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# Ramanujan number and other magical numbers

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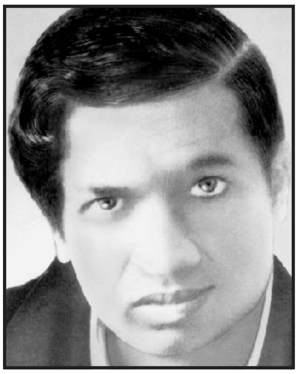
Rama Jain, Department of mathematics,
M.V.P.G. College, Lucknow

Mathematics is something beautiful, we can see how interesting each natural number is. All numbers are interesting, but some numbers are more interesting than others.
Some of such interesting numbers are
Ramanujan Number, Narcissistic Numbers, , Munchausen Number, Palindromes , beauty of numbers,

## Ramanujan Number

When we love mathematics we can see magic in numbers. Our face gets lit up when we observe something new about a number. Something similar and very interesting happened with great Indian mathematician Ramanujan.



***Srinivasa Ramanujan (1887-1920)***

He stumbled upon some very interesting number with some peculiar characteristics. Ramanujan number is 1729. 1729 is also known as the Hardy - Ramanujan number. This number is also called the Taxicab number.

Ramanujan number is so named after a famous anecdote of the British mathematician G. H. Hardy regarding a hospital visit to the Indian mathematician

Srinivasa Ramanujan.
In Hardy's own words:
*"I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number...1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two positive cubes in two different ways."*
The numbers are such that,

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

It is also identically the product of three prime numbers as

$$1729 = 7 \times 13 \times 19$$

. The largest known similar number is;

$$\begin{aligned} 885623890831 &= 7511^3 + 7730^3 \\ &= 8759^3 + 5978^3 \\ &= 3943 \times 14737 \times 15241 \end{aligned}$$

**Some More Facts**
**1.** If negative cubes are allowed, 91 is the smallest possible number with similar quality

$$91 = 6^3 + (-5)^3 = 4^3 + 3^3.$$

**2.** Interestingly 91 is also a factor of 1729. (91x19=1729)
**3.** If taking "positive cubes" would not have been a condition, Ramanujan number could have been -91, -189, -1729, and further negative numbers.
**4.** 1729 is also the third Carmichael number and the first absolute Euler pseudoprime.
**5.** Masahiko Fujiwara showed that 1729 is one of four positive integers (with the others being 81, 1458, and the trivial case 1) which, when its digits are added together, produces a sum which, when multiplied by its reversal, yields the original number:

$$1 + 7 + 2 + 9 = 19; 19 \times 91 = 1729$$

**6.** Till date only 10 Taxicab numbers are known. Subsequent Taxicab numbers are found using computers.

There are many other properties attached to this great number suggested by various great mathematicians.
Later on, numbers having similar property were all called as Ramanujan Numbers & the problem of finding such numbers came to be known as TaxiCab(2) problem. That is, solution to TaxiCab(2) yielded numbers of the kind

$$a^3 + b^3 = c^3 + d^3$$

Here are a few numbers of this kind :

$$\begin{aligned} 1729 &= 12^3 + 1^3 = 10^3 + 9^3 \\ 4104 &= 16^3 + 2^3 = 15^3 + 9^3 \\ 13832 &= 24^3 + 2^3 = 20^3 + 18^3 \\ 20683 &= 27^3 + 10^3 = 24^3 + 19^3 \end{aligned}$$

Numbers of the kind $a^3 + b^3 = m^3 + n^3 = x^3 + y^3$ are now known as Ramanujan Triples. The corresponding problem is known as TaxiCab(3).
Numbers of the kind $a^3 + b^3 = c^3 + d^3 = w^3 + x^3 = y^3 + z^3$ are known as Ramanujan Quadruples or corresponding problem is known as TaxiCab(4), and so on.

**Armstrong number or a plus perfect number or a narcissistic number**

Narcissus, according to Greek mythology, fell in love with his own image, seen in a pool of water, and changed into the flower now called by his name. Since this section deals with numbers "in love with themselves", narcissistic numbers will be defined as those that are representable, in some way, by mathematically manipulating the digits of the numbers themselves.
A plusperfect digital invariant (PPDI), an Armstrong number or a plus perfect number or a narcissistic number , is a number that is the sum of its own digits each raised to the power of the number of digits.
One such very interesting number is 153. Here are few properties of 153 . It is a fact that each number is a study in itself.
Below is a list of some attention-grabbing and curious property of 153.

**Property 1**

153 is the smallest number which can be expressed as the sum of cubes of its digits -

$$153 = 1^3 + 5^3 + 3^3$$

Because of this property 153 is called a narcissistic number (also known as a plus perfect digital invariant (PPDI), an Armstrong number or a plus perfect number)

**Property 2**

153 can be expressed as the sum of all integers from 1 to 17.

$$153 = 1 + 2 + 3 + 4 + ... + 16 + 17$$

Hence it can also be called a triangular number. Reverse of 153, i.e. 351 is also a triangular number, 153 can be termed as a reversible triangular number.

**Property 3**

153 is equal to the sum of factorials of number from 1 to 5 -

$$153 = 1! + 2! + 3! + 4! + 5!$$

**Property 4**

Sum of the digits of 153 is a perfect square-

$$1 + 5 + 3 = 9 = 3^2$$

Sum of all the divisors of 153 (except itself) is also a perfect square-

$$1 + 3 + 9 + 17 + 51 = 81 = 9^2$$

**Property 5**

153 can be expressed as the product of two numbers formed from its own digits-

$$153 = 3 \times 51$$

**Property 6**

A very interesting property of 153 is that-

Square root of 153 = 12.369 is the amount of full moons in one year.

**Other Observations on 153**

$$\begin{aligned} 1^0 + 5^1 + 3^2 &= 15 \\ 1^0 + 5^2 + 3^3 &= 53 \end{aligned}$$

153 if added to its reverse 351, we get 504.

153 + 351 = 504

Square of 504 is the smallest square which can be expressed as the product of two different non-square numbers which are reverse of one another:

$$504^2 = 288 \times 882$$

**Munchausen Number**
Munchausen Number is a number that is equal to the sum of its digits each raised to a power equal to the digit.
Munchausen number is also called perfect digit-to-digit invariant (PDDI) because of the above feature.
The only Munchausen numbers are 1 and 3435.
Specialty of 3435 -
also like 101, 131, 151, 181, and 191.

$$33 + 44 + 33 + 55 = 27 + 256 + 27 + 3125 = 3435$$

though, $0^0$ is undefined but if we consider it as 0, then there is one more Munchausen number founded till date - that number is 438579088 founded till date - that number is 438579088

$$4^4 + 3^3 + 8^8 + 5^5 + 7^7 + 9^9 + 0^0 + 8^8 + 8^8$$

$$= 256{+}27{+}16777216{+}3125{+}823543{+}387420489{+}0{+}16777216{+}16777216 = 438579088$$

**Palindromes**
A palindrome is something, which when written backwards or forwards, is the same thing. A palindrome may be a word such as mom, pop, sis, toot, or radar. Some more ardent palindromists have developed long phrases or sentences that are palindromes. Some examples are:
**1.** Madam, I'm Adam.
**2.** "Sir, I'm Iris."
**Palindrome Numbers**
Palindromes are very special kind of numbers. Typically a palindrome can be described as a number, word, sentence, etc. which reads same forward and backward. Specifically with regards to numbers, Palindromes are numbers which are symmetrical, i.e. they remain the same even when their digits are reversed. For example 14641 is a Palindrome.
In fact all the single digit numbers and numbers with same digit repeated are palindromes. So all numbers like 1,2,38,9,11,22,99,111,etc. are palindromes.
**Features of Palindrome Numbers**
**(1)** Reverse a non-palindromic number and add it to the original number. We will get a palindromic number by repeating this process. We may even get a palindromic number in first go.
**(i)** For example, let the original number be 37 (non-palindromic). Add reverse of it 73 to 37, we get 110 (not a palindromic number). Therefore repeat the process. 110 + 011 = 121 (palindromic number).

**(ii)** One more Example:

$$78 + 87 = 165, \qquad 165 + 561 = 726$$

$$726627 = 1353, \qquad 1353 + 3531 = 4884 \text{ (Which is a palindrome.)}$$

**(iii)** Another example, 16+61 = 77 (palindromic number).
Any number that never becomes palindromic in this way is known as Lychrel Number.
**(2)** A palindromic number in one base may or may not be palindromic in any other base. For example, 1991 is palindromic in both decimal and hexadecimal (7C7)
**(3)** Certain powers of palindromes made up of digit 1,2 and at times 3 are mostly palindromes. For example,

$$\begin{aligned} 11^2 &= 121 \\ 22^2 &= 484 \\ 101 \times 101 &= 10201 \\ 111 \times 111 &= 12321 \\ 121 \times 121 &= 14641 \\ 201 \times 201 &= 40804 \\ 212 \times 212 &= 44944 \end{aligned}$$

There are, however, an infinite number of cases as demonstrated here:
$11^2 = 121, 101^2 = 10201, 1001^2 = 1002001, 10001^2 = 100020001, ...$
$22^2 = 484, 202^2 = 40804, 2002^2 = 4008004, 20002^2 = 400080004, ...$
**(4)** All even digit palindromes are divisible by 11. There are many prime palindrome numbers

**Beauty Of Numbers**

Here are certain other ways to get Palindrome Numbers

$$\begin{aligned} 1 \times 9 + 2 &= 11 \\ 12 \times 9 + 3 &= 111 \\ 123 \times 9 + 4 &= 1111 \\ 1234 \times 9 + 5 &= 11111 \\ 12345 \times 9 + 6 &= 111111 \\ 123456 \times 9 + 7 &= 1111111 \\ 1234567 \times 9 + 8 &= 11111111 \\ 12345678 \times 9 + 9 &= 111111111 \\ 123456789 \times 9 + 10 &= 1111111111 \end{aligned}$$

Another way is as below-

$$
\begin{aligned}
9 \times 9 + 7 &= 88 \\
98 \times 9 + 6 &= 888 \\
987 \times 9 + 5 &= 8888 \\
9876 \times 9 + 4 &= 88888 \\
98765 \times 9 + 3 &= 888888 \\
987654 \times 9 + 2 &= 8888888 \\
9876543 \times 9 + 1 &= 88888888 \\
98765432 \times 9 + 0 &= 888888888
\end{aligned}
$$

We can list the sequence of numbers 1x1, 11x11, 111x111, 1111x1111, and so on until a palindrome results .

$$
\begin{aligned}
1 \times 1 &= 1 \\
11 \times 11 &= 121 \\
111 \times 111 &= 12321 \\
1111 \times 1111 &= 1234321 \\
11111 \times 11111 &= 123454321 \\
111111 \times 111111 &= 12345654321 \\
1111111 \times 1111111 &= 1234567654321 \\
11111111 \times 11111111 &= 123456787654321 \\
111111111 \times 111111111 &= 12345678987654321
\end{aligned}
$$

After this, the Palindrome doesn't occur.
List the sequence of numbers 11, 11 squared, 11 cubed, 11 to the fourth power, and so on until you do not get a palindrome. Here are the first four computations:

$$
\begin{aligned}
1111 \times 11 &= 121 \\
11 \times 11 \times 11 &= 1331 \\
11 \times 11 \times 11 \times 11 &= 14741
\end{aligned}
$$

It's a must read for all math lovers.

$$
\begin{aligned}
1 \times 8 + 1 &= 9 \\
12 \times 8 + 2 &= 98 \\
123 \times 8 + 3 &= 987 \\
1234 \times 8 + 4 &= 9876 \\
12345 \times 8 + 5 &= 98765 \\
123456 \times 8 + 6 &= 987654 \\
1234567 \times 8 + 7 &= 9876543 \\
12345678 \times 8 + 8 &= 98765432 \\
123456789 \times 8 + 9 &= 987654321
\end{aligned}
$$

**Kaprekar's Constant 6174**

6174 is known as Kaprekar's constant named after the Indian mathematician D.R.Kaprekar. 6174 has got a very interesting property.

To know what that mysterious property is, let us consider any four digit number. Arrange the digits in ascending and then in descending order to get two four- digit numbers. Then subtract from bigger number from smaller number. If we keep on repeating this process we will end up in 6174. This process is called Kaprekar's routine. All the numbers will yield 6174 in 7 or less than 7 iterations.

Example

Let's randomly choose any number, say 4732:

Now, arranging the digits in ascending and then in descending order to get two four digit numbers. Then taking difference of these two and proceeding as follows-

$$
\begin{aligned}
7432 - 2347 &= 5085 \\
8550 - 0558 &= 7992 \\
9972 - 2799 &= 7173 \\
7731 - 1377 &= 6354 \\
6543 - 3456 &= 3087 \\
8730 - 0378 &= 8352 \\
8532 - 2358 &= 6174
\end{aligned}
$$

Hence, we get 6174 in just 7 iterations.

We can take another example, by taking 7253 and proceeding as follows

$$\begin{aligned} 7532 - 2357 &= 5175 \\ 7551 - 1557 &= 5994 \\ 9954 - 4599 &= 5355 \\ 5553 - 3555 &= 1998 \\ 9981 - 1899 &= 8082 \\ 8820 - 0288 &= 8532 \\ 8532 - 2358 &= 6174. \end{aligned}$$

Again we get 6174 in 7 iterations.
Now, here is an example in which we get 6174 in less than 7 iterations.
Consider, a no. 3214 and then and proceeding as follows;

$$\begin{aligned} 4321 - 1234 &= 3087 \\ 8730 - 0378 &= 8352 \\ 8532 - 2358 &= 6174 \end{aligned}$$

Again, we get 6174 in 3 iterations.
Now, if we proceed the same procedure with 6174, we get the number 6174 repeatedly infinite number of times. Thus, we can say that if we take any four digit number and arrange it in descending and ascending orders to get two different numbers. Now, taking the difference of these two numbers and repeating the same process, we get 6174 infinite number of times after 7 or less than 7 iterations.
In this presentation, we have given only a few Magical Numbers with their properties. In fact, there are so many others which we can discuss in the forthcoming article.

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