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Motivation towards learning mathematics:

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Abstract:

Learning mathematics is the art of systematic planning of acquiring knowledge. Before you start learning mathematics, you must know what particular part you are going to learn. What is your primary information about that part? The way in which you are trying to learn mathematics is most important. Solving mathematical exercises is not complete learning of mathematics.

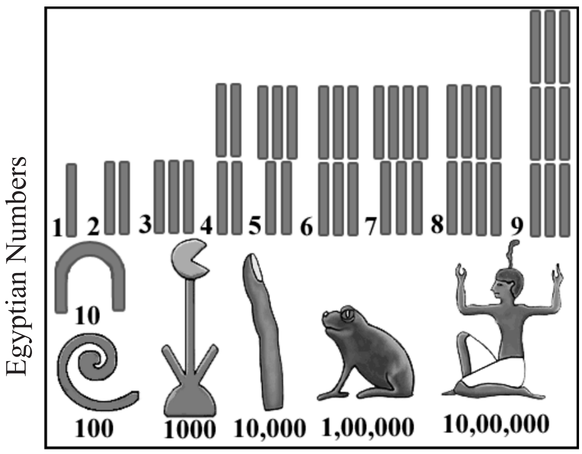
If you do not learn mathematics properly at basic or primary level, then you lost interest and charm in this subject. For proper learning of mathematics, in fact, nobody can confidently make any the best way out.

In this paper let us try and hope something good for the learners.

Some beautiful mathematical facts:

History of numerals coming from India:

It is now universally accepted that our decimal numbers derive from forms, which were invented in India and transmitted via Arab culture to Europe, undergoing a number of changes on the way. We also know that several different ways of writing numbers evolved in India before it became possible for existing decimal numerals to be marred with the place-value principle of the Babylonians to give birth to the system which eventually became the one which we use today.



Contrast between Greek and Hindu Mathematics

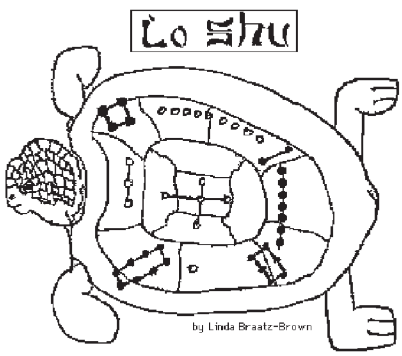
There are many differences between the Greek and the Hindu mathematics. In the first place, the Hindus who worked in mathematics regarded themselves primarily as astronomers. Hindu mathematics remained largely a handmaiden to astronomy. With the Greeks, mathematics attained an independent existence and was studied for it owns sake.

Hindu mathematics is largely empirical, with proofs or derivations seldom offered; an outstanding characteristic of Greek mathematics is its insistence on rigorous demonstration. Hindu mathematics is of very uneven quality, good and poor mathematics often appearing side by side; the Greeks seemed to have an instinct that led them to distinguish well from poor quality.

Some of the contrasts between Greek and Hindu mathematics is perpetuated today in the differences between many of our elementary geometry and algebra textbooks, because the former are deductive and the latter are often collections of rules.

Conclusion: In its final form the Hindu system of writing numerals is fundamentally different from that of the Egyptians, the Ancient Greeks and the Romans. The Hindus have special symbols for the individual numbers from one to nine, whose rank are indicated by their positions. The Hindu system is to some extent a continuation of the early Chinese system, although it has not been proved that there was direct influence. The Hindu system is a pure place-value system. Only a pure place-value system needs a symbol for a missing amount, for a non-existent rank, the zero. Only the Hindus within the context of Indo-European civilizations have consistently used a zero.

- **Magic square**: Magic squares have been around for over 3,000 years. They are descendants of the oldest known number mystery, the legend of Lo Shu, found in China in a book entitled Yih King.



- **March 14 is Albert Einsteins birthday. This day is also celebrated as Pi day.**

- There are just four numbers, after unity, which are the sums of the cubes of their digits: $153 = 1^3 + 5^3 + 3^3,\ 370 = 3^3 + 7^3 + 0^3,\ 371 = 3^3 + 7^3 + 1^3$ and $407 = 4^3 + 0^3 + 7^3$

- $2646798 = 2^1 + 6^2 + 4^3 + 6^4 + 7^5 + 9^6 + 8^7$

- 6174 is known as Kaprekar's constant. This number is notable for the following property:

1. Take any four-digit number, using at least two different digits. (Leading zeros are allowed.)
2. Arrange the digits in ascending and then in descending order to get two four-digit numbers, adding leading zeros if necessary.
3. Subtract the smaller number from the bigger number.
4. Go back to step 2.

The above process, known as Kaprekar's routine, will always reach its fixed point, 6174, in at most 7 iterations. Once 6174 is reached, the process will continue yielding 7641 - 1467 = 6174. For example, choose 3524:

$$5432 - 2345 = 3087$$

$$8730 - 0378 = 8352$$

$$8532 - 2358 = 6174$$

Interesting number 240:

With 20 divisors total (1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 40, 48, 60, 80, 120, and 240), more than any previous number, 240 is a highly composite number, and it is also a refactorable number or tau number, since it has 20 divisors and 20 divides 240. And since it has 31 totient answers, more than any previous integer, 240 is a highly totient number.

240 is a semiperfect number.

240 can be expressed as a sum of consecutive primes in two different ways: 240 = 53 + 59 + 61 + 67 = 17 + 19 + 23 + 29 + 31 + 37 + 41 + 43. It can also be expressed as the product of two consecutive integers, 15 and 16, thus 240 is a

pronic number.
Also, being divisible by the sum of its digits, 240 is a Harshad number, and not only that, it is also a concatenation of two of its proper divisors (see Sloane's A50480).
E_8 has 240 roots.
There are 240 distinct solutions of the Soma cube puzzle.
In mathematics, the sieve of Sundaram is a simple deterministic algorithm for finding all prime numbers up to a specified integer. It was discovered in 1934 by S. P. Sundaram, an Indian student from Sathyamangalam.

Brahmagupta's formula

In Euclidean geometry, Brahmagupta's formula finds the area of any quadrilateral given the lengths of the sides and some of the angles. In its most common form, it yields the area of quadrilaterals that can be inscribed in a circle.

Basic form

Brahmagupta's formula gives the area K of a cyclic quadrilateral whose sides have lengths a, b, c, d as

$$K = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

This formula generalizes Heron's formula for the area of a triangle

Heron's formula

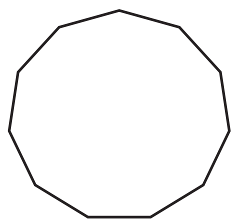
A triangle with sides a, b, and c.
In geometry, Heron's (or Hero's) formula, named after Heron of Alexandria, states that the area T of a triangle whose sides have lengths a, b, and c is

$$T = \sqrt{s(s-a)(s-b)(s-c)}$$

where s is the semi-perimeter of the triangle;

$$s = \frac{a+b+c}{2}$$

- In geometry, a hendecagon (also undecagon) is an 11-sided polygon.



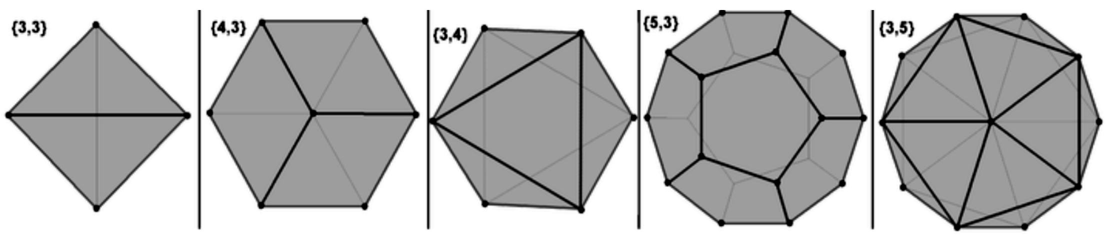
A regular hendecagon cannot be constructed using compass and straightedge. Because 11 is not a Pierpont prime, construction of a regular hendecagon is still

impossible even with the usage of an angle tri-sector
A regular hendecagon has internal angles of 147.27 degrees. The area of a regular hendecagon with side length a is given by

$$A = \frac{11}{4}a^2 cot\frac{\pi}{11} \simeq 9.36564a^2.$$

A regular hendecagon is not constructible with compass and straightedge; this animation shows an approximation:
History

John Flinders Petrie (1907-1972) was the only son of Egyptologist Sir W. M. Flinders Petrie. He was born in 1907 and as a schoolboy showed remarkable promise of mathematical ability. In periods of intense concentration he could answer questions about complicated four-dimensional objects by visualizing them. He first realized the importance of the regular skew polygons which appear on the surface of regular polyhedra and higher polytopes. He was a lifelong friend of Coxeter, who named these polygons after him.
The idea of Petrie polygons was later extended to semi regular polytopes. In 1972, a few months after his retirement, Petrie was killed by a car while attempting to cross a motorway near his home in Surrey.



The ancient Greeks founded Western mathematics, but as ingenious as they were, they could not solve three problems:

- Trisect an angle using only a straightedge and compass
- Construct a cube with twice the volume of a given cube
- Construct a square with the same area as a given circle.

Some interesting mathematical events:

- The Bridges of Konigsberg - This problem inspired the great Swiss mathematician Leonhard Euler to create graph theory, which led to the development of topology.
- The Value of Pi - Throughout the history of civilization various mathematicians have been concerned with discovering the value of and different expressions for the ratio of the circumference of a circle to its diameter.
- Puzzling Primes - To fully comprehend our number system, mathematicians need to understand the properties of the prime numbers. Finding them isn't so easy, either.
- Famous Paradoxes - In the history of mathematical thought, several paradoxes have challenged the notion that mathematics is a self-consistent system of knowledge. Presented here are Zeno's Paradox and Cantor's Infinities.
- The Problem of Points - An age-old gambling problem led to the development of probability by French mathematicians Pascal and Fermat in the seventeenth century.
- A Proof that e is irrational - A proof by contradiction that relies on the expression of e as a power series.

Brief and important outlines of learning conventional mathematics at different level.

High school

Pythagoras theorem ; Euclid's proof of the infinitude of primes; $\sqrt{2}$ is irrational; $sin^2\theta + cos^2\theta = 1$

Undergraduate

e is irrational; π is irrational; Fermat's little theorem; Fermat's theorem on sums of two squares; Sum of the reciprocals of the primes diverges; Bertrand's postulate; Law of large numbers; Spectral Theorem; L'Hopital's rule; Four color theorem; $e^{\pi i} + 1 = 0$

Postgraduate

Fermat's last theorem; Brouwer fixed-point theorem; Jordan Curve Theorem; Prime number theorem; Riemann hypothesis;

The permutations of Rubik's Cube have a group structure; the group is a fundamental concept within abstract algebra.

Here is a fairly comprehensive list of the branches of Mathematics:

1. **Foundations**
   Logic & Model Theory; Computability Theory & Recursion Theory; Set Theory; Category Theory

2. **Algebra**
   Group Theory, Symmetry; Ring Theory, Polynomials ; Field Theory; Module Theory, Linear Algebra ; Galois Theory, The Theory of Equations ; Number Theory; Combinatorics ; Algebraic Geometry

3. **Mathematical Analysis**
   Real Analysis & Measure Theory, Calculus; Complex Analysis; Tensor & Vector Analysis; Differential & Integral Equations ; Numerical Analysis ; Functional Analysis & The Theory of Functions

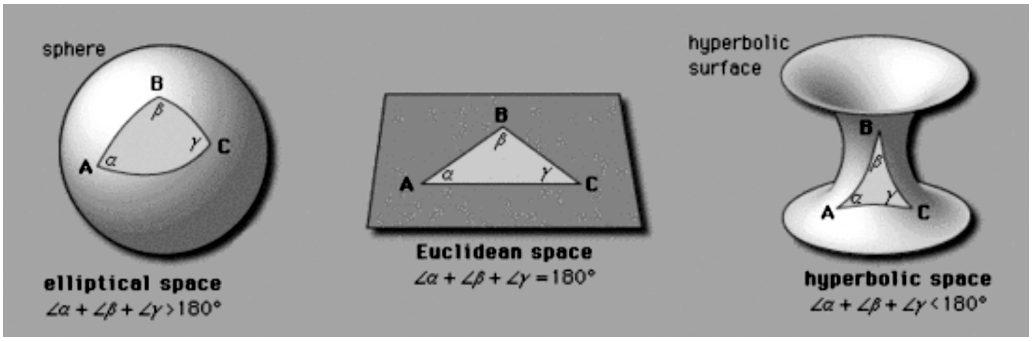
4. Geometry & Topology; Euclidean Geometry; Non-Euclidean Geometry; (Hyperpolic & Elliptic) ; Absolute Geometry; Metric Geometry; Projective Geometry; Affine Geometry; Discrete Geometry & Graph Theory; Differential Geometry; Point-Set or General Topology ; Algebraic Topology

5. **Applied Mathematics**
   Probability Theory; Statistics ; Computer Science; Mathematical Physics; Game Theory ; Systems & Control Theory

An important and interesting concept in mathematics:

A number of geometers made attempts to prove the parallel postulate by assuming its negation and trying to derive a contradiction, including Proclus, Ibn al-Haytham (Alhacen), Omar Khayym, Nasir al-Din al-Tusi, Witelo, Gersonides, Alfonso, and later Giovanni Gerolamo Saccheri, John Wallis, Johann Heinrich Lambert, and Legendre. Their attempts failed, but their efforts gave birth to hyperbolic geometry. Hyperbolic geometry is one of the richest areas of mathematics, with connections not only to geometry but to dynamical systems, chaos theory, number theory, relativity, and many other areas of mathematics and physics.

In mathematics, hyperbolic geometry (also called Lobachevskian geometry or BolyaiLobachevskian geometry) is a non-Euclidean geometry, meaning that the parallel postulate of Euclidean geometry is replaced. The parallel postulate in Euclidean geometry is equivalent to the statement that, in two-dimensional space, for any given line R and point P not on R, there is exactly one line through P that does not intersect R; i.e., that is parallel to R. In hyperbolic geometry there are at least two distinct lines through P which do not intersect R, so the parallel postulate is false. Models have been constructed within Euclidean geometry that obey the axioms of hyperbolic geometry, thus proving that the parallel postulate is independent of the other postulates of Euclid.



The five axioms for Euclidean geometry are:

1. Any two points can be joined by a straight line. (This line is unique given that the points are distinct)
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. Through a point not on a given straight line, one and only one line can be drawn that never meets the given line.

The five axioms for spherical geometry are:

1. Any two points can be joined by a straight line.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. There are No parallel lines.

The five axioms for hyperbolic geometry are:

1. Any two points can be joined by a straight line.

2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. Through a point not on a given straight line, infinitely many lines can be drawn that never meet the given line.

Sacred geometry is the geometry used in the planning and construction of religious structures such as churches, temples, mosques, religious monuments, altars, tabernacles; as well as for sacred spaces such as temenoi, sacred groves, village greens and holy wells, and the creation of religious art. In sacred geometry, symbolic and sacred meanings are ascribed to certain geometric shapes and certain geometric proportions, according to Paul Calter

The Mathematical Problems of David Hilbert

About Hilbert's address and his 23 mathematical problems Hilbert's address of 1900 to the International Congress of Mathematicians in Paris is perhaps the most influential speech ever given to mathematicians, given by a mathematician, or given about mathematics. In it, Hilbert outlined 23 major mathematical problems to be studied in the coming century. Some are broad, such as the axiomatization of physics (problem 6) and might never be considered completed. Others, such as problem 3, were much more specific and solved quickly. Some were resolved contrary to Hilbert's expectations, as the continuum hypothesis (problem 1).

The Greatest Theorems

1. The Irrationality of the Square Root of 2; Pythagoras and his school; 500 B.C.
2. Fundamental Theorem of Algebra; Karl Frederich Gauss; 1799
3. The Denumerability of the Rational Numbers; Georg Cantor; 1867
4. Pythagorean Theorem; Pythagoras and his school; 500 B.C.
5. Prime Number Theorem; Jacques Hadamard and Charles-Jean de la Vallee Poussin(separately); 1896
6. Godels Incompleteness Theorem; Kurt Godel; 1931
7. Law of Quadratic Reciprocity; Karl Frederich Gauss; 1801

8. The Impossibility of Trisecting the Angle and Doubling the Cube; Pierre Wantzel; 1837

9. The Area of a Circle; Archimedes; 225 B.C.

10. Eulers Generalization of Fermats Little Theorem; Leonhard Euler; 1760

11. (Fermats Little Theorem); (Pierre de Fermat); (1640)

Research aptitude: An illustration

Patterns in the Fibonacci Numbers

The Final Digits

Here are some patterns people have already noticed in the final digits of the Fibonacci numbers:

- Look at the final digit in each Fibonacci number - the units digit:
  0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...

Is there a pattern in the final digits?
0, 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, ...
Yes!
It takes a while before it is noticeable. In fact, the series is just 60 numbers long and then it repeats the same sequence again and again all the way through the Fibonacci series - for ever. We say the series of final digits repeats with a cycle length of 60.

- Suppose we look at the final two digits in the Fibonacci numbers. Do they have a pattern?
  0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...

Yes, there is a pattern here too. After Fib(300) the last two digits repeat the same sequence again and again. The cycle length is 300 this time.
So what about the last three digits?
and the last four digits?
and so on??

- For the last three digits, the cycle length is 1,500
- for the last four digits, the cycle length is 15,000 and
- for the last five digits the cycle length is 150,000
- and so on...

Many early writers felt (incorrectly) that if p was prime, then so was $M_p = 2^p - 1$. These numbers, now called the Mersenne Numbers, were the focus of most of the early searches for large primes. The early history of these numbers is strewn with many false claims of primality, even by such notables as Mersenne, Leibniz, and Euler. So we give credit to our first record holder with some doubt:

- 1588 Pietro Cataldi had correctly verified that $2^{17} - 1 = 131071$ and $2^{19} - 1 = 524287$ are both prime.
- 1772 Euler had used clever reasoning and trial division to show $2^{31} - 1 = 2147483647$ is prime.
- In 1876 Lucas proved that $2^{127} - 1 = 170141183460469231731687303715884105727$ was prime.
- In 1951 Ferrier found the prime $(2^{148} + 1)/17 = 20988936657440586486151264256610222593863921$.

Predictions

When will we have a one billion digit prime? Let's look more closely at the recent data: Using the regression line (on the graph) we might guess that someone will discover:

- a 100,000,000 digit prime by late 2015, and
- a 1,000,000,000 digit prime by 2024.

However, extrapolating with a regression line is of questionable value and the recent records seem to be coming at a far slower rate. On the other hand, eventually there could be major technological or social changes.