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A Mathematical Study of Recurrent-Vector Field in a Kaehlerian Space

Rajeev Kumar Singh and Maninder Singh Arora*,
Department of Mathematics,
P.B.P.G. College, Pratapgarh (U.P.) India
E-mail: profrajeevsingh@gmail.com
*Department of Mathematics,
P.P.N. College, Kanpur (U.P.) India
E-mail: maninder_arora4@rediffmail.com

Abstract: In this paper we have discussed about recurrent vector field V^h with respect to complex conformal connection Γ^h_{ji} . Also we have obtained results about a vector field recurrent with respect to special semi-symmetric metric F-connection.
Keywords and phrases: Recurrent vector field, complex conformal connection.

1. Introduction

Various connections like conformal complex connections, special semi-symmetric metric F-connections, besides the well known Christoffel symbols are introduced and studies by several mathematicians. We have mentioned the basic result and the covariant derivatives of a vector field with respect to them are shown to be connected. We have discussed the most general analytic k-torse forming vectors existing in Kaehlerian spaces. We have devoted to the study of parallel and recurrent vector fields in Kaehlerian space.

In real 2n-dimensional Kaehlerian space with F^h_i as structure tensor and g_{ji} as Hermitian metric, several connection parameters other than the Christoffel symbols are known. Since structure tensor is covariant with respect of $\left\{ \begin{matrix} h \\ ij \end{matrix} \right\}$, all these connections Γ^h_{ji} with respect to whom the covariant derivatives of structure tensor vanishes identically are called F-connection.

A connection parameter having Γ^h_{ji} as its components given by

$$\Gamma^h_{ji} = \left\{ \begin{matrix} h \\ ij \end{matrix} \right\} + \delta^h_j p_i + \delta^h_i p_j - g_{ji} p^h + F^h_j q_i - F_{ji} q^h$$

where $p_i = \partial_{ip}, p^h = g^{th} p_t, q_i = -p_t F^t_i, q^h = -q_t g^{th}$ and P is some scalar point function, has been called complex conformal connection.

2. Recurrent Field

A vector field V^h satisfying

$$\nabla_j V^h = \lambda_j V^h$$

will be called recurrent vector field in a Kaehelrian space where λ_j are the components of a non null vector. This idea of recurrency shall be extended with respect to the connections mentioned in this paper.

Thus for the vector field V^h to be recurrent with respect to complex conformal connection Γ^h_{ji} be must have
(2.1) $D_j V^h = \mu_j V^h$ where $\mu_j \neq 0$
On substituting from (2.1) in the equation
$D_j V^h = \overset{1}{D}{}_j V^h + p_j V^h$ we find,
$\overset{1}{D}_j V^h = (\mu_j - p_j)V^h$ and so if
$\mu_j \neq p_j$
we find that
$\overset{1}{D}_j V^h = t_j V^h$
where $t_j \neq 0$
and consequently vector field becomes a recurrent vector field respect to $\overset{1}{\Gamma}{}^h_{ji}$ also and thus we have

Theorem 2.1

A vector field recurrent with respect to conformal complex connections or recurrent with respect to special semi symmetric metric F-connection $\overset{1}{\Gamma}{}^h_{ji}$ also.
Similarly on putting from (2.1) in equation

$$D_j V^h = \overset{2}{D}_j V^h + p_j V^h + q_j \tilde{V}^h$$

we find

$$\overset{2}{D}_j V^h = (\mu_j - p_j)V^h - q_j \tilde{V}^h$$

and so this vector field is recurrent with respect to $\overset{2}{\Gamma}{}^h_{ji}$ also provided there exist a vector field S_j such that

$$(\mu_j - p_j)V^h - q_j \tilde{V}^h = S_j V^h$$

which after multiplication by V_h and using

$$V_h V^h = |V|^2, V_h \tilde{V}^h = 0$$

gives

$$(\mu_j - p_j - S_j)|V|^2 = 0$$

or (2.2) $S_j = \mu_j - p_j$ thus we conclude that
Theorem 2.2

If a vector field recurrent with to respect complex conformal connection is again recurrent with respect to semi-symmetric metric F- connection $\overset{2}{\Gamma}{}^h_{ji}$ also then the vectors of recurrence are related by (2.2).

On the other hand from the given equation

$$\overset{1}{D}_j V^h = \overset{2}{D}_j V^h + q_j \tilde{V}^h$$

we find if $\overset{2}{D}_j{}^h = a_j V^h$
Then $\overset{2}{D}_j{}^h = a_j V^h - q_j \tilde{V}^h$
and so for V^h to be recurrent with respect to $\overset{2}{\Gamma}{}^h_{ji}$ also we must have that there exist some vector field b_j such that

$$a_j V^h - q_j \tilde{V}^h = b_j V^h$$

The above equation on contraction with V_h gives

$$a_j = b_j$$

and thus the vectors of recurrence coincide and hence we conclude in the following theorem.
Theorem 2.3

A vector field recurrent with respect to special semi symmetric metric F-connection of both kinds as the same recurrence if and only if $q_j \tilde{V}^h = 0$.

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