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ANALYTICAL CHARACTERISTIC OF SPIRALLIKE FUNCTIONS DEFINED BY γ^{th} ORDER DIFFERINTEGRAL TYPE OPERATOR

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Abstract: In this paper we have obtained some necessary and sufficient conditions for the following classes:
1) $\mathbf{SVP}_{\psi}(\nu, \mathbf{p})$
A function $I(z)$ of the class $\mathcal{A}_p$ also contained in the subclass $SVP_{\psi}(\nu, p)$ if it satisfies the inequality

$$\left|\frac{I^{(p-1)}(z)}{(cos\psi + isin\psi)zI^{(p)}(z)} - \frac{1}{3\nu}\right| < \frac{2}{3\nu} \quad where \ \ \psi \in \mathbb{R} \ \ and \ \ 0 < \nu < 1.$$

2) $\mathbf{CVP}_{\psi}(\nu, \mathbf{p})$
A function $I(z) \in \mathcal{A}_p$ is said to be in the class $CVP_{\psi}(\nu, p)$ if it satisfies the inequality

$$\left|\frac{I^{(p)}(z)}{(cos\psi + isin\psi)zI^{(p+1)}(z)} - \frac{1}{3\nu}\right| < \frac{2}{3\nu} \quad where \ \ \psi \in \mathbb{R}and \ \ 0 < \nu < 1.$$

We have extended the previous results and derived some corollaries.

Keywords and Phrases: Differintegral operator, Pre-starlike functions, Spiral-like functions, Starlike functions.

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1. Preliminaries and Essentials

Let a p-valent analytic function $f(z)$ is characterized by

$$f(z) = z^p + \sum_{j=1}^{\infty} a_{j+p} z^{j+p} \qquad p \in \mathbb{N} \tag{1.1}$$

Belongs to the class $\mathcal{A}_p$ and all these functions are characterized in the disc represented by $D = z \in \mathbb{C} : |z| < 1$. The subclasses of p-valent star-like function of order ν and p-valent convex function are given by

$$S^*(\nu, p) = \left\{ f \in \mathcal{A}_p : \mathbb{R}\left(\frac{zf'(z)}{f(z)} \right) > \nu, \ z \in D \right\}$$
$$C^*(\nu, p) = \left\{ f \in \mathcal{A}_p : \mathbb{R}\left(1 + \frac{zf'(z)}{f(z)} \right) > \nu, \ z \in D \right\}$$

respectively, provided that ν is a real number such that $0 \le \nu < 1$.
a function $f \in \mathcal{A}_p$ is called ψ- spiral-like of order ν in the unit disc D if

$$\mathbb{R}\left\{ e^{i\psi} \frac{zf'(z)}{f(z)} \right\} > \nu cos\psi$$

Such functions are defined by the class $S_\psi(\nu)$ [8],(also see [9], [18]). In previous years many attractive classes of univalent, multivalent, spiral-like functions and many special cases were investigated, see for example [2, 6, 10-14].
Patel et al. [19] defined the operator $\Omega_z^{(\lambda,p)} : \mathcal{A}_p \to \mathcal{A}_p$ by

$$\Omega_z^{(\lambda,p)} f(z) = z^p + \sum_{j=1}^{\infty} \frac{\Gamma(j+p+1)\Gamma(p+1-\lambda)}{\Gamma(p+1)\Gamma(j+p+1-\lambda)} a_{p+j} z^{p+j} \tag{1.2}$$

Following the idea of Aouf et al. [3]. We define the operator $V_{\gamma,j}^{\lambda,p} : \mathcal{A}_p \to \mathcal{A}_p$ provided that $\gamma \in \mathbb{N}_0$, $\kappa \ge 0$, as follows:

$$\begin{aligned}
V_{0,j}^{\lambda,p} f(z) &= \Omega_z^{(\lambda,p)} f(z) \\
V_{1,j}^{\lambda,p} f(z) &= V_j^{\lambda,p} f(z) \\
V_{1,j}^{\lambda,p} f(z) &= (1-\kappa)\Omega_z^{(\lambda,p)} f(z) + \frac{\kappa z}{p}[\Omega_z^{(\lambda,p)} f(z)] \\
&= z^p + \sum_{j=1}^{\infty} \frac{p+\kappa j}{p} \frac{\Gamma(j+p+1)\Gamma(p+1-\lambda)}{\Gamma(p+1)\Gamma(j+p+1-\lambda)} a_{p+j} z^{p+j} \\
V_{2,j}^{\lambda,p} f(z) &= (1-\kappa)V_{1,j}^{\lambda,p} f(z) + \frac{\kappa z}{p}[V_{1,j}^{\lambda,p} f(z)]
\end{aligned}$$

$$V_{2,j}^{\lambda,p}f(z) = z^p + \sum_{j=1}^{\infty}\left(\frac{p+\kappa j}{p}\right)^2\frac{\Gamma(j+p+1)\Gamma(p+1-\lambda)}{\Gamma(p+1)\Gamma(j+p+1-\lambda)}a_{p+j}z^{p+j}$$

The general expression is

$$V_{\gamma,j}^{\lambda,p}f(z) = V_{1,j}^{\lambda,p}V_{\gamma-1,j}^{\lambda,p}f(z)$$

$$V_{\gamma,j}^{\lambda,p}f(z) = z^p + \sum_{j=1}^{\infty}\left(\frac{p+\kappa j}{p}\right)^\gamma\frac{\Gamma(j+p+1)\Gamma(p+1-\lambda)}{\Gamma(p+1)\Gamma(j+p+1-\lambda)}a_{p+j}z^{p+j} \tag{1.3}$$

$$V_{\gamma,j}^{\lambda,p}f(z) = z^p + \sum_{j=1}^{\infty}\tau_{p,j}^{\gamma}a_{p+j}z^{p+j} \approx I(z). \tag{1.4}$$

$$where, \tau_{p,j}^{\gamma} = \left(\frac{p+\kappa j}{p}\right)^\gamma\frac{\Gamma(j+p+1)\Gamma(p+1-\lambda)}{\Gamma(p+1)\Gamma(j+p+1-\lambda)}. \tag{1.5}$$

Motivated and inspired by the above mention work, we here define the following:

Definition 1.1. *A function $I(z)$ of the class $\mathcal{A}_p$ also contained in the subclass $SVP_\psi(\nu,p)$ if it satisfies the inequality*

$$\left|\frac{I^{(p-1)}(z)}{(cos\psi + isin\psi)zI^{(p)}(z)} - \frac{1}{3\nu}\right| < \frac{2}{3\nu} \quad where \;\; \psi \in \mathbb{R} \;\; and \;\; 0 < \nu < 1. \tag{1.6}$$

The p^{th} derivative of $I(z)$ is denoted by $I^{(p)}(z)$. For $\lambda = 0$, $\gamma = 0$ this class reduces to the class $S_\psi(\nu,p)$ introduced by Khan et al. [7].

Assertion 1. *It is easily seen that, for $\lambda = 0,\ \gamma = 0,\ p = 1$ and $\psi = 0$, $S_0(\nu,1) = M(\nu)$ this class studied in [15] and for $\lambda = 0, \gamma = 0, p = 1, S_\psi(\nu,1) = S_\psi(\nu)$, see [16].*

Definition 1.2. *A function $I(z) \in \mathcal{A}_p$ is said to be in the class $CVP_\psi(\nu,p)$ if it satisfies the inequality*

$$\left|\frac{I^{(p)}(z)}{(cos\psi + isin\psi)zI^{(p+1)}(z)} - \frac{1}{3\nu}\right| < \frac{2}{3\nu} \quad where \;\; \psi \in \mathbb{R} and \;\; 0 < \nu < 1. \tag{1.7}$$

For $\lambda = 0, \gamma = 0, p = 1, C_\psi(\nu,1) = K_\psi(v)$, see [16].

2. The necessary and sufficient conditions for the class $SVP_\psi(\nu,p)$

Theorem 2.1. *A function $I(z) \in SVP_\psi(\nu,p)$ if and only if,*

$$\mathbb{R}\left\{\frac{(cos\psi + isin\psi)zI^{(p)}(z)}{I^{(p-1)}(z)}\right\} > \nu. \tag{2.1}$$

Proof. Let $I(z) \in SVP_\psi(\nu, p)$, then we have

$$\left|\frac{1}{e^{i\psi}Y(z)} - \frac{1}{3\nu}\right| < \frac{2}{3\nu} \quad , z \in D, \quad where Y(z) = \frac{zI^{(p)}(z)}{I^{(p-1)}(z)}. \tag{2.2}$$

From the above, we have

$$\begin{aligned}
\left|\frac{3\nu - e^{i\psi}Y(z)}{3\nu e^{i\psi}Y(z)}\right| &< \frac{2}{3\nu}. \qquad (2.3)\\
\Leftrightarrow |3\nu - e^{i\psi}Y(z)|^2 &< |2e^{i\psi}Y(z)|^2\\
\Leftrightarrow [3\nu - e^{i\psi}Y(z)]\overline{[3\nu - e^{i\psi}Y(z)]} &< [2e^{i\psi}Y(z)]\overline{[2e^{i\psi}Y(z)]}\\
\Leftrightarrow [3\nu - e^{i\psi}Y(z)][3\nu - e^{-i\psi}\overline{Y(z)}] &< [2e^{i\psi}Y(z)][2e^{-i\psi}\overline{Y(z)}]\\
\Leftrightarrow 9\nu^2 - 3\nu e^{-i\psi}\overline{Y(z)} - 3\nu e^{i\psi}Y(z) + 4Y(z)\overline{Y(z)} &< 4Y(z)\overline{Y(z)}\\
\Leftrightarrow 9\nu^2 - 3\nu(e^{-i\psi}\overline{Y(z)} + e^{i\psi}Y(z)) &< 0\\
\Leftrightarrow 3\nu - 2\mathbb{R}e^{i\psi}Y(z) &< 0\\
\Leftrightarrow -2\mathbb{R}e^{i\psi}Y(z) &< -3\nu\\
\Leftrightarrow \mathbb{R}\left(e^{i\psi}\frac{zI^{(p)}(z)}{I^{(p-1)}(z)}\right) &> \frac{3\nu}{2}. \qquad (2.4)\\
\Leftrightarrow \mathbb{R}\left(e^{i\psi}\frac{zI^{(p)}(z)}{I^{(p-1)}(z)}\right) &> \nu. \qquad (2.5)
\end{aligned}$$

This achieves the result.

Assertion 2. *For $\lambda = 0,\ \gamma = 0$, this result reduced to the result of Khan et al. [7].*

Corollary 2.1. *For $p = 1, \lambda = 0, \gamma = 0$, the above result converts in to the following known result proved by Owa and Kamali [17].*

$$I(z) \in S_\psi(\nu) \Leftrightarrow \mathbb{R}\left(e^{i\psi}\frac{zI'(z)}{I(z)}\right) > \nu. \tag{2.6}$$

Theorem 2.2. *If $I(z) \in SVP_\psi(\nu, p)$ satisfies*

$$\sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}\left\{2(j+1) + |(j+1) - 3\nu e^{-i\psi}|\right\}\ |\tau_{j,p}^{\gamma}|\ |a_{j+p}| \le 2 - |(1 - 3\nu e^{-i\psi})|. \tag{2.7}$$

for some $|\psi| < \frac{\pi}{2}$ *and* $0 < \nu < cos\psi$*, then* $I(z) \in SVP_\psi(\nu, p)$.
Proof. If $I(z) \in SVP_\psi(\nu, p)$ then it suffices to show that

$$\left| \frac{3\nu - e^{i\psi} Y(z)}{e^{i\psi} Y(z)} \right| < 2. \tag{2.8}$$

for some $|\psi| < \frac{\pi}{2}$ and $0 < \nu < cos\psi$, where $Y(z) = \frac{zI^{(p)}(z)}{I^{(p-1)}(z)}$.
Now we have

$$\left| \frac{3\nu - e^{i\psi} Y(z)}{e^{i\psi} Y(z)} \right| = \left| \frac{3\nu e^{-i\psi} I^{(p-1)}(z) - zI^{(p)}(z)}{zI^{(p)}(z)} \right|. \tag{2.9}$$

$$= \left| \frac{(3\nu e^{-i\psi} - 1) + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!} [3\nu e^{-i\psi} - (j+1)]\ \tau_{j,p}^{\gamma}\ a_{j+p} z^j}{1 + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!} (j+1) \tau_{j,p}^{\gamma}\ a_{j+p} z^j} \right|$$

$$\leq \frac{|(3\nu e^{-i\psi} - 1)| + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!} |[3\nu e^{-i\psi} - (j+1)]|\ |\tau_{j,p}^{\gamma}|\ |a_{j+p}||z^j|}{1 - \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!} |(j+1)||\tau_{j,p}^{\gamma}|\ |a_{j+p}||z^j|}$$

$$< \frac{|(1 - 3\nu e^{-i\psi})| + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!} |[(j+1) - 3\nu e^{-i\psi}]|\ |\tau_{j,p}^{\gamma}|\ |a_{j+p}|}{1 - \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!} |(j+1)||\tau_{j,p}^{\gamma}|\ |a_{j+p}|}. \tag{2.10}$$

The last expression in (2.10) is bounded above by 2 if

$$|(1 - 3\nu e^{-i\psi})| + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!} |[(j+1) - 3\nu e^{-i\psi}]|\ |\tau_{j,p}^{\gamma}|\ |a_{j+p}|$$
$$\leq 2 \left\{ 1 - \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!} |(j+1)||\tau_{j,p}^{\gamma}|\ |a_{j+p}| \right\}. \tag{2.11}$$

After simplification of (2.11) we have

$$\sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!} \left\{ 2(j+1) + |(j+1) - 3\nu e^{-i\psi}| \right\}\ |\tau_{j,p}^{\gamma}|\ |a_{j+p}| \leq 2 - |(1 - 3\nu e^{-i\psi})|$$

Therefore,$I(z) \in SVP_\psi(\nu, p)$ for some $|\psi| < \frac{\pi}{2}$ and $0 < \nu < cos\psi$.

Assertion 3. *For* $\lambda = 0, \gamma = 0$, *this result reduces into the result introduced by Khan et al. [7]*

Corollary 2.2. *For* $\lambda = 0, \gamma = 0, p = 1, \psi = 0$ *and* $0 < \nu < 1$ *then* $I(z) \in \mathcal{A}_p$

satisfies the following coefficient inequality

$$\sum_{j=2}^{\infty}(j-\nu)|\tau_{j,p}^{\gamma}||a_{j+p}| \leq \frac{1}{3}[2-|(1-3\nu)|]. \tag{2.12}$$

Corollary 2.3. *If* $I(z) \in \mathcal{A}_p$ *satisfies*

$$\sum_{j=1}^{\infty}\frac{(j+p)!}{(j+1)!(p)!}\left\{2(j+1)+\sqrt{(j+1)^2-3\sqrt{2}\nu(j+1)+9\nu^2}\right\}|\tau_{j,p}^{\gamma}||a_{j+p}|$$
$$\leq 2-\sqrt{1+9\nu^2-3\sqrt{2}\nu}. \tag{2.13}$$

For some $\nu \in (0, \frac{1}{\sqrt{2}})$*, then* $I(z) \in SVP_{\frac{\pi}{4}}(\nu, p)$.

3. The necessary and sufficient conditions for the class $CVP_{\psi}(\nu, p)$

Theorem 3.1. *A function* $I(z) \in CVP_{\psi}(\nu, p)$ *if and only if,*

$$\mathbb{R}\left\{(cos\psi + isin\psi)\left(1+\frac{zI^{(p+1)}(z)}{I^{(p)}(z)}\right)\right\} > \nu. \tag{3.1}$$

Proof. Let $I(z) \in CVP_{\psi}(\nu, p)$, then we have

$$\left|\frac{1}{e^{i\psi}L(z)}-\frac{1}{3\nu}\right| < \frac{2}{3\nu} \quad , z \in D, \quad where L(z) = 1+\frac{zI^{(p+1)}(z)}{I^{(p)}(z)}. \tag{3.2}$$

From the above, we have

$$\left|\frac{3\nu - e^{i\psi}L(z)}{3\nu e^{i\psi}L(z)}\right| < \frac{2}{3\nu}. \tag{3.3}$$

$$\Leftrightarrow |3\nu - e^{i\psi}L(z)|^2 < |2e^{i\psi}L(z)|^2$$

$$\Leftrightarrow [3\nu - e^{i\psi}L(z)]\overline{[3\nu - e^{i\psi}L(z)]} < [2e^{i\psi}L(z)]\overline{[2e^{i\psi}L(z)]}$$

$$\Leftrightarrow [3\nu - e^{i\psi}L(z)][3\nu - e^{-i\psi}\overline{L(z)}] < [2e^{i\psi}L(z)][2e^{-i\psi}\overline{L(z)}]$$

$$\Leftrightarrow 9\nu^2 - 3\nu e^{-i\psi}\overline{L(z)} - 3\nu e^{i\psi}L(z) + 4L(z)\overline{L(z)} < 4L(z)\overline{L(z)}$$

$$\Leftrightarrow 9\nu^2 - 3\nu(e^{-i\psi}\overline{L(z)} + e^{i\psi}L(z)) < 0$$

$$\Leftrightarrow 3\nu - 2\mathbb{R}e^{i\psi}L(z) < 0$$

$$\Leftrightarrow -2\mathbb{R}e^{i\psi}L(z) < -3\nu$$

$$\Leftrightarrow \mathbb{R}\left(e^{i\psi}\left(1+\frac{zI^{(p+1)}(z)}{I^{(p)}(z)}\right)\right) > \frac{3\nu}{2}. \tag{3.4}$$

$$\Leftrightarrow \mathbb{R}\left(e^{i\psi}\left(1+\frac{zI^{(p+1)}(z)}{I^{(p)}(z)}\right)\right) > \nu. \tag{3.5}$$

This achieves the result.

Theorem 3.2. *If* $I(z) \in CVP_\psi(\nu, p)$ *satisfies*

$$\sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}(j+1)\left\{2(j+1) + |(j+1) - 3\nu e^{-i\psi}|\ |\tau_{j,p}^{\gamma}|\ |a_{j+p}|\right\}$$
$$\leq 2 - |1 - 3\nu e^{-i\psi}|. \tag{3.6}$$

For some $|\psi| < \frac{\pi}{2}$ *and* $0 < \nu < cos\psi$, *then* $I(z) \in CVP_\psi(\nu, p)$.
Proof. To prove that $I(z) \in CVP_\psi(\nu, p)$ we need to prove that

$$\left|\frac{3\nu - e^{i\psi}L(z)}{e^{i\psi}L(z)}\right| < 2. \tag{3.7}$$

for some $|\psi| < \frac{\pi}{2}$ and $0 < \nu < cos\psi$, where $L(z) = 1 + \frac{zI^{(p+1)}(z)}{I^{(p)}(z)}$.
Now we have

$$\left|\frac{3\nu - e^{i\psi}L(z)}{e^{i\psi}L(z)}\right| = \left|\frac{3\nu e^{-i\psi}I^{(p)}(z) - I^{(p)}(z) + zI^{(p+1)}(z)}{I^{(p)}(z) + zI^{(p+1)}(z)}\right|. \tag{3.8}$$

$$= \left|\frac{(3\nu e^{-i\psi} - 1) + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}(j+1)[3\nu e^{-i\psi} - (j+1)]\ \tau_{j,p}^{\gamma}\ a_{j+p}z^j}{1 + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}(j+1)^2\tau_{j,p}^{\gamma}\ a_{j+p}z^j}\right|$$
$$\leq \frac{|(3\nu e^{-i\psi} - 1)| + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}|(j+1)||[3\nu e^{-i\psi} - (j+1)]|\ |\tau_{j,p}^{\gamma}|\ |a_{j+p}||z^j|}{1 - \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}|(j+1)|^2|\tau_{j,p}^{\gamma}|\ |a_{j+p}||z^j|}$$

$$< \frac{|(1 - 3\nu e^{-i\psi})| + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}|(j+1)||[(j+1) - 3\nu e^{-i\psi}]|\ |\tau_{j,p}^{\gamma}|\ |a_{j+p}|}{1 - \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}|(j+1)|^2|\tau_{j,p}^{\gamma}|\ |a_{j+p}|}. \tag{3.9}$$

The last expression in (3.9) is bounded above by 2 if

$$|(1 - 3\nu e^{-i\psi})| + \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}(j+1)|[(j+1) - 3\nu e^{-i\psi}]|\ |\tau_{j,p}^{\gamma}|\ |a_{j+p}| \quad \leq$$

$$2\left\{1 - \sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}|(j+1)|^2|\tau_{j,p}^{\gamma}|\ |a_{j+p}|\right\}. \tag{3.10}$$

After simplification of (3.10) we have

$$\sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}(j+1)\left\{2(j+1)+|(j+1)-3\nu e^{-i\psi}|\,|\tau_{j,p}^{\gamma}|\,|a_{j+p}|\right\}$$
$$\leq 2-|(1-3\nu e^{-i\psi})|. \tag{3.11}$$

Therefore, $I(z) \in CVP_{\psi}(\nu, p)$ for some $|\psi| < \frac{\pi}{2}$ and $0 < \nu < cos\psi$.

Corollary 3.1. *If $I(z) \in \mathcal{A}_p$ satisfies*

$$\sum_{j=1}^{\infty} \frac{(j+p)!}{(j+1)!(p)!}(j+1)\left\{2(j+1)+\sqrt{(j+1)^2-3\sqrt{2}\nu(j+1)+9\nu^2}\right\}|\tau_{j,p}^{\gamma}||a_{j+p}|$$
$$\leq 2-\sqrt{1+9\nu^2-3\sqrt{2}\nu}.$$

For some $\nu \in (0, \frac{1}{\sqrt{2}})$, then $I(z) \in CVP_{\frac{\pi}{4}}(\nu, p)$.

4. Conclusion

In this paper we have derived some necessary and sufficient conditions for the subclasses of starlike and convex spirallike multivalent functions using γ^{th} order differintegral type operator. These results are new and will help researchers in the field of Geometric function theory.

References

[1] Ahmad Bakhtiar, Khan Muhammad Ghaffar, Frasin Basem Aref, Aouf Mohamed Kamal, Thabet Abdeljawad, Wali Khan Mashwani, Muhammad Arif, On q-analogue of meromorphic multivalent functions in lemniscate of Bernoulli domain [J], AIMS Mathematics, 6(4) (2021): 3037-3052. doi: 10.3934 / math. 2021185

[2] Arif M. et al., On products of multivalent close-to-star functions, J. Inequal. Appl., 2015: 5 (2015), 14 pp. MR3359751

[3] Aouf M. K., Mostafa A. O. and Zayed H. M., Subordination and superordination properties of p-valent functions defined by a generalized fractional differintegral operator, Quaest. Math., 39 (2016), no. 4, 545-560. MR3521174

[4] Khan, B., Liu, Z. G., Srivastava, H. M., Khan, N., Darus, M. and Tahir, M., A study of some families of multivalent q-starlike functions involving higher-order q-derivatives. Mathematics, 8(9) (2020), p. 1470.

[5] Khan, B., Srivastava, H., Khan, N., Darus, M., Tahir, M., Samad, A., Applications of Higher-Order Derivatives to the Subclasses of Meromorphic Starlike Functions, Journal of Applied and Computational Mechanics, 7(1) (2021), pp. 321-333. doi: 10.22055/jacm.2020.34948.2518

[6] Khan, N., Khan, B., Ahmad, Q.Z. and Ahmad, S., Some Convolution properties of multivalent analytic functions, AIMS Math, 2(2) (2017), pp. 260-268.

[7] Khan, N., Ahmad, Q. Z., Khalid, T. and Khan, B., Results on spirallike p-valent functions, AIMS Math, 3 (2017), pp. 12-20.

[8] Libera, R. J., Univalent α-spiral functions, Canadian J. Math., 19 (1967), 449-456. MR0214750

[9] Noor, K. I., Khan N. and Noor, M. A., On generalized spiral-like analytic functions, Filomat, 28 (2014), no. 7, 1493-1503. MR3360055

[10] Noor, K. I. and Khan, N., Some convolution properties of a subclass of p-valent functions, Maejo International Journal of Science and Technology, 9(2) (2015), p. 181.

[11] Noor, K. I., Khan, N. and Ahmad, Q. Z., Coefficient bounds for a subclass of multivalent functions of reciprocal order, AIMS Mathematics, 2(2) (2017), pp.322-335.

[12] Noor, K. I., Ahmad, Q. Z. and Khan, N., On some subclasses of meromorphic functions defined by fractional derivative operator, Italian J. Pure. App Math, (2017), pp. 1-8.

[13] Nunokawa M. and Sokół, J., On the multivalency of certain analytic functions, J. Inequal. Appl., 2014, 2014:357, 9 pp. MR3317504

[14] Nunokawa M. et al., A subclass of analytic functions related with conic domain, J. Class. Anal., 9 (2016), no. 2, 137-149. MR3588362

[15] Obradović M. and Owa S., On some results for λ-spiral functions of order α, Internat. J. Math. Math. Sci., 9 (1986), no. 3, 439-446. MR0859112

[16] Owa, S., Ochiai, K. and Srivastava, H. M., Some coefficient inequalities and distortion bounds associated with certain new subclasses of analytic functions, Math. Inequal. Appl., 9 (2006), no. 1, 125-135. MR2198556

[17] Owa, S., Sağsöz, F. and Kamali, M., On some results for subclass of β-spirallike functions of order α, Tamsui Oxf. J. Inf. Math. Sci., 28 (2012), no. 1, 79-93. MR3087258

[18] Polatoğlu, Y. and Şen, A., Some results on subclasses of Janowski λ-spirallike functions of complex order, Gen. Math., 15 (2007), no. 3, 88-97. MR2369070

[19] Patel, J. and Mishra, A. K., On certain subclasses of multivalent functions associated with an extended fractional differintegral operator, J. Math. Anal. Appl., 332 (2007), no. 1, 109-122. MR2319647

[20] Rehman, M. S. U., Ahmad, Q. Z., Srivastava, H. M., Khan, N., Darus, M. and Khan, B., Applications of higher-order q-derivatives to the subclass of q-starlike functions associated with the Janowski functions [J], AIMS Mathematics, 6(2) (2021), pp. 1110-1125.

[21] Srivastava, H. M., Ahmad, Q. Z., Khan, N., Kiran, S. and Khan, B., Some applications of higher-order derivatives involving certain subclasses of analytic and multivalent functions, J. Nonlinear Var. Anal, 2 (2018), pp. 343-353.