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CYCLIC DISTANCE IN GRAPHS

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Abstract: In this paper the concept of cyclic distance is introduced. For $u, v \in V(G)$ of a connected graph G, the cyclic distance between u and v is defined as the minimum number of cycles to be traversed from a cycle containing u to a cycle containing v. Using this notion, cyclic radius and cyclic diameter of a graph are defined. Cyclic distance matrix of a graph is also introduced and some of its properties are studied.

Keywords and Phrases: Cycle neighbor set, maximal cyclic component, cyclic radius, cyclic diameter, shrinked graph, cyclic distance matrix.

2020 Mathematics Subject Classification: 05C12.

1. Introduction

All graphs $G = (V(G), E(G))$ discussed in this paper are simple, finite, connected and undirected. For notation and terminology we refer to [2, 3]. Different types of distance concepts like detour distance [4], superior distance [7], etc., can be found in the literature of graph theory. Recently M. P Jalsiya and Raji Pilakkat [6] introduced the concept of transitively tracked graphs. This motivated the authors to define a new distance concept called cyclic distance in graphs.

Generally, graphs representing many real life situations are very complicated, large and contains plenty of cycles, circuits etc,. Some popular examples are social networking systems and electric circuits. The advantage of defining cyclic distance is that such graphs can be studied in a smaller frame using this notion. Cyclic distance reduces the distance between two vertices in a graph. This concept enables us to treat two distinct vertices as a single unit when they belong to a subgraph which is at least two connected.

2. Cyclic Distance in Graphs

In this section cycle neighbor sets and maximal cyclic components are defined and using them the concept of cyclic distance between two vertices of a graph is introduced.

Definition 2.1. *Let $G(V, E)$ be any graph. A subset C of $V(G)$ is called a cycle neighbor set of G, if for any two vertices u and v in C, there is a cycle in G, which contains both u and v. So that there is at least two distinct paths connecting any two vertices of a cycle neighbor set in G.*

Definition 2.2. *A cycle neighbor set C of a graph $G(V, E)$ is said to be a maximal cycle neighbor set, if for all vertices $u \in V \setminus C$, $C \cup \{u\}$ is not a cycle neighbor set.*

Definition 2.3. *For a graph G, maximal cyclic components are the subgraphs induced by maximal cycle neighbor sets of G.*

We use the abbreviation MCCs to denote the maximal cyclic components of a graph G.

Proposition 2.4 is a direct consequence of the definition of MCCs of G.

Proposition 2.4. *Let G be any graph. Then any two MCCs of G can have at most one vertex in common.*

Definition 2.5. *Two MCCs of a graph G are said to be neighbors if they have either a vertex in common or they are connected by a bridge between them. Two MCCs of G are disjoint if there is no vertex common to them and two MCCs are distinct if they have at most one common vertex.*

Definition 2.6. *For $u, v \in V(G)$ of a graph G, the cyclic distance between u and v is defined as the minimum number of MCCs to be traversed from the MCC containing u to the MCC containing v other than the one containing u. It can be written as $cd_G(u, v)$ or $cd(u, v)$.*

For example in the graph in figure 1, $cd(u, v) = 3$ while $d(u, v) = 6$.

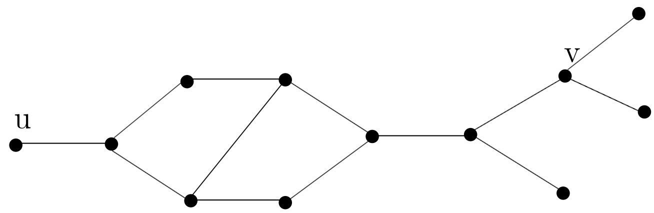

Figure 1

Definition 2.7. *For $u, v \in V(G)$ of a graph G, a cyclic path from u to v is a finite sequence of distinct MCCs containing the vertex u to the one containing v.*

Proposition 2.8. *Let G be any graph. Then for any two vertices $u, v \in V(G)$, there is a unique cyclic path joining u and v.*

Proof. Assume that $u, v \in V(G)$ such that $cd(u, v) \geq 1$. Suppose if possible, the cyclic path between them is not unique. Then there are more than one cyclic paths between u and v in G. Let there be two cyclic paths between u and v such that whose internal maximal cyclic components are disjoint. Combining these two cyclic paths, we get a cyclic component containing both u and v so that $cd(u, v) = 0$, a contradiction to $cd(u, v) \geq 1$.

Definition 2.9. *For a graph G, $u, v \in V(G)$ are said to be cyclic similar vertices if the cyclic distance between u and v that is, $cd(u, v) = 0$ and a graph G is called a cyclic similar graph, if $cd(u, v) = 0$ for every pair of vertices in G.*

Theorem 2.10. *Let G be any connected graph. Then the following statements are equivalent.*

1. *G is a cyclic similar graph.*
2. *G is at least two-connected.*
3. *$V(G)$ is a cycle neighbor set.*

Proof. Suppose that G is a cyclic similar graph. Then $cd(u, v) = 0$ for all vertices $u, v \in V(G)$. That is for every pair u, v of vertices in G, there is a cycle containing these vertices. Hence there are at least two internally disjoint paths joining every pair of vertices in G. Therefore, G is at least two-connected.

Now let us prove that if (2) does not hold then (3) cannot hold. Suppose that, G is at most one connected. Then since G is connected, there is at least one cut vertex say w in G and there are vertices u and v in G such that $u...w...v$ is the only path connecting u and v and therefore both u and v together cannot belong to any cycle of G. Hence $V(G)$ is not a cycle neighbor set.

If $V(G)$ is a cycle neighbor set, then by definition, every vertex of G belongs to a cycle of G and therefore G is a cyclic similar graph. Hence (1).

The concept of cyclic distance helps to develop a topological structure on connected graphs. Thus, we have Theorem 2.11.

Theorem 2.11. *Let $G(V, E)$ be any graph. Then on the set of vertices of G a pseudo metric is induced by cyclic distance.*

Proof. Let $u, v \in V(G)$. From the definition of cyclic distance, it is clear that $cd(u, v) \geq 0$ for all $u, v \in V(G)$ and $cd(u, v) = cd(v, u)$. Now let u, v, w be any three vertices in G.

Claim: $cd(u, w) \leq cd(u, v) + cd(v, w)$

If G is acyclic or if all of u, v and w belong to the same maximal cyclic component of G, then the inequality holds trivially. Now consider the following cases.

Case (i). Only two among u, v and w are in the same maximal cyclic component. Then the cyclic distance between two pairs will be the same and the cyclic distance between the other pair is zero. So the claim holds.

Case (ii). All the vertices u, v and w are in distinct maximal cyclic components of G. By Proposition 2.8, the cyclic path between u and w is unique. Therefore, $cd(u, w) = cd(u, v) + cd(v, w)$ or $cd(u, w) < cd(u, v) + cd(v, w)$ according as v is in between the cyclic path connecting u and w or not. Hence the proof.

3. Cyclic radius and cyclic diameter

In this section, cyclic radius, cyclic diameter, cyclic center, cyclic periphery etc., of a graph with respect to cyclic distance are defined analogue to radius, diameter, center and periphery of a graph with respect to the classical distance between vertices.

Definition 3.1. *For a graph G, the cyclic eccentricity of a vertex v is denoted by $ce_G(v)$ or simply $ce(v)$ and is defined as $ce(v) = max_{u \in V(G)} cd(u, v)$. Let $u, v \in V(G)$ then v is called a cyclic eccentric vertex of u if $ce(u) = cd(u, v)$.*

Definition 3.2. *For any graph G, cyclic diameter (denoted by $cdiam(G)$) and cyclic radius (denoted by $crad(G)$) are respectively defined as the largest and smallest cyclic eccentricities of the vertices of the graph G. That is, $cdiam(G) = max_{v \in V(G)} ce(v)$ and $crad(G) = min_{v \in V(G)} ce(v)$.*

Definition 3.3. *Cyclic center (denoted by $CC(G)$) and cyclic periphery (denoted by $CP(G)$) of a graph G are defined as the set of all vertices for which cyclic eccentricity is equal to the cyclic radius and those vertices for which cyclic eccentricity is equal to the cyclic diameter respectively.*

Definition 3.4. *Let G be any graph. Then G is said to be cyclic self centered if*

$cdiam(G) = crad(G)$.

Remark 3.5. *For any graph G, $cdiam(G) = 0$ if and only if G is cyclic similar.*

Definition 3.6. *A graph G is called a cyclic flower if there are at least two blocks in G such that each block is a cyclic similar graph of order greater than or equal to three and all these blocks has exactly one common vertex. Hence a cyclic flower has at least two MCCs and there is a unique common vertex for all MCCs. The common vertex of a cyclic flower graph is called the flower centric vertex. If a path on k vertices is attached to the flower centric vertex of a cyclic flower (or to any one vertex of a cyclic similar graph) by a bridge, then it is called cyclic flower with k-stem (or a cyclic similar graph with k-stem).*

Theorem 3.7. *Let G be a graph. Then $cdiam(G) = 1$ if and only if*

1. *G has two MCCs connected by a bridge or*
2. *G is a cyclic flower or*
3. *G is either a graph containing a cyclic flower and a cyclic similar graph connected by a bridge between the flower centric vertex of the cyclic flower and any vertex of the cyclic similar graph or G contains two cyclic flowers connected by a bridge through their flower centric vertices.*

Proof. If G is any one of the graphs as in the statement of the theorem, then clearly $cdiam(G) = 1$.

Now let $cdiam(G) = 1$. If there is only one MCC, then G is cyclic similar and hence $cdiam(G) = 0$. Therefore G has at least two MCCs say G_1 and G_2. Therefore either G contains two MCCs connected by a bridge or G contains a cyclic flower with two MCCs as a subgraph.

Consider the first case that G contains two MCCs G_1 and G_2 connected by a bridge. If there are exactly two MCCs then (1) holds. When there are more MCCs in G other than G_1 and G_2, then since G is connected, either these extra MCCs will have a vertex in common with $G_1 \cup G_2$ or they will be connected to $G_1 \cup G_2$ by bridges. But here if the vertex in $G_1 \cup G_2$ which is shared by these MCCs is different from the connecting vertices of G_1 and G_2, then $cdiam(G) > 1$. Therefore every additional MCCs in G which has a vertex in common with $G_1 \cup G_2$ will be sharing either the connecting vertex of G_1 to G_2 or that of G_2 to G_1. If so, (3) holds. No MCCs can be connected to $G_1 \cup G_2$ by bridges, since then $cdiam(G)$ will be increased at least by one.

Now consider the second case that G contains a cyclic flower with two MCCs say G_1 and G_2. If there are more MCCs which have a vertex in common with $G_1 \cup G_2$, then the common vertex is the same as that of G_1 and G_2, otherwise cyclic diameter will be increased. On the other hand if these MCCs are connected by bridges to $G_1 \cup G_2$, then the connecting edge should be unique, otherwise $cdiam(G) > 1$. Therefore (2) or (3) holds. Hence the proof.

Let $u, v \in A$, where $A \subseteq V(G)$ is a maximal cycle neighbor set of a connected graph G. Then it is clear that $|cd(u,x) - cd(v,x)| \leq 1$ for any vertex $x \in V(G) \setminus A$. Also when the subgraphs induced by $A, B \subseteq V(G)$ are two neighboring MCCs of G, then $|cd(u,x) - cd(v,x)| \leq 1$ for all $u \in A$, $v \in B$ and $x \in V(G) \setminus A \cup B$. Hence it follows that;

Lemma 3.8. *Let the subgraphs induced by $A, B \subseteq V(G)$ be any two neighboring MCCs of a graph G. Then $|ce(u) - ce(v)| \leq 1$ for all $u, v \in A$ for every maximal cycle neighbor set A of G and $|ce(u) - ce(v)| \leq 1$ for all $u \in A$ and $v \in B$.*

Theorem 3.9. *Let G be a graph. If there is a positive integer k such that $crad(G) < k < cdiam(G)$ then there exists a vertex $v \in V(G)$ with $ce(v) = k$.*

Proof. Let $u, v \in V(G)$ such that $ce(u) = crad(G)$ and $ce(v) = cdiam(G)$. Consider the cyclic path connecting u and v. Let S and W be the set of all vertices in that cyclic path with $ce(s) < k$ for every $s \in S$ and $ce(w) \geq k$ for every $w \in W$. From the definition of cyclic paths it is clear that the vertices of S and W are connected through a vertex y in W which is common to two neighboring MCCs or by a bridge with one end in S and the other end say y in W. In both cases $ce(y) \geq k$. Then by the Lemma 3.8, $|ce(w) - ce(y)| \leq 1$. Therefore we have, $ce(y) = k$. Hence the proof.

Theorem 3.10. *Let G be any graph. Then G is cyclic self centered if and only if*

1. *G is cyclic similar or*

2. *G is a graph with $cdiam(G) = 1$ which is not a cyclic flower.*

Proof. G is cyclic similar if and only if $cdiam(G) = 0$. Therefore, cyclic similar graphs are cyclic self centered.

Now Suppose that $cdiam(G) = 1$. Then G is any of the graphs as in the statement of Theorem 3.7 Among them when G is a cyclic flower, $crad(G) = 0$ and $cdiam(G) = 1$ and in all other cases, G is cyclic self centered.

Now let $crad(G) = cdiam(G) = k$ where $k \geq 2$. Then $ce(v) = k$ for all $v \in V(G)$. Let w be a cyclic eccentric vertex of v. Then $cd(v, w) = k$. By Proposition 2.8, there is a unique cyclic path connecting v and w. In that cyclic path there are $k + 1$ maximal cyclic components say $G_0, G_1, G_2, ..., G_k$ such that $v \in V(G_0)$ and $w \in V(G_k)$. Let $u \in V(G_l)$ where $1 \leq l \leq k - 1$. From the definition of cyclic paths, it is clear that there is at least one vertex say u in $V(G_l)$ such that u does not belong to $V(G_{l-1})$ and u does not belong to $V(G_{l+1})$ and $ce(u) = k$. Correspondingly there is a cyclic eccentric vertex $y \in V(G)$ with $cd(u, y) = k$. Consider the cyclic path connecting v and y. Since the cyclic path joining any two vertices of a graph is unique, either u lies interior to the cyclic path between v and y or v lies interior to the cyclic path between u and y. In the first case, it is clear that $cd(v, y) = cd(v, u) + cd(u, y) > k$ and in the second case, $cd(y, w) = cd(y, v) + cd(v, w) > k$, a contradiction to $cdiam(G) = k$. Hence $k < 2$ whenever $crad(G) = cdiam(G) = k$. Hence the proof.

Corollary 3.11. *If G is a cyclic self centered graph then $cdiam(G) \leq 1$.*

Definition 3.12. *A collection of m cyclic flowers and a collection of n cyclic similar graphs (where $m \geq 0, n \geq 0$ and both m and n are finite) are attached to the vertices of a cyclic similar graph H through bridges then the resulting graph G is called a cyclic bouquet if $m + n \geq 2$. The cyclic similar graph H to which all these cyclic flowers and cyclic similar graphs are attached is called the central cyclic component of the cyclic bouquet.*

Theorem 3.13. *Let G be any graph which is not a tree. If order of G is $n \geq 4$. Then $|CC(G)| = 1$ and $|CP(G)| = n - 1$ if and only if*

1. *G is cyclic flower or*
2. *G is cyclic flower with 1-stem or G is cyclic similar graph with 1-stem or*
3. *G is a cyclic bouquet whose central cyclic component is K_1, holds.*

Proof. If G is any one of the graphs as in the statement of the theorem, then it is clear that $|CC(G)| = 1$ and $|CP(G)| = n - 1$.

To prove the converse, let G be a graph with $|CC(G)| = 1$ and $|CP(G)| = n-1$. Let $CC(G) = \{u\} \subseteq V(G)$. Then $ce(u) < ce(v)$ for all $v \in V(G) \setminus \{u\}$. Also since every vertex in $V(G) \setminus \{u\}$ is a cyclic peripheral vertex, $cd(u, v) \leq 1$ for all $v \in V(G) \setminus \{u\}$. Otherwise, there is some vertex $v \in V(G) \setminus \{u\}$ such that $cd(u, v) \geq 2$. Then we can find a vertex x in the cyclic path between u and v such that $cd(u, x) = 1$, which contradicts the fact that x is a cyclic peripheral vertex. Therefore, $cd(u, v) \leq 1$. So that we have,

Case (i). $cd(u, v) = 1$, for all $v \in V(G) \setminus \{u\}$. Then G is a cyclic bouquet with K_1 as central cyclic component, other than a star graph $K_{n,1}$, since G is not a tree.

Case (ii). $cd(u, v) < 1$. In this case, $cd(u, v) = 0$, for all $v \in V(G) \setminus \{u\}$. Hence either G is cyclic similar or u is a cut vertex which belongs to every maximal cyclic components of G. When G is cyclic similar, $|CC(G)| = |CP(G)| = n$. Therefore u is a cut vertex satisfying the above condition and in this case G is a cyclic flower.

Case (iii). $cd(u, v) \leq 1$. Let $cd(u, v) = 0$ for every $v \in A$ for a nonempty subset $A \subseteq V(G) \setminus \{u\}$ and $Cd(u, v) = 1$ for all $v \in B$ for a nonempty subset $B \subseteq V(G) \setminus \{u\}$ with $A \cup B = V(G) \setminus \{u\}$. Then either $\langle A \cup \{u\} \rangle$, the graph induced by $A \cup \{u\}$ is cyclic similar or $\langle A \cup \{u\} \rangle$ is a cyclic flower with u as the flower centric vertex. In both cases, $|B| = 1$, otherwise $|CC(G)| \neq 1$. Therefore G is a cyclic flower with 1-stem or a cyclic similar graph with 1-stem

Corollary 3.14. *If $|CC(G)| = 1$ and $|CP(G)| = n - 1$ for a connected graph G of order $n \geq 4$, then $1 \leq cdiam(G) \leq 2$.*

4. Cyclic distance matrix of a graph

In this section we deal with a method of condensing a graph using the concept of cyclic distance. A new graph matrix called cyclic distance matrix of a graph is also introduced.

Definition 4.1. *Let G be any graph. The shrinked graph of G, denoted by $S(G)$ is the graph obtained from G by contracting (or shrinking) each MCCs of G to a vertex and connecting two vertices of $S(G)$ by an edge if they correspond to neighboring MCCs of G.*

Shrinked graph of a graph G has the following trivial properties.

Proposition 4.2.

1. *The shrinked graph of a tree T is T itself. That is, $S(T) \cong T$.*
2. *The shrinked graph of a cyclic similar graph is K_1.*
3. *The order of $S(G)$ = the number of MCCs of G.*
4. *When G is not a tree, then $|V(S(G))| \leq |V(G)| - 2$.*

If G contains a cyclic flower with more than two MCCs then by shrinking these MCCs of the cyclic flower to vertices and connecting them by edges, we get a complete graph K_n, $n \geq 3$ in $S(G)$, so we have Theorem 4.3.

Theorem 4.3. *The shrinked graph of graph G is a tree if and only if G contains no cyclic flowers with more than two MCCs.*

Definition 4.4. *Let G be any graph. Then G is called a cyclic tree if no two MCCs have a vertex in common.*

Since a cyclic tree contains no cyclic flowers, we have the following corollary;

Corollary 4.5. *For a cyclic tree G, $S(G)$ is a tree.*

Let G be a graph containing $k \geq 2$ MCCs viz, $m_1, m_2, ..., m_k$. Let us use $M(G)$ to denote the set of all MCCs in G.

Definition 4.6. *Let G be a graph with $M(G) = \{m_1, m_2, ..., m_k\}$. Consider any two MCCs m_i, m_j, where $1 \leq i, j \leq k$ with set of vertices $\{v_1, v_2, ..., v_n\}$ and $\{u_1, u_2, ..., u_m\}$ in m_i and m_j respectively. Then the distance between the MCCs m_i and m_j is denoted by $dist(m_i, m_j)$ and is defined as*

$dist(m_i, m_j) = max_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} \{cd(v_i, u_j)\}$.

Theorem 4.7 follows directly from the definition of distance between maximal cyclic components.

Theorem 4.7. *Let G be a connected graph with $M(G) = \{m_1, m_2, ..., m_k\}$. Then $max_{1 \leq i,j \leq k} dist(m_i, m_j) = cdiam(G)$, the cyclic diameter of G.*

Definition 4.8. *The cyclic distance matrix of a graph G with $M(G) = \{m_1, m_2, ..., m_k\}$ is a $k \times k$ matrix denoted by $CD(G)$ and is defined as follows. The rows and columns of $CD(G)$ are indexed by the set $M(G)$. The (i, j)-th entry of $CD(G)$ is $dist(m_i, m_j)$, the distance between the maximal cyclic components m_i and m_j.*

Proposition 4.9 gives some properties of cyclic distance matrix of a graph G.

Proposition 4.9.

1. *For any connected graph G, the cyclic distance matrix $CD(G)$ is a zero diagonal, symmetric matrix with nonnegative entries. Hence trace of $CD(G) = 0$.*

2. *The maximum value among all entries in $CD(G)$ of a graph G is $cdiam(G)$, the cyclic diameter of G.*

3. *Let G be a graph with k MCCs. Then the entries in $CD(G)$ is a subset of $\{0, 1, 2, ..., k-1\}$ of the form $\{0, 1, 2, ..., l\}$,where $0 \leq l \leq k-1$*

4. *$CD(G)$ of a cyclic similar graph G is O.*

5. *The number of ones in the i-th row of of $CD(G)$ of a graph G is the number of neighboring MCCs of the i-th MCC.*

The MCCs of a tree are the vertices itself, so we have;

Theorem 4.10. *$CD(T)$ of a tree T is the distance matrix [1] of the tree itself.*

Theorem 4.11. *$CD(G)$ of a graph G is a binary matrix if and only if*

1. *G has two MCCs connected by a bridge or*

2. *G is a cyclic flower or*

3. *G is either a graph containing a cyclic flower and a cyclic similar graph connected by a bridge between the flower centric vertex of the cyclic flower and any vertex of the cyclic similar graph or G contains two cyclic flowers connected by a bridge through their flower centric vertices, holds.*

Proof. Suppose that $CD(G)$ of a graph G is a binary matrix. Let $M(G) = \{m_1, m_2, ..., m_k\}$. Then $k \neq 1$. Otherwise, G will be cyclic similar. In that case, $CD(G)$ cannot be a binary matrix. Hence $k \geq 2$. Since G contains at least two MCCs and $CD(G)$ is a binary matrix, for all $i \neq j$ with $1 \leq i, j \leq k$, $dist(m_i, m_j) = 1$. Therefore all the MCCs in G are neighbors to each other. So that $cdiam(G) = 1$. Hence it follows from Theorem 3.7 that one of the statements in the theorem holds.

Converse is obvious.

Definition 4.12. *The set of eigen values of the cyclic distance matrix $CD(G)$ of a graph G is called the cyclic distance spectrum of G. It is denoted by cd-spectrum of G.*

$CD(G)$ of a graph G is not unique. It depends on the labelling of the MCCs of G. A relabelling of the MCCs of G will result in a permutation of the rows and columns simultaneously. Hence for any labeling the eigen values of the graph will be the same. Since $CD(G)$ is a symmetric matrix, the eigen values of $CD(G)$ are real. Also the sum of the eigen values of $CD(G)$ equal to trace of $CD(G) = 0$, and determinant of $CD(G)$ equal to the product of the eigen values.

Let G be a connected graph with number of MCCs as $k \geq 2$ and $cdiam(G) = 1$. Then $CD(G)$ is a matrix with diagonal elements as zero and all other entries as one. Which is the same as the adjacency matrix of a complete graph K_k on k vertices. It is clear that rank of this matrix is k. For any positive integer K; the eigenvalues of K_k are $k-1$ and 1 with multiplicities 1 and $k-1$ respectively. [1] Hence we have;

Theorem 4.13. *Let G be any graph containing $k \geq 2$ MCCs and $cdiam(G) = 1$, then*

1. *Rank $CD(G) = k$ = number of MCCs in G.*

2. *The cd-spectrum of G consists of $k-1$ and 1 with multiplicities 1 and $k-1$ respectively.*

3. *The determinant of $CD(G)$ equals $(-1)^{(k-1)}(k-1)$.*

In 1971, R. L Graham and H. O Pollak [5] Proved that if T is a tree of order n, then the determinant of the distance matrix of T, $det\ D(T) = (-1^n)(n-1)2^{(n-2)}$. We will use this result to show that the determinant of the $CD(G)$ of a graph G, not containing any cyclic flower with more than two MCCs depends only on the number of MCCs of G.

Theorem 4.14. *Let G be a connected graph with number of MCCs as k, which contains no cyclic flowers with more than two MCCs. Then the determinant of $CD(G)$ of G depends only on the number of MCCs in G and it is given by $det\ CD(G) = (-1^k)(k-1)2^{(k-2)}$.*

Proof. Let G be any graph and let $S(G)$ be the shrinked graph of G. By Proposition 4.9, the number of MCCs in G and the order of $S(G)$ are the same. Also from the definition of distance between MCCs in a graph, it is clear that for any two MCCs m_i, m_j with $1 \leq i, j \leq k$ in G, $dist(m_i, m_j) = dist(v_i, v_j)$ where v_i and v_j are the vertices in $S(G)$ representing to the MCCs m_i and m_j in G and $dist(v_i, v_j)$ is the distance between the vertices v_i and v_j in $S(G)$. By Theorem 4.3, the shrinked graph $S(G)$ of a graph G is a tree if and only if G contains no cyclic flowers with more than two MCCs. Hence for any such graph G, $S(G)$ is a tree. Therefore $CD(G)$ of G and the distance matrix of $S(G)$ are the same. Hence by using the classical result of Graham and Pollak [5], $det\ CD(G) = det\ D(S(G)) = (-1^k)(k-1)2^{(k-2)}$, depends only on the number of MCCs in G.

Corollary 4.15. *Let G be a connected graph with number of MCCs k, $k \geq 2$ which does not contain cyclic flowers with more than two MCCs. Then,*

1. *$det\ CD(G)$ is independent of the structure of the graph G*
2. *The rank of $CD(G) = k$*
3. *$CD(G)$ is nonsingular*

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