South East Asian J. of Mathematics and Mathematical Sciences
Vol. 17, No. 1 (2021), pp. 471-480

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

AN ANISOTROPIC AND INHOMOGENEOUS BIANCHI TYPE-III COSMOLOGICAL MODEL WITH ELECTROMAGNETIC FIELD IN GR

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(**Received:** *Apr. 18, 2020* **Accepted:** *Feb. 19, 2021* **Published:** *Apr. 30, 2021)*

Abstract: Investigate an anisotropic and an inhomogeneous Bianchi type-III cosmological model in presence of electromagnetic field in GR. Here we assume that F_{12} is only non-zero element of electromagnetic field tensorF_{ij}. We have been find out a exact solutions by considering the metric potentials and displacement field are functions of coordinates t and x and the scale of expansion(θ) in the model is proportional to the particular eigen value σ_2^2 of the shear tensor σ_i^j , which leads to $B = (AC)^n$. The geometrical and bodily properties of the model are discussed in the occurrence of electromagnetic field.

Keywords and Phrases: Bianchi type-III, Electromagnetic field, Inhomogeneous universe, GR.

2020 Mathematics Subject Classification: 83C50, 83F05.

1. Introduction

Study of the cosmological models clear that the accessible theories for the constitution of the universe fall in to two categories, based either leading the amplification of quantum fluctuations in a scalar field during price rises or leading equilibrium breaking phase conversion in the primary universe. It's leading to the development of topological flaws like as area walls, monopoles, cosmic strings textures and other creatures. Space time admitting three parameter groups of automorphisms are significant in discussing the Bianchi models. In case of group is purely transitive above the three dimensional, particularly stable times subspace is helpful for two reasons. First, there are merely nine sets of separate arrangement constants for groups of these types. So for the classification of homogeneous space time we can utilize algebra and other reason for significance of this, is the effortlessness of the space-time field equations. Interstellar strings are observing topologically stable objects, which might be found during a segment change in the premature cosmos. Rising of the density fluctuation is conforms that the survival of the strings in cosmos, Strings play an important role in the construction of galaxies. These strings have stress energy and couple to gravitational field. So it is motivating to learn the gravitation properties of strings. The Letelier [13 14] and Stachel [23] gives the general relativistic treatment of strings. Tikekar and Patel [25] have discussed some exact solution in Bianchi type-VI0 string cosmology. Inactive rotating world model is discussed by Patel and Maharaj [16] in presence of magnetic field. Jain and Bali [3] have investigated a non-static magnetized astrophysical model with Bianchi type-III in G R. Bali, Dave and Upadhaya [4-7] have deliberate I, III and IX Bianchi type string cosmological models in G R. Asseo and Sol [2], Pudritz and Silk [22], Perley and Taylor [17], Wolfe, Lanzetta and Oren [26], Bali, Pradhan and Amirhashchi [8], Pradhan, Rai and Jotania [18], Amirhashchi, Pradhan and Zainuddin [1] have studied of the existence of primordial magnetic field in the premature stages of the growth of the space. Bali and Tyagi [9], Pradhan, Singh, and Jotania [19] have discussed inhomogeneous anisotropic cosmological models in the occurrence of electromagnetic field. Subsequently, many famous others [10, 11, 12, 15, 20, 21, 24], who have discussed several cosmological models with electromagnetic field with various Bianchi in different aspects. We have been find out a exact solutions by considering the metric potentials and displacement field are functions of coordinates t and x and the scale of expansion(θ) in the model is proportional to the particular eigen value σ_2^2 of the shear tensor σ_i^j , which leads to $B = (AC)^n$. The geometrical and bodily properties of the model are discussed in the occurrence of electromagnetic field.

2. The Metric and Field Equations

We considers spatially inhomogeneous Bianchi type-III can be written in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2ax}dy^2 + C^2dz^2 \tag{1}$$

With the conference $x^1 = x, x^2 = y, x^3 = z, x^4 = t$ B is the function of t only and A and C are functions of x and t. The volume element is given as for the model (1)

$$V = \sqrt{-g} = ABCe^{-ax} \tag{2}$$

The observational quantity deceleration parameter q is defined as

$$q = -\frac{V\ddot{V}}{V^2} \tag{3}$$

The energy momentum tensor with electromagnetic field for the string is given by

$$T_i^j = \rho u_i u^j - \lambda x_i x^j + E_i^j \tag{4}$$

where co-moving coordinates are as $u^1 = u^2 = u^3 = 0, u^4 = 1$ and $x^1 = \frac{1}{A}, x^2 = x^3 = x^4 = 0$
i.e.

$$u^i u_i = -x^i x_i = -1 \; and \; u^i x_i = 0 \tag{5}$$

In equation (4), electromagnetic energy tensorE_i^j is given by Lichnerowicz[24]

$$E_i^j = \bar{\mu}[h_l h^l (u_i u^j + \frac{1}{2} g_i^j) - h_i h^j] \tag{6}$$

where h_i are the magnetic flux vector and $\bar{\mu}$ is the magnetic permeability defined as

$$h_i = \frac{1}{\bar{\mu}} * F_{ji} u^j \tag{7}$$

where $*F_{ij}$ is defined by Syange [24]

$$*F_{ij} = \frac{\sqrt{-g}}{2} \varepsilon_{ijkl} F^{kl} \tag{8}$$

If we take for granted that the current flow along z-axis, then F_{12} is the only non vanishing component of F_{ij} with the intention that $h_3 \neq 0$.
The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{k;ij} = 0 \tag{9}$$

$$\lceil \frac{1}{\bar{\mu}} F^{ij} \rceil_{;j} = 0 \tag{10}$$

require that F_{12} is the function of x and t both and the magnetic permeability is also function of x and t both. Where the semicolons are shows a covariant differentiation.
The Einstein's field equation

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G T_i^j \tag{11}$$

Where R_i^j Ricci tensor and $R = g^{ij}R_{ij}$ is the Ricci scalar and T_i^j is energy momentum tensor. For the line element (1), Einstein's field equation (11) leads to the subsequent arrangement of equations.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{B}}{AB} + \frac{C'a}{CA^2} = 8\pi G[\lambda - \frac{(F_{12})^2 e^{2ax}}{2\bar{\mu}A^2B^2}] \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{1}{A^2}[\frac{A'C'}{AC} - \frac{C''}{C}] = -8\pi G[\frac{(F_{12})^2 e^{2ax}}{2\bar{\mu}A^2B^2}] \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2}[\frac{A'a}{A} + a^2] = 8\pi G[\frac{(F_{12})^2 e^{2ax}}{2\bar{\mu}A^2B^2}] \tag{14}$$

$$\frac{1}{A^2}[a^2 - \frac{A'C'}{AC} + \frac{C''}{C} + \frac{A'a}{A} - \frac{C'a}{C}] - \frac{\dot{A}\dot{C}}{AC} = -8\pi G[\lambda + \frac{(F_{12})^2 e^{2ax}}{2\bar{\mu}A^2B^2}] \tag{15}$$

$$\frac{\dot{A}C'}{AC} - \frac{\dot{C}'}{C} + a[\frac{\dot{B}}{B} - \frac{\dot{A}}{A}] = 0 \tag{16}$$

Where the over head dot represents differentiation with respect to t and the over head prime represents differentiation with respect to x.
The expansion scalar θ for the model (1) is given by

$$\theta = u^i_{;i} = (\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) \tag{17}$$

The shear tensor is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}[(\frac{\dot{A}}{A} - \frac{\theta}{3})^2 + (\frac{\dot{B}}{B} - \frac{\theta}{3})^2 + (\frac{\dot{C}}{C} - \frac{\theta}{3})^2] \tag{18}$$

3. Solution of the Field Equations

The field equations (12) to (16) are system of five highly non-linear differential equations with six unknown variables, A, B, C, ρ, λ and F_{12}. Therefore one physically reasonable condition between these unknowns is required to obtain explicit solution of the field equations. So let us assume that the expansion scalar (θ) is proportional to the value σ_2^2 of the shear tensor σ_j^i.
This condition leads to

$$B = (AC)^n \tag{19}$$

where $n = \frac{3+\delta}{2\delta-3}$ is a constant.
Equations (13) and (14) lead to

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2}[\frac{A'a}{A} + a^2] + \frac{1}{A^2}[\frac{A'C'}{AC} - \frac{C''}{C}] = 0 \tag{20}$$

Using equation (19) in (16), we get

$$\frac{\dot{A}}{A}(\frac{C'}{C} + an - a) - \frac{\dot{C}'}{C} + an\frac{\dot{C}}{C} = 0 \tag{21}$$

From the condition (19) we can make the following assumptions

$$A(x,t) = f(x)k(t) \ and \ C(x,t) = \frac{l(t)}{f(x)} \tag{22}$$

Other choices also can be made and thus the above assumptions in equation (22) are by no means unique.
Now by using above assumptions in equation (21), we get

$$\frac{\dot{k}/k}{\dot{l}/l} = \frac{(f'/f + an)}{(f'/f + a - an)} = N(Constant) \tag{23}$$

This leads to

$$\frac{\dot{k}}{k} = N\frac{\dot{l}}{l} \tag{24}$$

And

$$\frac{f'}{f} = \frac{N(a-an) - an}{1-N} \tag{25}$$

The solutions of the equations (24) and (25) are as follows

$$k(t) = \alpha[l(t)]^N \tag{26}$$

$$f(t) = \beta e^{bx} \tag{27}$$

Where $b = \frac{N(a-an)-an}{1-N}$ is a constant and α and β are constant of integration.
Now from equation (20) and (21), we get

$$A^2[\frac{\dot{A}^2}{A^2}n^2 + \frac{\dot{C}^2}{C^2}(n^2-n) + \frac{\dot{A}\dot{C}}{AC}(2n^2+n+1) + \frac{\ddot{A}}{A}(2+n) + \frac{\ddot{C}}{C}(1+n)] = M(Constant\ w.r.to\ t) \tag{28}$$

If we substitute the value of A and C from (22) in above equation, then we get

$$\frac{\ddot{l}}{l} + \frac{\dot{l}^2}{l^2} p = r l^{-2N} \tag{29}$$

Where p and r are constants w.r.to t
The solution of equation (29), we obtain

$$l(t) = [c_1 t + c_2]^{\frac{1}{N}} \tag{30}$$

Where $c_1 = N\sqrt{\frac{r}{1+p-N}}, c_2$ are constants.
Consequently, equation (26) becomes

$$k(t) = \alpha[c_1 t + c_2)] \tag{31}$$

Hence, we get

$$A(x,t) = c_3 e^{bx} T \tag{32}$$

$$B(t) = \alpha^n T^{(\frac{n}{N}+n)} \tag{33}$$

$$C(x,t) = c_4 e^{-bx} T^{\frac{1}{N}} \tag{34}$$

Where $c_3 = \alpha\beta, c_4 = \frac{1}{\beta}$ *and* $T = c_1 t + c_2$
The metric potentials A, B and C can be singular at only $t \to \infty$. Hence the line element (1) with these coefficient is singular free even at $t = 0$, and it can be written in the following form

$$ds^2 = -dt^2 + c_3^2 e^{2bx} T^2 dx^2 + \alpha^{2n} T^{2(\frac{n}{N}+n)} e^{-2ax} dy^2 + c_4^2 e^{-2bx} T^{\frac{2}{N}} dz^2 \tag{35}$$

4. Geometrical and Material Properties of the Model

The electromagnetic field F_{12}, the energy density (ρ), the string tension density (λ), the particle density ρ_p, the scalar of expansion (θ)and shear tensor (σ). For the model (35) geometrical and material properties are as follows.

$$(F_{12})^2 = \frac{\bar{\mu}\alpha^{2(n+1)}\beta^2 T^{2(\frac{n}{N}+n)}}{4\pi G e^{2(a-b)x}}\left[\frac{n(n-1)(N+1)^2}{N^2} c_1^2 - (ab + a^2)\right] \tag{36}$$

The string tension density

$$\lambda = \frac{T^{-2}}{8\pi G}\left[\frac{c_1^2}{N^2}\{(N+1)^2(2n^2 - n) - 2nN(N+1) + 1 - N\} - \{ab + a^2 + \frac{abe^{-2bx}}{c_3^2}\}\right] \tag{37}$$

The energy density

$$\rho = \frac{T^{-2}}{8\pi G}[\frac{c_1^2}{N} - \frac{n(n-1)(N+1)^2}{N^2}c_1^2 - \frac{(2ab+a^2+2b^2)}{c_3^2 e^{2bx}} + ab + a^2] \tag{38}$$

The particle density

$$\begin{aligned}\rho_p =& \rho - \lambda \\ =& \frac{T^{-2}}{8\pi G}[\frac{c_1^2}{N} + \frac{c_1^2}{N^2}\{(N+1)^2(2n-3n^2) + 2nN(N+1) + N - 1\} \\ & - 2(ab+a^2) - \frac{(ab+a^2+2b^2)}{c_3^2 e^{2bx}}]\end{aligned} \tag{39}$$

From the above equations we found that the energy conditions $\rho \geq 0, \rho_p \geq 0$ are satisfied under these conditions

$$\frac{(2ab+a^2+2b^2)}{c_3^2 e^{2bx}} \leq \frac{c_1^2}{N}[1 - \frac{n(n-1)(N+1)^2}{N}] + ab + a^2 \tag{40}$$

$$2(ab+a^2) + \frac{(ab+a^2+2b^2)}{c_3^2 e^{2bx}} \leq \frac{c_1^2}{N^2}[(N+1)^2(2n-3n^2) + 2N(nN+n+1) - 1] \tag{41}$$

We also detected that the string tension density $\lambda \geq 0$ under this condition

$$\{ab + a^2 + \frac{abe^{-2bx}}{c_3^2}\} \leq \frac{c_1^2}{N^2}\{(N+1)^2(2n^2-n) - 2nN(N+1) + 1 - N\} \tag{42}$$

The expansion

$$\theta = \frac{c_1(n+1)(N+1)T^{-1}}{N} \tag{43}$$

The shear tensor

$$\sigma^2 = \frac{c_1^2 T^{-2}}{6N^2}[3N^2 + (N+1)^2(2n^2+2n-1)] \tag{44}$$

Volume element of the model is

$$V^3 = c_3 c_4 \alpha^n e^{-ax} T^{\frac{(n+1)(N+1)}{N}} \tag{45}$$

From equation (43) and (44), we find

$$\frac{\sigma^2}{\theta^2} = \frac{3N^2 + (N+1)^2(2n^2+2n-1)}{6(n+1)^2(N+1)^2} = constant \tag{46}$$

Since $\frac{\sigma}{\theta} = constant$, so model (35) does not approach isotropy for any value of t.
Since $\dot{u}_i = 0$, so this model does not admit acceleration.
Here we can see that

$$\frac{\sigma_2^2}{\theta} = \frac{2n-1}{3(n+1)} = \delta \tag{47}$$

Where δ is a constant of proportional.
Substituting the value from equation, (45) to (3) than we have deceleration parameter q as

$$q = -1 + \frac{3N}{(n+1)(N+1)} \tag{48}$$

The deceleration parameter q approaches the value -1 as in the case of de-sitter universe at $N = 0$.

5. Conclusion

Here we studied an anisotropic and inhomogeneous cosmological model of Bianchi type-III with presence of electromagnetic field in GR. And behavior of string in above model with electromagnetic field applying equations (40) and (41) we find that energy conditions $\rho \geq 0, \rho_p \geq 0$ are satisfied. The string tension $\lambda \geq 0$ applying equation (42) and this model is singular free even at the initial epoch $t = 0$ and has vanishing acceleration.
This model starts expanding at $T > 0$ and it stops expanding as $T \to \infty$. We found that $\frac{\sigma}{\theta} = constant$, this means that for any value of t the model is not moving towards isotropy. We also find that $\rho, \lambda, \rho_p, \theta, \sigma^2$ tends to ∞ when $T \to 0, x \to 0$. That is the cosmos starts from primary spectacle with infinity energy, infinity internal tension, infinity speed of shear and expansion. The material and geometrical performance of the model is also discussed.

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