{ x^{\lambda-1} I \left(\begin{array}{l|l} z_1 x^{\rho_1} & A : C \\ \vdots \quad ; q & \vdots \\ z_r x^{\rho_r} & B : D \end{array} \right) \right\} = (1-q)^{-\mu} x^{\lambda-\mu-1}$$

$$I_{p+1,q'+1;V}^{0,n+1;X} \left(\begin{array}{l|l} z_1 x^{\rho_1} & (1-\lambda; \rho_1, \cdots, \rho_r; 1), A : C \\ \vdots \quad ; q & \vdots \\ z_r x^{\rho_r} & B, (1-\lambda+\mu; \rho_1, \cdots, \rho_r; 1) : D \end{array} \right), \tag{3.6}$$

where $Re(\lambda + \mu) > 0, Re(t_i log(z_i) - log \sin \pi t_i) < 0, (i = 1, \cdots, r)$.

The deduction of the theorem 3.2 is similar that of theorem 3.1.

4. Leibniz's Application

In view of Agarwal [3], we have basic extension of leibniz rule of fractional q-derivatives for a multiplication of two basic functions in form of an infinite series including the fractional q-derivatives as follows:

Lemma 4.1.

$$D_{x,q}^{\alpha}\{U(x)V(x)\} = \sum_{n=0}^{\infty} \frac{(-)^n q^{\frac{n(n+1)}{2}} [q^{-\mu}; q]_n}{(q;q)_n} D_{x,q}^{\mu-n}\{U(xq^n)\} D_{x,q}^{n}\{W(x)\}, \tag{4.1}$$

where $U(x)$ and $V(x)$ are regular functions.

We have the formula

Theorem 4.1. *For $Re(\mu) < 0, \rho_i (i = 1, \cdots, r)$ being any positive integers, the Riemann Liouville fractional q-integral of multiplication of two basic functions exists and which gives*

$$I_{p+1,q'+1;V}^{0,n+1;X} \left(\begin{array}{l|l} z_1 x^{\rho_1} & (1-\lambda; \rho_1, \cdots, \rho_r; 1) A : C \\ \vdots \quad ; q & \vdots \\ z_r x^{\rho_r} & B, (1-\lambda+\mu; \rho_1, \cdots, \rho_r; 1) : D \end{array} \right)$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n q^{n\lambda+\frac{n(n-1)}{2}}[q^{-\mu};q]_n}{(q;q)_n} I^{0,n+1;X}_{p+1,q'+1;V} \left(\begin{array}{c|l} z_1 x^{\rho_1} & (0;\rho_1,\cdots,\rho_r;1), A : C \\ \vdots \quad ;q & \vdots \\ z_r x^{\rho_r} & B,(n;\rho_1,\cdots,\rho_r;1) : D \end{array} \right), \tag{4.2}$$

where $Re(t_i log(z_i) - log \sin \pi t_i) < 0, (i = 1, \cdots, r).$

Proof. On taking in the q-Leibniz rule $U(x) = x^{\lambda-1}$ and

$$V(x) = I^{0,n;X}_{p,q';V} \left(\begin{array}{c|l} z_1 x^{\rho_1} & A : C \\ \vdots \quad ;q & \vdots \\ z_r x^{\rho_r} & B : D \end{array} \right).$$

We get (see Lemma 4.1).

$$D^{\mu}_{x,q} \left\{ x^{\lambda-1} I \left(\begin{array}{c|l} z_1 x^{\rho_1} & A : C \\ \vdots \quad ;q & \vdots \\ z_r x^{\rho_r} & B : D \end{array} \right) \right\} = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n\lambda+\frac{n(n-1)}{2}}[q^{-\mu};q]_n}{(q;q)_n}$$

$$\times D^{\mu-n}_{x,q}(xq^n)^{\lambda-1} D^n_{x,q}\{I(z_1 x^{\rho_1},\cdots,z_r x^{\rho_r};q)\}. \tag{4.3}$$

Using the theorem 3.1 and let $\lambda = 1$, we obtain

$$D^n_{x,q}\{I(z_1 x^{\rho_1},\cdots,z_r x^{\rho_r};q)\} = (1-q)^{-\mu} x^{-\mu}$$

$$\times I^{0,n+1;X}_{p+1,q';V} \left(\begin{array}{c|l} z_1 x^{\rho_1} & (0;\rho_1,\cdots,\rho_r;1), A : C \\ \vdots \quad ;q & \vdots \\ z_r x^{\rho_r} & B,(n;\rho_1,\cdots,\rho_r;1) : D \end{array} \right). \tag{4.4}$$

Now using the equations (4.4) and (1.8), we obtain the theorem 4.1 .

5. Particular Cases

Remark 5.1. *If* $C^{(i)}_j = D^{(i)}_j = 1$, *then q- analogue of multivariable I-function changes in q- analogue of multivariable H-function by Srivastava and Panda [28, 29].*

If $r = 2$, the q- analogue of multivariable I-function changes in q- analogue of I-function of two variables [15]. Let

$$A = \{(a_i;\alpha_i, A_i;\mathbf{A}_i)\}_{1,p_1}; A' = \{(e_i;E_i;\mathbf{E}_i)\}_{1,p_2}, \{(g_i;G_i;\mathbf{G}_i)\}_{1,p_3}. \tag{5.1}$$

$$B = \{(b_i;\beta_i, B_i;\mathbf{B}_i)\}_{1,q_1}; B' = \{(f_i;F_i;\mathbf{F}_i)\}_{1,q_2}, \{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4}. \tag{5.2}$$

We have,

Corollary 5.1. *For Re $(\mu) > 0$, ρ and σ being any positive integers, the Riemann Liouville fractional q-integral of a product of two basic function is*

$$I^{0,n_1;m_2,n_2,m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^\rho \\ . \\ z_2x^\sigma \end{array};q\left|\begin{array}{l}(1-\lambda;\rho,\sigma;1),A:A' \\ \\ B,(1-\lambda+\mu;\rho,\sigma;1):B'\end{array}\right.\right)$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n q^{n\lambda+\frac{n(n-1)}{2}}[q^{-\mu};q]_n}{(q;q)_n} I^{0,n_1+1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^\rho \\ . \\ z_2x^\sigma \end{array};q\left|\begin{array}{l}(0;\rho,\sigma;1),A:A' \\ \\ B,(n;\rho,\sigma;1):B'\end{array}\right.\right), \tag{5.3}$$

where $Re(slog(z_1) - logsin\pi s) < 0$ and $Re(tlog(z_2) - logsin\pi t) < 0$.

Remark 5.2. *We obtain the same relations with the fractional operators I^μ_q and K^μ_q. If $\mathbf{A_i} = \mathbf{E_i} = \mathbf{F_i} = \mathbf{G_i} = \mathbf{H_i} = 1$, then the q - analogue of generalized I-function of two variables given by Kumari et al. [15] reduces to basic of generalized of H-function of two variables [12] defined by Saxena et al. [25], we get the same results given by Yadav et al. [33].*

$$C_2 = \{(a_i;\alpha_i,A_i)\}_{1,p_1}; D_2 = \{(e_i;E_i)\}_{1,p_2}, \{(g_i;G_i)\}_{1,p_3}. \tag{5.4}$$

$$E_2 = \{(b_i;\beta_i,B_i)\}_{1,q_1}; F_2 = \{(f_i;F_i)\}_{1,q_2}, \{(h_i;H_i)\}_{1,q_3}. \tag{5.5}$$

This gives

Corollary 5.2.

$$H^{0,n_1;m_2,n_2,m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^\rho \\ . \\ z_2x^\sigma \end{array};q\left|\begin{array}{l}(1-\lambda;\rho,\sigma;1),C_2;D_2 \\ \\ E_2,(1-\lambda+\mu;\rho,\sigma;1):F_2\end{array}\right.\right)$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n q^{n\lambda+\frac{n(n-1)}{2}}[q^{-\mu};q]_n}{(q;q)_n} H^{0,n_1+1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^\rho \\ . \\ z_2x^\sigma \end{array};q\left|\begin{array}{l}(0;\rho,\sigma),C_2:D_2 \\ \\ E_2,(n;\rho,\sigma):F_2\end{array}\right.\right), \tag{5.6}$$

under the conditions verified by the corollary 5.1 and $\mathbf{A_i} = \mathbf{E_i} = \mathbf{F_i} = \mathbf{G_i} = \mathbf{H_i} = 1$.

We suppose

$$(\alpha_j)_{1,p_1} = (A_j)_{1,p_1} = (E_j)_{1,p_2} = (G_i)_{1,p_3} = (\beta_j)_{1,q_1} = (B_j)_{1,q_1} = (F_j)_{1,q_2} = (H_j)_{1,q_3} = 1. \tag{5.7}$$

The basic q-analogue generalized H-function of two variables reduces to basic generalized q-analogue of two variables Meijer's G-function defined by Agarwal [2], we use the following notations:

$$A_2' = (a_j)_{1,p_1} : B_2' = (e_j)_{1,p_2}, (g_j)_{1,p_3}. \tag{5.8}$$

$$C_2' = (b_j)_{1,q_1} : D_2' = (f_j)_{1,q_2}, (h_j)_{1,q_3}. \tag{5.9}$$

We obtain the formulas as follows:

Corollary 5.3.

$$G^{0,n_1;m_2,n_2,m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^\rho \\ \cdot \\ z_2x^\sigma \end{array} ;q \left| \begin{array}{l} (1-\lambda;\rho,\sigma;1), A_2'; B_2' \\ \\ C_2', (1-\lambda+\mu;\rho,\sigma;1) : D_2' \end{array}\right.\right)$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n q^{n\lambda + \frac{n(n-1)}{2}} [q^{-\mu};q]_n}{(q;q)_n} G^{0,n_1+1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^\rho \\ \cdot \\ z_2x^\sigma \end{array} ;q \left| \begin{array}{l} (0;\rho,\sigma), A_2' : B_2' \\ \\ C_2', (n;\rho,\sigma) : D_2' \end{array}\right.\right). \tag{5.10}$$

If $r = 1$, the q - analogue of multivariable I-function turns in q - analogue of I-function given by Rathie [21].

Let $A_1 = (a_j, \alpha_j : A_j)_{1,p}; B_1 = (b_j, \beta_j : B_j)_{1,q'}$, we obtain the following result:

Corollary 5.4.

$$I^{m,n'}_{p,q'}\left(z_1x^\rho \; ;q \left| \begin{array}{l} (1-\lambda,\rho;1), A_1 \\ \\ B_1, (1-\lambda+\mu;1) \end{array}\right.\right) = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n\lambda + \frac{n(n-1)}{2}} [q^{-\mu};q]_n}{(q;q)_n}$$

$$I^{m,n'+1}_{p+1,q'+1}\left(z_1x^\rho \; ;q \left| \begin{array}{l} (0,\rho;1), A_1 \\ \\ B_1, (n,\rho;1) \end{array}\right.\right), \tag{5.11}$$

under the condition verified by the theorem 4.1 and $r = 1$.

If $r = 1$ and $(A_j)_p = (B_j)_{q'} = 1$, the basic analogue I-function of one variable reduces to q - analogue H-function of one variable given by Saxena et al. [24], we get the same equation.

Corollary 5.5.

$$H^{m,n'}_{p,q'}\left(z_1x^\rho \; ;q \left| \begin{array}{l} (1-\lambda,\rho), A_1 \\ \\ B_1, (1-\lambda+\mu) \end{array}\right.\right) = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n\lambda + \frac{n(n-1)}{2}} [q^{-\mu};q]_n}{(q;q)_n}$$

$$H^{m,n'+1}_{p+1,q+1}\left(z_1 x^{\rho}\ ;q \middle| \begin{array}{l} (0,\rho), A_1 \\ \\ B_1, (n,\rho) \end{array} \right), \tag{5.12}$$

under the condition verified by the corollary 5.4 and $(A_j)_p = (B_j)'_q = 1$.

Now, $(\alpha_j)_{1,p} = (\beta_j)_{1,q'} = 1$, the basic of one variable H-function changes to basic analogue of Meijer's G- function. Let $A''_1 = (a_j)_{1,p}$ and $B''_1 = (b_j)_{1,q'}$, we get the following results:

Corollary 5.6.

$$G^{m,n'}_{p,q'}\left(z_1 x^{\rho}\ ;q \middle| \begin{array}{l} (1-\lambda,\rho), A''_1 \\ \\ B''_1, (1-\lambda+\mu) \end{array} \right) = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n\lambda + \frac{n(n-1)}{2}} [q^{-\mu};q]_n}{(q;q)_n}$$

$$G^{m,n'+1}_{p+1,q+1}\left(z_1 x^{\rho}\ ;q \middle| \begin{array}{l} (0,\rho), A''_1 \\ \\ B''_1, (n,\rho) \end{array} \right), \tag{5.13}$$

under the condition verified by the corollary 5.5 and $(\alpha_j)_{1,p} = (\beta_j)_{1,q'} = 1$.

6. Conclusion

The results obtained in this paper are most general results, therefore a result obtained gives various results by putting specific values to the parameters and variables. These results can be written as q - analogue of various functions [35] in the field of mathematics and mathematical physics.

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