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Some Transformation Formulae of Ordinary Bilateral Hypergeometric Series

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Abstract: In the present paper, using the bilateral Bailey transform due to Andrews and Warnaar and certain known summation formulae, we have established some transformation formulae of ordinary bilateral hypergeometric series which are believed to be new. We have also discussed few special cases.
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1. Introduction

Bailey [1] established a simple but a very useful transform called as Bailey transform: if

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$$

and

$$\gamma_n = \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{r+n},$$

then

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n$$

subject to conditions on the four sequences $\alpha_n, \beta_n, \gamma_n$ and δ_n which make all the relevant infinite series absolutely convergent.

The celebrated Bailey [1] transform was extensively used to obtain transformation formulae of ordinary hypergeometric series and basic hypergeometric series with help of known summation formulae. The technique provided by Bailey [1] and Slater [3, 4] was extensively exploited by number of mathematicians notably Andrews [6, 7], Verma and Jain [8, 9],Verma[10], U.B.Singh [11], Agarwal [12], S.P.Singh [13], Denis[14], Denis et. al. [15, 16, 17], Srivastav and Rudravarapu[18] and S.Singh [19] to establish number of transformation formulae of ordinary and basic hypergeometric series and also Rogers-Ramanujan type identities of different moduli. Recently, Denis et.al. [17] have established transformation formulae involving bilateral poly-basic hypergeometric functions with the help of bilateral Bailey transform introduced by Andrews and Warnaar [2]. We have applied the idea of Denis et.al. [17] to obtain transformation formulae between ordinary bilateral hypergeometric series and ordinary hypergeometric series.

2. Definitions and Notations:

For 'a' real or complex and 'n' be a positive integer, we define

$$(a)_k = \begin{cases} 1 & \text{if } k = 0; \\ a(a+1)(a+2)(a+3)\dots(a+k-1) & \text{if } k \geq 1. \end{cases}$$

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$$

and

$$(a)_{n-r} = \frac{(-1)^r (a)_n}{(1-a-n)_r}, \quad (a)_{2n} = \left(\frac{1}{2}a\right)_n \left(\frac{1}{2}a + \frac{1}{2}\right)_n 2^{2n}.$$

Now, we define a generalized hypergeometric function,

$$_AF_B[(a);(b);z] = \sum_{n=0}^{\infty} \frac{((a)_A)_n z^n}{((b)_B)_n} \tag{2.1}$$

where there are always A of a parameters and B of the b parameters. The meaning of (a) and (b) are sequences of parameters $a_1, a_2, a_3, ...a_A$ and $b_1, b_2, b_3, ..., b_B$ respectively.

The series (2.1) is convergent if

(I) $Rl\left(\sum_{v=1}^{B} b_v - \sum_{v=1}^{A} a_v\right) > 0$ when $z = 1$

(II) $Rl\left(\sum_{v=1}^{B} b_v - \sum_{v=1}^{A} a_v\right) > -1$ when $z = -1$

(III) $A = B + 1$ when $|z| < 1$

(IV) $A > B + 1$ when $z = 0$

We also define an ordinary bilateral hypergeometric function by

$$ {}_AH_B \left[\begin{array}{l} a_1, a_2, a_3, ..., a_A; z \\ b_1, b_2, b_3, ..., b_B \end{array} \right] = \sum_{n=-\infty}^{\infty} \frac{(a_1)_n (a_2)_n (a_3)_n ... (a_A)_n}{(b_1)_n (b_2)_n (b_3)_n ... (b_B)_n} z^n \tag{2.2} $$

which holds for all real or complex values of the parameters except zero or integers and for all values of argument z such that $|z| = 1$.

Further, the series (2.2) is convergent if

(I) $Rl \left(\sum_{v=1}^{B} b_v - \sum_{v=1}^{A} a_v \right) > 1$ when $z = -1$

(II) $Rl \left(\sum_{v=1}^{B} b_v - \sum_{v=1}^{A} a_v \right) > 0$ when $z = 1$

The series terminates above if any one of a parameters is negative integer and terminates below if any one of b parameters is positive integer.

In 2007, Andrews and Warnaar [2] extended the Bailey transform to bilateral Bailey transform and introduced two bilateral versions of Bailey's transform for proving certain identities related to false theta functions due to Ramanujan.

(a) Symmetric Bilateral Bailey's Transform:

If

$$ \beta_n = \sum_{r=-n}^{n} \alpha_r u_{n-r} v_{n+r} \tag{2.3} $$

and

$$ \gamma_n = \sum_{r=|n|}^{\infty} \delta_r u_{r-n} v_{r+n} \tag{2.4} $$

then

$$ \sum_{n=-\infty}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n \tag{2.5} $$

subject to conditions on the four sequences $\alpha_n, \beta_n, \gamma_n$ and δ_n which make all the relevant infinite series uniformly and absolutely convergent.

(b) Asymmetric Bilateral Bailey's Transform:

Let $m = max.(n, -n-1)$, then,

If

$$ \beta_n = \sum_{r=-n-1}^{n} \alpha_r u_{n-r} v_{n+r+1} \tag{2.6} $$

and

$$\gamma_n = \sum_{r=m}^{\infty} \delta_r u_{r-n} v_{r+n+1} \tag{2.7}$$

then

$$\sum_{n=-\infty}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n \tag{2.8}$$

subject to conditions on the four sequences $\alpha_n, \beta_n, \gamma_n$ and δ_n which make all the relevant infinite series uniformly and absolutely convergent.
We shall also make use of following summation formulae

$${}_2H_2 = \left[\begin{array}{l} a, b; 1 \\ c, d \end{array} \right] = \Gamma \left[\begin{array}{l} c, d, 1-a, 1-b, c+d-a-b-1 \\ c-a, d-a, c-b, d-b \end{array} \right] \tag{2.9}$$

[Slater 5; p.181]

On putting a=-n and c=n+1 in (2.9), we have

$${}_2H_2 = \left[\begin{array}{c} -n, b; 1 \\ 1+n, d \end{array} \right] = \frac{(1)_n (1)_n (d-b)_{2n}}{(d)_n (1-b)_n (1)_{2n}} \tag{2.10}$$

Also,

$$\begin{aligned} &{}_3H_3 \left[\begin{array}{l} b, c, d; 1 \\ 1+a-b, 1+a-c, 1+a-d \end{array} \right] \\ &= \Gamma \left[\begin{array}{l} 1-b, 1-c, 1-d, 1+a-b, 1+a-c, 1+a-d, \\ \\ 1+a-c-d, 1+a-b-d, 1+a-b-c, 1+\frac{1}{2}a-b, 1+\frac{1}{2}a-c, \end{array} \right. \\ &\qquad \left. \begin{array}{l} 1-\frac{1}{2}a, 1+\frac{1}{2}a, 1+\frac{3}{2}a-b-c-d \\ \\ 1+\frac{1}{2}a-d, 1+a, 1-a \end{array} \right] \end{aligned} \tag{2.11}$$

[Slater 5; p.182]

On putting a=0, and b=-n in (2.11), we have

$${}_3H_3 \left[\begin{array}{l} -n, c, d; 1 \\ 1-b, 1-c, 1-d \end{array} \right] = \frac{(1)_n (1-c-d)_n}{(1-d)_n (1-c)_n} \tag{2.12}$$

$$
{}_7F_6\left[\begin{array}{l} a, 1+\frac{1}{2}a, b, c, d, 1+2a-b-c-d+n, -n; 1 \\ \frac{1}{2}a, 1+a-b, 1+a-c, 1+a-d, b+c+d-a-n, 1+a-n \end{array}\right]
$$

$$
= \frac{(1+a)_n(1+a-b-c)_n(1+a-b-d)_n(1+a-c-d)_n}{(1+a-b)_n(1+a-c)_n(1+a-d)_n(1+a-b-c-d)_n} \qquad (2.13)
$$

[Slater 5; App. III 14, p. 244]

As $a \to 0$, we get

$$
{}_5H_5\left[\begin{array}{l} b, c, d, 1-b-c-d+n, -n; 1 \\ 1-b, 1-c, 1-d, b+c+d-n, 1+n \end{array}\right]
$$

$$
= \frac{(1)_n(1-b-c)_n(1-b-d)_n(1-c-d)_n}{(1-b)_n(1-c)_n(1-d)_n(1-b-c-d)_n} \qquad (2.14)
$$

On putting $a = -n, c = n+2$ in (2.9), we have

$$
{}_2H_2\left[\begin{array}{l} -n, b; 1 \\ n+2, d \end{array}\right] = \frac{(d-b)}{(2-b)}\frac{(1)_n(2)_n(1+d-b)_{2n}}{(d)_n(2-b)_n(2)_{2n}} \qquad (2.15)
$$

Gauss's theorem

$$
{}_2F_1\left[\begin{array}{l} a, b; 1 \\ c \end{array}\right] = \Gamma\left[\begin{array}{l} c, c-a-b \\ c-a, c-b \end{array}\right] \qquad (2.16)
$$

[Slater 5; App. III 3, p. 243]

Watson's theorem

$$
{}_3F_2\left[\begin{array}{l} a, b, c; 1 \\ \frac{1}{2}+\frac{1}{2}a+\frac{1}{2}b, 2c \end{array}\right] = \Gamma\left[\begin{array}{l} \frac{1}{2}, c+\frac{1}{2}, \frac{1}{2}+\frac{1}{2}a+\frac{1}{2}b, \frac{1}{2}-\frac{1}{2}a-\frac{1}{2}b+c \\ \frac{1}{2}+\frac{1}{2}a, \frac{1}{2}+\frac{1}{2}b, \frac{1}{2}-\frac{1}{2}a+c, \frac{1}{2}-\frac{1}{2}b+c \end{array}\right] \qquad (2.17)
$$

[Slater 5; App. III 23, p. 245]

3. Main Results:

Our main results are as under -

$$
{}_3H_3\left[\begin{array}{l} b, p, q; -1 \\ d, 1-p, 1-q \end{array}\right] = \frac{\Gamma(1-p)\Gamma(1-q)}{\Gamma(1-p-q)}\; {}_4F_3\left[\begin{array}{l} p, q, \frac{d-b}{2}, \frac{d-b+1}{2}; 1 \\ \frac{1}{2}, d, 1-b \end{array}\right] \qquad (3.1)
$$

$$\sum_{n=-\infty}^{\infty} \frac{(b)_n(2p)_n(2q)_n\left(\frac{1}{2}+p+q\right)_n(-4)^{-n}}{(d)_n\Gamma\left(\frac{1}{2}+\frac{n}{2}+p\right)\Gamma\left(\frac{1}{2}+\frac{n}{2}+q\right)\Gamma\left(1-p+\frac{n}{2}\right)\Gamma\left(1-q+\frac{n}{2}\right)}$$

$$= \frac{1}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}+p+q\right)\Gamma(1-p-q)}\ {}_4F_3\left[\begin{matrix} 2p, 2q, \frac{d-b}{2}, \frac{d-b+1}{2}; 1 \\ d, 1-b, \frac{1}{2}+p+q \end{matrix}\right] \tag{3.2}$$

$${}_4H_4\left[\begin{matrix} c, d, p, q; -1 \\ 1-c, 1-d, 1-p, 1-q \end{matrix}\right] = \frac{\Gamma(1-p)\Gamma(1-q)}{\Gamma(1-p-q)}\ {}_3F_2\left[\begin{matrix} p, q, 1-c-d; 1 \\ 1-c, 1-d \end{matrix}\right] \tag{3.3}$$

$$\sum_{n=-\infty}^{\infty} \frac{(p)_n(q)_n(c)_n(d)_n\left(-\frac{1}{4}\right)^n}{(1-c)_n(1-d)_n\Gamma\left(\frac{1}{2}+\frac{p}{2}+\frac{n}{2}\right)\Gamma\left(\frac{1}{2}+\frac{q}{2}+\frac{n}{2}\right)\Gamma\left(1-\frac{p}{2}+\frac{n}{2}\right)\Gamma\left(1-\frac{q}{2}+\frac{n}{2}\right)}$$

$$= \frac{1}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-\frac{p}{2}-\frac{q}{2}\right)\Gamma\left(\frac{1}{2}+\frac{p}{2}+\frac{q}{2}\right)}\ {}_4F_3\left[\begin{matrix} \frac{1}{2}, p, q, 1-c-d; 1 \\ 1-c, 1-d, \frac{1}{2}+\frac{p}{2}+\frac{q}{2} \end{matrix}\right] \tag{3.4}$$

$${}_5H_5\left[\begin{matrix} b, c, d, \frac{1-b-c-d}{2}, 1-\frac{(b+c+d)}{2}; 1 \\ 1-b, 1-c, 1-d, \frac{(b+c+d)}{2}, \frac{(1+b+c+d)}{2} \end{matrix}\right]$$

$$= \frac{\Gamma(b+c+d)\Gamma(b+c+d)}{\Gamma(2b+2c+2d-1)}\ {}_4F_3\left[\begin{matrix} 1-b-c, 1-b-d, 1-c-d, 1-b-c-d; 1 \\ 1-b, 1-c, 1-d \end{matrix}\right] \tag{3.5}$$

$${}_4H_4\left[\begin{matrix} b, c, d, \frac{1-b-c-d}{2}; 1 \\ 1-b, 1-c, 1-d, \frac{(1+b+c+d)}{2} \end{matrix}\right]$$

$$= \frac{\left(\Gamma\left(1-\frac{b}{2}-\frac{c}{2}-\frac{d}{2}\right)\right)^2\left(\Gamma\left(\frac{1+b+c+d}{2}\right)\right)^2}{\Gamma\left(\frac{1}{2}\right)\Gamma(b+c+d)\Gamma\left(\frac{3}{2}-b-c-d\right)}\times$$

$$\times\ {}_5F_4\left[\begin{matrix}\frac{1}{2}, 1-b-c, 1-b-d, 1-c-d, 1-b-c-d; 1\\ 1-b, 1-c, 1-d, \frac{3}{2}-b-c-d\end{matrix}\right] \tag{3.6}$$

$${}_3H_3\left[\begin{matrix}b, p, q; -1\\ d, 2-p, 2-q\end{matrix}\right]$$

$$= \frac{(d-b)\Gamma(2-p)\Gamma(2-q)}{(2-b)\Gamma(2-p-q)}\ {}_4F_3\left[\begin{matrix}p, q, \frac{1+d-b}{2}, 1+\frac{d-b}{2}; 1\\ \frac{3}{2}, 2-b, d\end{matrix}\right] \tag{3.7}$$

provided $Re(4+d-b-2p-2q) > 1$

4. Proof:

(i) Proof of (3.1):

Setting $\alpha_r = \frac{(b)_r(-1)^r}{(d)_r}, u_r = \frac{1}{(1)_r}, v_r = \frac{1}{(1)_r}$ in (2.3) and making use of (2.10), we have

$$\beta_n = \frac{\left(\frac{d-b}{2}\right)_n \left(\frac{d-b+1}{2}\right)_n}{(d)_n(1-b)_n\left(\frac{1}{2}\right)_n(1)_n} \tag{4.1}$$

Again, taking $\delta_r = (p)_r(q)_r$ in (2.4) and using (2.16), we get

$$\gamma_n = \frac{\Gamma(1-p-q)}{\Gamma(1-p)\Gamma(1-q)}\frac{(p)_n(q)_n}{(1-p)_n(1-q)_n} \tag{4.2}$$

putting these values of (4.1) and (4.2) in (2.5), we get required result (3.1).

(ii) Proof of (3.2):

Taking $\alpha_r = \frac{(b)_r(-1)^r}{(d)_r}, u_r = \frac{1}{(1)_r}, v_r = \frac{1}{(1)_r}$ $\delta_r = \frac{(2p)_r(2q)_r\left(\frac{1}{2}\right)_r}{\left(\frac{1}{2}+p+q\right)}$ in (2.4) and making use of (2.17), we have

$$\gamma_n = \Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}+p+q\right)\Gamma(1-p-q)\times$$

$$\times\left(\frac{(2p)_n(2q)_n\left(\frac{1}{2}+p+q\right)_n 4^{-n}}{\Gamma\left(\frac{1}{2}+\frac{n}{2}+p\right)\Gamma\left(\frac{1}{2}+\frac{n}{2}+q\right)\Gamma\left(1+p-\frac{n}{2}\right)\Gamma\left(1+q-\frac{n}{2}\right)}\right) \tag{4.3}$$

putting these values of (4.1) and (4.3) in (2.5), we get required result (3.2).

(iii) Proof of (3.3):

Taking $\alpha_r = \dfrac{(c)_r(d)_r(-1)^r}{(1-c)_r(1-d)_r}, u_r = \dfrac{1}{(1)_r}, v_r = \dfrac{1}{(1)_r}$ in (2.3) and making use of (2.12), we have

$$\beta_n = \frac{(1-c-d)_n}{(1-c)_n(1-d)_n(1)_n} \tag{4.4}$$

Again, taking $\delta_r = (p)_r(q)_r$, we get γ_n same as (4.2)
Now, putting these values of (4.2) and (4.4) in (2.5), we get required result (3.3).

(iv) Proof of (3.4):

Taking $\alpha_r = \dfrac{(c)_r(d)_r(-1)^r}{(1-c)_r(1-d)_r}, u_r = \dfrac{1}{(1)_r}, v_r = \dfrac{1}{(1)_r}$ in (2.3) and making use of (2.12), we get β_n is same as (4.4)

Again, taking $\delta_r = \dfrac{(p)_r(q)_r\left(\frac{1}{2}\right)_r}{\left(\frac{1}{2}+\frac{1}{2}p+\frac{1}{2}q\right)_r}$ in (2.4) and using (2.17), we have

$$\gamma_n = \Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-\frac{p}{2}-\frac{q}{2}\right)\Gamma(1+\frac{p}{2}+\frac{q}{2})\times$$

$$\times\left(\frac{(p)_n(q)_n 4^{-n}}{\Gamma\left(\frac{1}{2}+\frac{n}{2}+\frac{p}{2}\right)\Gamma\left(\frac{1}{2}+\frac{n}{2}+\frac{q}{2}\right)\Gamma\left(1+\frac{n}{2}-\frac{p}{2}\right)\Gamma\left(1+\frac{n}{2}-\frac{q}{2}\right)}\right) \tag{4.5}$$

putting these values of (4.4) and (4.5) in (2.5), we get required result (3.4).

(v) Proof of (3.5):

Taking $\alpha_r = \dfrac{(b)_r(c)_r(d)_r}{(1-b)_r(1-c)_r(1-d)_r}, u_r = \dfrac{(1-b-c-d)_r}{(1)_r},\ v_r = \dfrac{(1-b-c-d)_r}{(1)_r}$ in (2.3) and making use of (2.14), we have

$$\beta_n = \frac{(1-b-c)_n(1-b-d)_n(1-c-d)_n(1-b-c-d)_n}{(1-b)_n(1-c)_n(1-d)_n(1)_n} \tag{4.6}$$

Again, taking $\delta_r = 1$ in (2.4) and using (2.16), we get

$$\gamma_n = \frac{\Gamma(2b+2c+2d-1)}{\Gamma(b+c+d)\Gamma(b+c+d)}\frac{\left(\frac{1-b-c-d}{2}\right)_n\left(1-\frac{(b+c+d)}{2}\right)_n}{\left(\frac{b+c+d}{2}\right)_n\left(\frac{1+b+c+d}{2}\right)_n} \tag{4.7}$$

provided $Re(b+c+d) > \frac{1}{2}$.
putting these values of (4.6) and (4.7) in (2.5), we get required result (3.5).

(vi) Proof of (3.6):

Taking $\alpha_r = \dfrac{(b)_r(c)_r(d)_r}{(1-b)_r(1-c)_r(1-d)_r}, u_r = \dfrac{(1-b-c-d)_r}{(1)_r}, \; v_r = \dfrac{(1-b-c-d)_r}{(1)_r}$ and $\delta_r = \dfrac{\left(\frac{1}{2}\right)_r}{\left(\frac{3}{2}-b-c-d\right)_r}$ in (2.4) and making use of (2.17), we have

Here, β_n is same as (4.6) and

$$\gamma_n = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma(b+c+d)}{\Gamma\left(1-\frac{b}{2}-\frac{c}{2}-\frac{d}{2}\right)\Gamma\left(1-\frac{b}{2}-\frac{c}{2}-\frac{d}{2}\right)} \times$$

$$\times \frac{\Gamma\left(\frac{3}{2}-b-c-d\right)\left(\frac{1-b-c-d}{2}\right)_n}{\Gamma\left(\frac{1+b+c+d}{2}\right)\Gamma\left(\frac{1+b+c+d}{2}\right)\left(\frac{1+b+c+d}{2}\right)_n} \tag{4.8}$$

putting these values of (4.6) and (4.8) in (2.5), we get required result (3.6).

(vii) Proof of (3.7):

Setting $\alpha_r = \dfrac{(b)_r(-1)^r}{(d)_r}, u_r = \dfrac{1}{(1)_r}, v_r = \dfrac{1}{(1)_r}$ in (2.6) and making use of (2.15), we get

$$\beta_n = \frac{(d-b)}{(2-b)} \frac{\left(\frac{1+d-b}{2}\right)_n \left(1+\frac{d-b}{2}\right)_n}{\left(\frac{3}{2}\right)_n (2-b)_n(d)_n(1)_n} \tag{4.9}$$

Again, taking $\delta_r = (p)_r(q)_r$ in (2.7), we get

$$\gamma_n = \frac{\Gamma(2-p-q)}{\Gamma(2-p)\Gamma(2-q)} \frac{(p)_n(q)_n}{(2-p)_n(2-q)_n} \tag{4.10}$$

putting these values of (4.9) and (4.10) in (2.8), we get required result (3.7).

5. Special Cases:

(i) Replacing p by d and $q = \frac{1}{2}$ in (3.1), we get

$$ {}_1H_1\left[\begin{matrix} b; -1 \\ 1-d \end{matrix}\right] = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma(1-b)\Gamma(1-d)}{\Gamma\left(1-\frac{1}{2}b-\frac{1}{2}d\right)\Gamma\left(\frac{1}{2}-\frac{1}{2}b-\frac{1}{2}d\right)} $$

After some simplification, we get

$$ {}_1H_1\left[\begin{matrix} b; -1 \\ 1-d \end{matrix}\right] = \frac{\Gamma(1-b)\Gamma(1-d)}{2^{b+d}\Gamma(1-b-d)} \tag{5.1} $$

(ii) Replacing b=-2p and d=2q in (3.2), we get

$$ \sum_{n=-\infty}^{\infty} \frac{(-2p)_n(2p)_n\left(\frac{1}{2}+p+q\right)_n(-4)^{-n}}{\Gamma\left(\frac{1}{2}+\frac{n}{2}+p\right)\Gamma\left(\frac{1}{2}+\frac{n}{2}+q\right)\Gamma\left(1-p+\frac{n}{2}\right)\Gamma\left(1-q+\frac{n}{2}\right)} $$

$$ = \frac{\Gamma(1+2p)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}+p+q\right)\Gamma(1+p-q)} \tag{5.2} $$

Several other special cases could also be deduced.

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