J. of Ramanujan Society of Mathematics and Mathematical Sciences Vol. 8, No. 1 (2020), pp. 85-96

> ISSN (Online): 2582-5461 ISSN (Print): 2319-1023

EXTENSION OF FIXED POINT THEOREMS TYPE T-ZAMFIRESCU MAPPING IN CONE METRIC SPACE

Archana Rajput and S. K. Malhotra

Department of Mathematics,

Govt. Dr. Shyama Prashad Mukharjee Science and Commerce College, Benazeer, Bhopal, Madhya Pradesh - 462008, INDIA

E-mail : archanaraje1988@gmail.com, skmalhotra75@gmail.com

(Received: Jul. 15, 2020 Accepted: Nov. 05, 2020 Published: Dec. 30, 2020)

Abstract: The objective of this paper is to obtain sufficient conditions for the existence of fixed point of T-Zamfirescu in complete cone metric spaces and we prove fixed point theorem for an extended Kannan and Chatterjea type T-contraction mapping in a cone metric space. Our results generalize recent results existing in the literature of T-Zamfirescu mappings in cone metric space.

Keywords and Phrases: Cone Metric Space, *T*-Zamfirescu mapping, Cone normed space.

2010 Mathematics Subject Classification: 47H10, 54H25, 46B25.

1. Introduction

In [4], Huang and Zhang introduced the concept of cone metric space as a generalization of metric space, in which they replace the set of real numbers with a real Banach space. After that, many others [1, 2, 5, 6, 7, 12] proved numerous fixed point theorems for contractive type mappings on a cone metric space. Morales and Rojas [10], [9], [11] have extended the concept of T-contraction mappings to cone metric space by proving fixed point theorems for T-Kannan, T-Zamfirescu, T-weakly contraction mappings. The purpose of this paper is to prove fixed point theorem for an extended Kannan and Chatterjea T-Zamfirescu type mapping in a cone metric space. Our results pull out and generalized fixed point theorems of [8].

2. Preliminaries and Definition

Definition 2.1. Let $(E, \|\cdot\|)$ be a real Banach space and R be set of real number. A subset $P \subseteq E$ is said to be a cone if and only if (i) P is closed, nonempty and $P \neq \{0\}$ (ii) $a, b \in R, a, b \ge 0, x, y \in P$ implies $ax + by \in P$ (iii) $P \cap (-P) = \{0\}$

For a given cone P subset of E, we define a partial ordering \leq with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write x < y to indicate that $x \leq y$ but $x \neq y$ while $x \ll y$ will stand for $y - x \in$ int P where int P denotes interior of **P** and is assumed to be nonempty.

Definition 2.2. [4] Let X be a nonempty set. Suppose that the mapping $d : X \times X \to E$ satisfies

(i) $0 \le d(x,y)$ for every $x, y \in X$, d(x,y) = 0 if and only if x = y.

(ii) d(x,y) = d(y,x) for every $x, y \in X$.

(iii) $d(x,y) \leq d(x,z) + d(z,y)$ for every $x, y, z \in X$.

Then d is a cone metric on X and (X, d) is a cone metric space.

Example 2.3. [3] Let $E = R^n$, $P = \{(x, y) \in E : x, y \ge 0\} \subset R^2$, X = R and $d: X \times X \to E$ such that $d(x, y) = (|x - y|, \alpha |x - y|)$, where $\alpha \ge 0$ is a constant. Then (X, d) is a cone metric space.

Definition 2.4. Let E be a Banach space and $P \subset E$ a cone. The cone P is called normal if there is a number K > 0 such that for all $x, y \in E$,

 $0 \le x \le y$ Implies $||x|| \le K ||y||$.

The least positive number satisfying the above inequality is called the normal constant of P.

Then $\|\cdot\|$ is called a norm on X, and $(X, \|\cdot\|)$ is called a cone normed space. Clearly each cone normed space is a cone metric space with metric defined by $d(x, y) = \|x - y\|$.

Definition 2.5. [3] Let (X, d) be a cone metric space, $x \in X$ and $\{x_n\}$ a sequence in X. Then

(i) $\{x_n\}$ converges to x if for every $c \in E$ with $0 \ll c$ there is a natural number N such that $d(x_n, x) \leq c$ for all $n \geq N$

We shall denote it by $\lim_{n\to\infty} x_n = x \text{ or } x_n \to x$.

(ii) $\{x_n\}$ is a Cauchy sequence, if for every $c \in E$ with $0 \ll c$ there is a natural number N such that

 $d(x_n, x) \leq c \text{ for all } n, m \geq N.$

(iii) (X, d) is a complete cone metric space if every Cauchy sequence is convergent in X.

Definition 2.6. Let $(X, \|\cdot\|)$ be a cone normed space, $x \in X$ and $\{x_n\}$ a sequence in X. Then

(i) $\{x_n\}$ converges to x if for every $c \in E$ with $0 \ll c$ there is a natural number N such that $||x_n - x|| \leq c$ for all $n \geq N$

We shall denote it by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$.

(ii) $\{x_n\}$ is a Cauchy sequence, if for every $c \in E$ with $0 \ll c$ there is a natural number N such that

$$||x_n - x_m|| \le c \text{ for all } n, m \ge N.$$

(iii) $(X, \|\cdot\|)$ is a complete cone normed space if every Cauchy sequence is convergent. A complete cone normed space is called a Cone Banach space.

Lemma 2.7. [3] Let (X, d) be a cone normed space. P be a normal cone with constant K. Let $\{x_n\}, \{y_n\}$ be a sequence in X and $x, y \in X$. Then (i) $\{x_n\}$ converges to x if and only if $\lim_{n\to\infty} d(x_n, x) = 0$. (ii) If $\{x_n\}$ converges to x and $\{x_n\}$ converges to y then x = y. (iii) If $\{x_n\}$ is a Cauchy sequence if and only $\lim_{n,m\to\infty} d(x_n, x_m) = 0$.

(iv) If the sequence $\{x_n\}$ converges to x and $\{y_n\}$ converges to y then

$$d(x_n, y_n) \to d(x, y).$$

Definition 2.8. Let (X, d) be a cone metric space, P be a normal cone with normal constant K Let $T : X \to X$. Then

(i) T is said to be continuous,

if
$$\lim_{n \to \infty} x_n = x$$
 implies that $\lim_{n \to \infty} Tx_n = Tx$ for every $\{x_n\}$ in X.

(ii) T is said to be sequentially convergent if we have, for every sequence $\{y_n\}$, if $T(y_n)$ is convergent, then $\{y_n\}$ also is convergent.

Now, following the ideas of T. Zamfirescu, we introduce the notion of T-Zamfirescu mappings.

Definition 2.9. [14] Let (X, d) be a cone metric space and $T, S : X \to X$ two mappings. S is called a T-Zamfirescu mapping, (TZ-mapping), if and only if, there are real numbers, $0 \le a < 1$, $0 \le b$, c < 1/2 such that for all $x, y \in X$, at least

one of the next conditions are true:

$$\begin{aligned} (TZ_1): & d(TSx, TSy) \leq ad(Tx, Ty). \\ (TZ_2): & d(TSx, TSy) \leq b \big[d(Tx, TSx) + d(Ty, TSy) \big]. \\ (TZ_1): & d(TSx, TSy) \leq c \big[d(Tx, TSy) + d(Ty, TSx) \big]. \end{aligned}$$

Corollary 2.10. [13] Let $a, b, c, u \in E$ the real Banach space (i) If $a \leq b$ and $b \ll c$ then $a \ll c$. (ii) If $a \ll b$ and $b \ll c$ then $a \ll c$. (iii) If $0 \leq u \ll c$ for each $c \in int P$, then u = 0.

3. Main Results

Lemma 3.1. Let (X, d) be a cone metric space and $T, S : X \to X$ two mappings with

$$d(TSx, TSy) \le \alpha \Big[d(Tx, TSx) + d(Ty, TSy) \Big] + \beta d(Tx, Ty)$$
(3.1)

for all $x, y \in X$ where $0 \le \alpha$ and $0 \le \beta \le 1$. Then S is a T-Zamfirescu mapping. **Proof.** Let (X, d) be a cone metric space and $T, S : X \to X$ two mappings with

$$d(TSx, TSy) \le \alpha \left[d(Tx, TSx) + d(Ty, TSy) \right] + \beta d(Tx, Ty)$$

for all $x, y \in X$. Where $0 \le \alpha$ and $0 \le \beta \le 1$

$$\begin{split} d(TSx, TSy) &\leq \alpha \Big[d(Tx, TSx) + d(Ty, TSy) \Big] + \beta d(Tx, Ty) \\ d(TSx, TSy) &\leq \alpha \Big[d(Tx, TSx) + d(Ty, TSy) \Big] + \beta \Big\{ d(Tx, TSx) \\ &+ d(TSx, TSy) + d(TSy, Ty) \Big\} \\ (1 - \beta) d(TSx, TSy) &\leq (\alpha + \beta) \Big[d(Tx, TSx) + d(Ty, TSy) \Big] \\ d(TSx, TSy) &\leq \frac{(\alpha + \beta)}{(1 - \beta)} \Big[d(Tx, TSx) + d(Ty, TSy) \Big] \\ d(TSx, TSy) &\leq b \Big[d(Tx, TSx) + d(Ty, TSy) \Big] \\ where \ b = \frac{(\alpha + \beta)}{(1 - \beta)} \geq 0. \end{split}$$

Hence by definition of T-Zamfirescu, S is T-Zamfirescu mapping.

Theorem 3.2. Let (X, d) be a complete cone metric space, P be normal cone with normal cone with normal constant K. Moreover, let $T : X \to X$ be a continuous and injective mapping and $S : X \to X$ a continuous mapping. If the mappings Tand S satisfy

$$d(TSx, TSy) \le \alpha \left[d(Tx, TSx) + d(Ty, TSy) \right] + \beta d(Tx, Ty)$$
(3.2)

for all $x, y \in X$, where $0 \le \alpha$ and $0 \le \beta \le 1$. Then S has a unique fixed point in X.

Proof. Let $x_0 \in X$ be arbitrary. Define a sequence $\{x_n\}$ in X such that $x_{n+1} = Sx_n$ for each $= 0, 1, 2, ..., \infty$. We have

$$d(TSx_{n}, TSx_{n-1}) \leq \alpha \left[d(Tx_{n}, TSx_{n}) + d(Tx_{n-1}, TSx_{n-1}) + \beta d(Tx_{n}, Tx_{n-1}) \right] \\ + \beta d(Tx_{n}, Tx_{n-1}) + d(Tx_{n-1}, Tx_{n}) \\ d(Tx_{n+1}, Tx_{n}) \leq \alpha \left[d(Tx_{n}, Tx_{n+1}) + d(Tx_{n-1}, Tx_{n}) \right] \\ + \beta d(Tx_{n-1}, Tx_{n}) \\ (1 - \alpha) d(Tx_{n+1}, Tx_{n}) \leq (\alpha + \beta) d(Tx_{n-1}, Tx_{n}). \\ d(Tx_{n+1}, Tx_{n}) \leq \frac{(\alpha + \beta)}{(1 - \alpha)} d(Tx_{n-1}, Tx_{n}).$$

Proceeding as above

$$d(Tx_{n+1}, Tx_n) \le \frac{(\alpha+\beta)^n}{(1-\alpha)} d(Tx_0, Tx_1).$$

Next, to claim that $\{Tx_n\}$ is a Cauchy sequence. Consider $m, n \in N$ such that m > n

$$d(Tx_n, Tx_m) \le d(Tx_n, Tx_{n+1}) + d(Tx_{n+1}, Tx_{n+2}) + \dots + d(Tx_{m-1}, Tx_m)$$

$$d(Tx_n, Tx_m) \le \left[\frac{(\alpha + \beta)^n}{(1 - \alpha)} + \frac{(\alpha + \beta)^{n+1}}{(1 - \alpha)} + \dots + \frac{(\alpha + \beta)^{m-1}}{(1 - \alpha)}\right] d(Tx_0, Tx_1) \quad (3.3)$$

We take $\frac{\alpha+\beta}{1-\alpha} = k$, the inequality (3.3) implies that for all $m, n \in N, n > m$

$$d(Tx_n, Tx_m) \le \frac{k^n}{1-k} d(Tx_0, Tx_1).$$

Since, P be normal cone, therefore

$$||d(Tx_n, Tx_m)|| \le \frac{k^n}{1-k} ||d(Tx_0, Tx_1)||.$$

Further, since $k \in (0, 1), k^n \to 0$ as $n \to \infty$. Therefore $||d(Tx_n, Tx_m)|| \to 0$ as $m, n \to \infty$.

Thus, $\{Tx_n\}$ is a Cauchy sequence in X. As X is a complete cone metric space, there exists $z \in X$ such that

$$\lim_{n \to \infty} Tx_n = z.$$

Since T is sub-sequentially convergent, $\{x_n\}$ has a convergent subsequence $\{x_m\}$ such that

$$\lim_{m \to \infty} Tx_m = u$$

Since, T is continuous implies that

$$\lim_{m \to \infty} Tx_m = Tu \tag{3.4}$$

By the uniqueness of the limit, z = Tu. Since S is continuous,

$$\lim_{m \to \infty} Sx_m = Su$$

Again as T is continuous,

$$\lim_{m \to \infty} TSx_m = TSu$$

Therefore

$$\lim_{m \to \infty} TSx_{m+1} = TSu \tag{3.5}$$

Now consider,

$$d(TSu, Tu) \leq d(TSu, Tx_{m}) + d(Tx_{m}, Tu) d(TSu, Tu) \leq \alpha \left[d(Tu, TSu) + d(Tx_{m-1}, Tx_{m}) \right] + \beta d(Tu, Tx_{m-1}) + d(Tx_{m}, Tu) (1 - \alpha)d(TSu, Tu) \leq \alpha d(Tx_{m-1}, Tx_{m}) + \beta d(Tu, Tx_{m-1}) + d(Tx_{m}, Tu) d(TSu, Tu) \leq \frac{\alpha}{1 - \alpha} d(Tx_{m-1}, Tx_{m}) + \frac{\beta}{1 - \alpha} d(Tu, Tx_{m-1}) + \frac{1}{1 - \alpha} d(Tx_{m}, Tu) d(TSu, Tu) \leq \frac{\alpha}{1 - \alpha} d(Tx_{m-1}, Tx_{m}) + \frac{\beta}{1 - \alpha} \left\{ d(Tu, Tx_{m}) + d(Tx_{m}, Tx_{m-1}) \right\} + \frac{1}{1 - \alpha} d(Tx_{m}, Tu) d(TSu, Tu) \leq \frac{\alpha + \beta}{1 - \alpha} d(Tx_{m-1}, Tx_{m}) + \frac{\beta + 1}{1 - \alpha} d(Tu, Tx_{m})$$
(3.6)

Let $0 \ll c$ be arbitrary, By (3.4), we have

$$d(Tu, Tx_m) \ll \frac{c(1-\alpha)}{2(1+\beta)}$$

And by (3.5) we have

$$d(Tx_{m-1}, Tx_m) \ll \frac{c(1-\alpha)}{2(\alpha+\beta)}$$

Then (3.6) becomes,

$$d(TSu, Tu) \ll c$$
 for each $c \in int P$

Now, Using Corollary (2.10-iii), it follows that d(TSu, Tu) = 0 which implies that Tu = TSu

Since T is one-to-one, Thus u is the fixed point of S.

We claim that, u is the fixed point of.

If w is another fixed point of S, then w = Sw

$$d(Tu, Tw) = d(TSu, TSw)$$

$$\leq \alpha \Big(d(Tu, TSu) + d(Tw, TSw) \Big) + \beta d(Tu, Tw)$$

$$\leq \beta d(Tu, Tw)$$

This is a contradiction. Hence $d(Tu, Tw) = 0 \Rightarrow Tu = Tw$. As T is injective, u = w. Therefore the fixed point of S is unique.

Theorem 3.3. Let (X, d) be a complete cone metric space, P be normal cone with normal cone with normal constant K. Moreover, let $T : X \to X$ be a continuous and injective mapping and $S : X \to X$ a continuous mapping. If the mappings Tand S satisfy

$$d(TSx, TSy) \le \alpha \Big[d(Ty, TSx) + d(Tx, TSy) \Big] + \beta d(Tx, Ty)$$
(3.7)

for all $x, y \in X$, where $\alpha > 0, \beta \ge 0, 2\alpha + \beta < 1$ then S has an unique fixed point in X.

Proof. Let $x_0 \in X$ be arbitrary. Define a sequence $\{x_n\}$ in X such that $x_{n+1} = Sx_n$ for each $= 0, 1, 2, ..., \infty$.

We have

$$d(TSx_{n}, TSx_{n-1}) \leq \alpha \left[d(Tx_{n-1}, TSx_{n}) + d(Tx_{n}, TSx_{n-1}) \right] + \beta d(Tx_{n-1}, Tx_{n}) d(Tx_{n+1}, Tx_{n}) \leq \alpha \left[d(Tx_{n-1}, Tx_{n+1}) + d(Tx_{n}, Tx_{n}) \right] + \beta d(Tx_{n-1}, Tx_{n}) d(Tx_{n+1}, Tx_{n}) \leq \alpha d(Tx_{n-1}, Tx_{n+1}) + \beta d(Tx_{n-1}, Tx_{n}) d(Tx_{n+1}, Tx_{n}) \leq \alpha \left\{ d(Tx_{n-1}, Tx_{n+1}) + d(Tx_{n}, Tx_{n+1}) \right\} + \beta d(Tx_{n-1}, Tx_{n}) 1 - \alpha) d(Tx_{n+1}, Tx_{n}) \leq (\alpha + \beta) d(Tx_{n-1}, Tx_{n}) d(Tx_{n+1}, Tx_{n}) \leq \frac{(\alpha + \beta)}{(1 - \alpha)} d(Tx_{n-1}, Tx_{n})$$

Proceeding as above

(

$$d(Tx_{n+1}, Tx_n) \le \frac{(\alpha + \beta)^n}{(1 - \alpha)} d(Tx_0, Tx_1)$$

Next, to claim that $\{Tx_n\}$ is a Cauchy sequence. Consider $m,n\in N$ such that m>n

$$d(Tx_n, Tx_m) \le d(Tx_n, Tx_{n+1}) + d(Tx_{n+1}, Tx_{n+2}) + \dots + d(Tx_{m-1}, Tx_m)$$

$$d(Tx_n, Tx_m) \le \left[\frac{(\alpha + \beta)^n}{(1 - \alpha)} + \frac{(\alpha + \beta)^{n+1}}{(1 - \alpha)} + \dots + \frac{(\alpha + \beta)^{m-1}}{(1 - \alpha)}\right] d(Tx_0, Tx_1) \quad (3.8)$$

We take $\frac{\alpha+\beta}{1-\alpha} = k$, $d(Tx_n, Tx_m) \leq \left[k^n + k^{n+1} + \dots + k^{m-1}\right] d(Tx_0, Tx_1)$ $d(Tx_n, Tx_m) \leq \frac{k^n}{1-k} d(Tx_0, Tx_1)$

Since, P be normal cone, therefore

$$||d(Tx_n, Tx_m)|| \le \frac{k^n}{1-k} ||d(Tx_0, Tx_1)||$$

Further, since $k \in (0,1), k^n \to 0$ as $n \to \infty$. Therefore $||d(Tx_n, Tx_m)|| \to 0$ as $m, n \to \infty$

Thus, $\{Tx_n\}$ is a Cauchy sequence in X. As X is a complete cone metric space, there exists $z \in X$ such that

$$\lim_{n \to \infty} Tx_n = z$$

Since T is sub-sequentially convergent, $\{x_n\}$ has a convergent subsequence $\{x_m\}$ such that

$$\lim_{m \to \infty} T x_m = u$$

Since, T is continuous implies that

$$\lim_{m \to \infty} T x_m = T u \tag{3.9}$$

By the uniqueness of the limit, z = Tu. Since S is continuous,

$$\lim_{m \to \infty} Sx_m = Su$$

Again as T is continuous,

$$\lim_{m \to \infty} TSx_m = TSu$$

Therefore

$$\lim_{m \to \infty} TSx_{m+1} = TSu \tag{3.10}$$

Now consider,

$$d(TSu, Tu) \leq d(TSu, Tx_{m}) + d(Tx_{m}, Tu) d(TSu, Tu) \leq \alpha \Big[d(Tx_{m-1}, TSu) + d(Tu, Tx_{m}) \Big] + \beta d(Tu, Tx_{m-1}) + d(Tx_{m}, Tu) d(TSu, Tu) \leq \alpha \Big[d(Tx_{m-1}, Tu) + d(Tu, Tx_{m}) + d(Tu, Tx_{m}) \Big] + \beta d(Tu, Tx_{m-1}) + d(Tx_{m}, Tu) d(TSu, Tu) \leq \frac{\alpha + \beta}{1 - \alpha} \Big\{ d(Tx_{m-1}, Tu) + d(Tu, Tx_{m}) \Big\} + \frac{\alpha + 1}{1 - \alpha} d(Tx_{m}, Tu) d(TSu, Tu) \leq \frac{\alpha + \beta}{1 - \alpha} \Big\{ d(Tx_{m-1}, Tu) \Big\} + \frac{2\alpha + \beta + 1}{1 - \alpha} d(Tx_{m}, Tu)$$
(3.11)

Let $0 \ll c$ be arbitrary, By (3.9), we have

$$d(Tu, Tx_m) \ll \frac{c(1-\alpha)}{2(2\alpha+\beta+1)}$$

And by (3.10) we have

$$d(Tx_{m-1}, Tx_m) \ll \frac{c(1-\alpha)}{2(\alpha+\beta)}$$

Then (3.11) becomes,

$$d(TSu, Tu) \ll c$$
 for each $c \in int P$

Now, Using Corollary (2.10-iii), it follows that d(TSu, Tu) = 0 which implies that Tu = TSu

Since T is one-to-one, Thus u is the fixed point of S.

We claim that, u is the fixed point of.

If w is another fixed point of S, then w = Sw

$$d(Tu, Tw) = d(TSu, TSw)$$

$$\leq \alpha \Big(d(Tw, TSu) + d(Tu, TSw) \Big) + \beta d(Tu, Tw)$$

$$\leq (2\alpha + \beta) d(Tu, Tw)$$

This is a contradiction. Hence $d(Tu, Tw) = 0 \Rightarrow Tu = Tw$. As T is injective, u = w. Therefore the fixed point of S is unique.

References

- Abdeljawad T., Karapinar E., A note on cone metric fixed point theory and its equivalance, [Nonlinear Analysis, 72(5) (2010), pp. 2259-2261], Gazi Univ. J. Sci., 24(2) (2011), pp. 233-234.
- [2] Alnafei S. H., Radenovic S., Shahzad N., Fixed point theorems for mappings with convex diminishing diameters on cone metric spaces Appl. Math. Lett., 24 (2011), pp. 2162-2166.
- [3] Huan L. G., Zhang X., Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl., 332 (2007), pp. 1468–1476.
- [4] Huang L. G., Zhang X., Cone metric space and fixed point theorems, Math. Anal. Appl., 332(2)(2007).
- [5] Kadelburg Z., Radenovic S., Rakocevic S., A note on the equivalence of some metric and cone metric fixed point results, Appl. Math. Lett., 24 (2011), pp. 370-374.
- [6] Khamsi M. A., Remarks on cone metric spaces and fixed point theorems of contractive mappings, Fixed Point Theory Appl., 2010, Article ID 315398 (2010).
- [7] Kumar A., Rathee S., Kumar N., The point of coincidence and common fixed point for three mappings in cone metric spaces, J. Appl. Math. 2013, Article ID 146794 (2013).
- [8] Malceski A., Malceski S., Anevska K. and Malceski R., New Extension of Kannan and Chatterjea Fixed Point Theorems on Complete Metric Spaces, British Journal of Mathematics and Computer Science, 17(1) (2016), 1-10.
- [9] Morales J. and Rojas E., Cone metric spaces and fixed point theorems of *T*-Kannan contractive mappings, arXiv:0907.3949v1 [math.FA].
- [10] Morales J. and Rojas E., Cone metric spaces and fixed point theorems of T-contractive mappings, preprint, (2009).
- [11] Morales J. and Rojas E., Some results on *T*-Zamfirescu operators, Revista Notas de mathematica, 5(1) (2009), pp. 64-71.

- [12] Radenovic S., A pair of nonself mappings in cone metric space, Kragujev. J. Math., 36(2) (2012), 189-198.
- [13] Rezapour Sh., A review on topological properties of cone metric spaces, Analysis, Topology and Applications(ATA08), Vrnjacka Banja, Serbia, May-June (2008).
- [14] Zamfirescu T., Fixed points theorems in metric spaces, Arch. Math., 23 (1972), pp. 292–298.