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COMMON FIXED POINT THEOREMS FOR WEAKLY COMPATIBLE MAPPINGS SATISFYING CLR PROPERTY ON PARTIAL METRIC SPACES

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Abstract: The purpose of this paper is to obtain common fixed point results for quadruple weakly compatible mappings using CLR property in partial metric spaces. We extended the very recent results which were presented by Farzaneh et al. [10]. We justify our result by a suitable example.

Keywords and Phrases: Fixed point, partial metric space, CLR-property.

2010 Mathematics Subject Classification: 47H10, 54H25.

1. Introduction and Preliminaries

In 1992, Matthews [17] introduced the notion of partial metric spaces. He extended Banach's contraction principle to partial metric spaces which is the extension of usual metric space. The existence of fixed point for mapping defined on complete metric spaces (X, d) satisfy general contractive inequality of integral type was established by Branciari [6]. This result which involves more general contractive condition of integral type, was used by many authors to obtain some fixed point and common fixed point theorems on various spaces see eg., ([2], [4], [7], [8], [9], [11], [14], [15], [16], [18], [22], [23], [24], [25]).

In 2002, Aamri and El-Moutawakil [1] defined the notion of (E.A) property for self mappings which contains he class of non-compatible mappings in metric spaces. It was pointed that (E.A) property allows replacing the completeness requirement of the space with a more natural condition of closedness of the range as well as relaxes the complexness of the whole space, continuity of one or more mappings and containment of the range of one mapping into the range of other which is utilized to construct the sequence of joint iterates. Subsequently, there are several results proved for contraction mappings satisfying (E.A) property in partial metric spaces. Most recently, Sintunavarat and Kumam [21] defined the notion of common limit range CLR property in fuzzy metric spaces. In [21], it is observed that the notion of CLR-property never requires the condition of the closedness of the subspace while (E.A) property requires this condition for the existence of the fixed point.

Definition 1.1. *A partial metric space (briefly PMS) is a pair (X, p) where* $p : X \times X \to \mathbb{R}^+$ *is continuous map and* $\mathbb{R}^+ = [0, \infty)$ *such that for all x, y,* $z \in X$

(p1) $p(x,x) = p(y,y) = p(x,y) \Leftrightarrow x = y$,

(p2) $p(x,x) \le p(x,y)$,

(p3) $p(x,y) = p(y,x)$,

(p4) $p(x,y) \le p(x,z) + p(z,y) - p(z,z)$.

Each partial metric p on X generates a T_0 topology τ_p on X which has the family of open p - balls

$$\{B_p(x, \epsilon) : x \in X, \epsilon > 0\},$$

as a base, where $B_p(x, \epsilon) = \{y \in X : p(x, y) < p(x, x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$.

1. A sequence $\{x_n\}$ in a PMS (X, p) converges to a point $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

2. A sequence $\{x_n\}$ in a PMS (X, p) converges to a point $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

3. A PMS (X, p) is said to be complete if every cauchy sequence in X converges, with respect to τ_p, to a point $x \in X$ such that

$$p(x, x) = \lim_{m,n\to\infty} p(x_m, x_n).$$

The following lemma states a new version of the continuity on partial metric.

Lemma 1.2. [10] *Assume that* $x_n \to x$ *and* $y_n \to y$ *in PMS (X, p). Then*

$$\lim_{n\to\infty} (p(x_m, x_n) - \min\{p(x_n, x_n), p(y_n, y_n)\}) = p(x, y) - \min\{p(x_n, x_n), p(y_n, y_n)\}.$$

Remark 1.3. *Let*

$$p^*(x,y) = p(x,y) - \min\{p(x,x), p(y,y)\} \ \ \forall x, y \in X. \tag{1.1}$$

Therefore by the above Lemma $\lim_{n\to\infty} p^*(x_n, y_n) = p^*(x,y)$*, when* $x_n \to x$ *and* $y_n \to y$ *in PMS.*

Let $\mathcal{L}(\mathbb{R}^+)$ denote the Lebesgue integrable functions with finite integral and $\mathrm{USC}(\mathbb{R}^+)$ denote the upper semi-continuous functions.

$$\Phi := \{\varphi : \mathbb{R}^+ \to \mathbb{R}^+ : \varphi \in \mathcal{L}(\mathbb{R}^+), \int_0^{\epsilon} \varphi(t)dt > 0, \epsilon > 0\}$$

and

$$\Psi := \{\psi : \mathbb{R}^+ \to \mathbb{R}^+ : \psi \in USC(\mathbb{R}^+), \psi(0) = 0 \ and \ \psi(t) < t; \ \ \forall t > 0\}.$$

A pair of self mappings F and G on X is weakly compatible if there exists a point $x \in X$ such that $Fx = Gx$ implies $FGx = GFx$ i.e., they commute at their coincidence points.

The following are partial metric version of metric ones in ([1], [12], [22]).

Definition 1.4. *Let* (X, p) *be a partial metric space for the self mappings* $F, G, S, T : X \to X$*. If there exist two sequences* x_n *and* y_n *in* X *such that*

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gx_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Ty_n = t \in X,$$

then the pair (F, G) *and* (S, T) *satisfy the common (E.A) property.*

Definition 1.5. *Let* (X, p) *be a partial metric space for the self mappings* $F, G, S, T : X \to X$*. If there exist two sequences* x_n *and* y_n *in* X *such that*

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gx_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Ty_n = t \in G(X) \cap T(X),$$

then the pair (F, G) *and* (S, T) *satisfy the common limit range property with respect to the mapping* G *and* T*, denoted by* CLR_{GT}*.*

The purpose of this paper to employ the common CLR-property to obtain common fixed point results for two pair of weakly compatible mappings satisfying generalized contractive condition of integral type on partial metric spaces.

2. Main Results

In this section, we establish common fixed point theorems for weakly compatible mappings using CLR and common (E.A) properties.

Theorem 2.1. *Let (X, p) be a partial metric space and* F*,* G*,* S *and* T *be four*

self-mappings on X satisfying the following conditions:
(1) The pair (F,G) and (S,T) share (CLR_{GT}) property;
(2)$\int_0^{p(Fx,Sy)} \varphi(t)dt \leq \psi\big(\int_0^{C^1_{F,G,S,T(x,y)}} \varphi(t)dt\big)$ $\forall x, y \in X$, *where* $(\varphi,\psi) \in \Phi \times \Psi$ *and*

$$C^1_{F,G,S,T(x,y)} = \max\Big\{p(Fx,Sy), p(Fx,Gx), p(Gy,Ty), \frac{1}{2}\big[p(Fx,Ty)+p(Sy,Gx)\big] \frac{p^*(Sy,Ty)p(Fx,Gx)}{1+p(Gx,Ty)}, \frac{p^*(Fx,Sy)p(Fx,Ty)}{1+p(Gx,Fx)}\Big\}.$$

If the pairs (F,G) and (S,T) are weakly compatible, then F, G, S and T have a unique common fixed point in X.

Proof. By (CLR_{GT}) property for (F,G) and (S,T), there exist two sequences x_n and y_n in X such that

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gx_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Ty_n = z, \tag{2.1}$$

for some $z \in G(X) \cap T(X)$.
Since $z \in G(X)$, then there exists a point $u \in X$ such that $Gu = z$. Now we claim that $Fu = Gu$. To prove the claim, let $Fu \neq Gu$. By putting $x = u$ and $y = y_n$ in condition (2) of above theorem, we have

$$\int_0^{p(Fu,Sy_n)} \varphi(t)dt \leq \psi\big(\int_0^{C^1_{F,G,S,T(u,y_n)}} \varphi(t)dt\big), \tag{2.2}$$

We have

$$C^1_{F,G,S,T(u,y_n)} = \max\Big\{p(Fu,Sy_n), p(Fu,Gu), p(Gy_n,Ty_n), \frac{1}{2}\big[p(Fu,Ty_n) + p(Sy_n,Gu)\big] \frac{p^*(Sy_n,Ty_n)p(Fu,Gu)}{1+p(Gu,Ty_n)}, \frac{p^*(Fu,Sy_n)p(Fu,Ty_n)}{1+p(Gu,Fu)}\Big\},$$

taking limit $n \to \infty$

$$\begin{aligned}\lim_{n\to\infty} C^1_{F,G,S,T(u,y_n)} &= \max\Big\{p(Fu,z), p(Fu,z), p(z,z), \frac{1}{2}\big[p(Fu,z)+p(z,z)\big] \\ &\qquad \frac{p^*(z,z)p(Fu,z)}{1+p(z,z)}, \frac{p^*(Fu,z)p(Fu,z)}{1+p(z,Fu)}\Big\} \\ &= \max\Big\{p(Fu,z), p(Fu,z), p(z,z), \frac{1}{2}[p(Fu,z)+p(z,z)] \\ &\qquad 0, \frac{p^*(Fu,z)p(Fu,z)}{1+p(z,Fu)}\Big\} \\ &= \quad p(Fu,z),\end{aligned} \tag{2.3}$$

because

$$p(Gu, Ty_n) = p(z, Ty_n) \to p(z, z)$$

$$p(Gu, Fu) = p(z, Fu)$$

$$p(Sy_n, Gu) = p(Sy_n, z) \to p(z, z)$$

$$p^*(ty_n, Sy_n) \to p(z, z) = 0$$

$$p^*(Fu, Ty_n) \to p^*(Fu, z) \leq p(Fz, z)$$

$$p^*(Fu, Sy_n) \to p^*(Fu, z) \leq p(Fz, z),$$

from equation (2.1)
$p^*(Fu, Ty_n) \to p^*(Fu, z) = p(Fu, z) - \min\{p(z, z), p(Fu, Fu)\}$. If $p(z, z) < p\ (Fu, Fu)$ then $p^*(Fu, z) = p(Fu, z) - p(z, z)$

$$\frac{(p(Fu, z) - p(z, z))p(Fu, z)}{1 + p(z, Fu)} \leq p(Fu, z)$$

and if $p(z, z) > p(Fu, Fu)$

$$\frac{(p(Fu, z) - p(Fu, Fu))p(Fu, z)}{1 + p(z, Fu)} \leq p(Fu, z).$$

So

$$\begin{aligned}
\int_0^{p(Fu,z)} \varphi(t)dt &= \limsup_{n\to\infty} \int_0^{p(Fu,Sy_n)} \varphi(t)dt \\
&\leq \limsup_{n\to\infty} \psi\Big(\int_0^{C^1_{F,G,S,T(u,y_n)}} \varphi(t)dt\Big) \\
&\leq \psi\Big(\limsup_{n\to\infty} \int_0^{C^1_{F,G,S,T(u,y_n)}} \varphi(t)dt\Big) \\
&= \psi\Big(\int_0^{p(Fu,z)} \varphi(t)dt\Big) \\
&< \int_0^{p(Fu,z)} \varphi(t)dt,
\end{aligned}$$

which is contradiction thus $Fu = Gu$ and hence

$$Fu = Gu = z. \tag{2.4}$$

Similarly we can show that $Sv = Tv$ and hence

$$Sv = Tv = z. \tag{2.5}$$

From (2.4) and (2.5),

$$Fu = Gu = z = Sv = Tv. \tag{2.6}$$

Now we have to show that z is a common fixed point of F, G, S and T. Since the pair (F, G) and (S, T) are weakly compatible, by using (2.6) we have

$$Fu = Gu \Rightarrow GFu = FGu \Rightarrow Fz = Gz, \tag{2.7}$$

$$Sv = Tv \Rightarrow TSv = STv \Rightarrow Sz = Tz, \tag{2.8}$$

Next we have to show that $Fz = z$. For this suppose $Fz \neq z$ using condition (2) in theorem 2.1, putting $x = z$ and $y = v$, we have

$$\int_0^{p(Fz,Sv)} \varphi(t)dt \leq \psi\Big(\int_0^{C^1_{F,G,S,T(z,v)}} \varphi(t)dt\Big).$$

By (2.6) and (2.7), we have

$$\begin{aligned} C^1_{F,G,S,T(z,v)} =& \max\Big\{p(Fz, z), p(Fz, z), p(z, z), \frac{1}{2}\big[p(Fz, z) + p(z, z)\big] \\ & \frac{p^*(z, z)p(Fz, z)}{1 + p(z, z)}, \frac{p^*(Fz, z)p(Fz, z)}{1 + p(z, Fz)}\Big\} \\ =& \ p(Fz, z), \end{aligned}$$

and $\int_0^{p(Fz,z)} \varphi(t)dt \leq \psi\big(\int_0^{p(Fz,z)} \varphi(t)dt\big) < \int_0^{p(Fz,z)} \varphi(t)dt$, which is contradiction. Thus $Fz = z$ and from (2.7), we can write

$$Fz = Gz = z. \tag{2.9}$$

Similarly, let $x = u$ and $y = z$ in condition (2) of theorem 2.1,

$$Sz = Tz = z. \tag{2.10}$$

Hence By (2.9) and (2.10), we get

$$Fz = Gz = Sz = Tz = z. \tag{2.11}$$

Shows that, z is a common fixed point of F, G, S and T. For uniqueness we assume that z_1 and z_2 are two distinct common fixed points of F, G, S and T. Then replacing x by z_1 and y by z_2 in condition (2) of above theorem, we have

$$\int_0^{p(z_1,z_2)} \varphi(t)dt = \int_0^{p(Fz_1,Sz_2)} \varphi(t)dt \le \psi\Big(\int_0^{C^1_{F,G,S,T(z_1,z_2)}} \varphi(t)dt\Big).$$

Since $C^1_{F,G,S,T(z_1,z_2)} = p(z_1,z_2)$ So

$$\int_0^{p(z_1,z_2)} \varphi(t)dt \le \psi\Big(\int_0^{p(z_1,z_2)} \varphi(t)dt\Big) < \int_0^{p(z_1,z_2)} \varphi(t)dt$$

Which is a contradiction and thus $z_1 = z_2$. Hence F, G, S and T have a unique common fixed point in X.

By the help of the theorem 2.1, we easily deduce the following corollaries.

Corollary 2.2. *Let (X,p) be a partial metric space and F, G and T be three self-mappings on X satisfying in the following conditions:*
(1) The pair (F,G) and (F,T) share (CLR_{GT}) property;
(2)$\int_0^{p(Fx,Fy)} \varphi(t)dt \le \psi\big(\int_0^{C^1_{F,G,F,T(x,y)}} \varphi(t)dt\big)$ $\forall x,y \in X$, where $(\varphi,\psi) \in \Phi\times\Psi$ and

$$C^1_{F,G,F,T(x,y)} = \max\Big\{p(Fx,Fy), p(Fx,Gx), p(Gy,Ty), \frac{1}{2}[p(Fx,Ty)+p(Fy,Gx)] \\ \frac{p^*(Fy,Ty)p(Fx,Gx)}{1+p(Gx,Ty)}, \frac{p^*(Fx,Fy)p(Fx,Ty)}{1+p(Gx,Fx)}\Big\}.$$

If the pairs (F,G) and (F,T) are weakly compatible, then F, G and T have a unique common fixed point in X.

Corollary 2.3. *Let (X,p) be a partial metric space and F and T be two self-mappings on X satisfying in the following conditions:*
(1) The pair (F,T) share (CLR_T) property;
(2)$\int_0^{p(Fx,Fy)} \varphi(t)dt \le \psi\big(\int_0^{C^1_{F,T,F,T(x,y)}} \varphi(t)dt\big)$ $\forall x,y \in X$, where $(\varphi,\psi) \in \Phi\times\Psi$ and

$$C^1_{F,T,F,T(x,y)} = \max\Big\{p(Fx,Fy), p(Fx,Tx), p(Ty,Ty), \frac{1}{2}[p(Fx,Ty)+p(Fy,Tx)] \\ \frac{p^*(Fy,Ty)p(Fx,Tx)}{1+p(Tx,Ty)}, \frac{p^*(Fx,Fy)p(Fx,Ty)}{1+p(Tx,Fx)}\Big\}.$$

If the pair (F,T) are weakly compatible, then F and T have a unique common fixed point in X.

In a similar way we establish the following result.

Theorem 2.4. *Let (X,p) be a partial metric space and F, G, S and T be four self-mappings on X satisfying in the following conditions:*
(1) The pair (F,G) and (S,T) share (CLR_{GT}) property;
(2) $\int_0^{p(Fx,Sy)} \varphi(t)dt \le \psi\big(\int_0^{C^2_{F,G,S,T(x,y)}} \varphi(t)dt\big)$ $\forall x,y \in X$, *where* $(\varphi,\psi) \in \Phi \times \Psi$ *and*

$$C^2_{F,G,S,T(x,y)} = \max\Big\{p(Fx,Sy), p(Fx,Gx), p(Gy,Ty), \frac{1}{2}[p(Fx,Ty)+p(Sy,Gx)] \frac{p^*(Sy,Ty)p(Fx,Gx)}{1+p(Gx,Ty)}, \frac{p^*(Fx,Sy)p(Fx,Ty)}{1+p(Gx,Fx)}\Big\}.$$

If the pairs (F,G) and (S,T) are weakly compatible, then F, G, S and T have a unique common fixed point in X.

In general CLR_{GT} property implies the common property (E.A) but the converse is not true. So replacing CLR_{GT} property by common property (E.A) in Theorem 2.1 and Theorem 2.4, we get the following results, because the (E.A) property together with the closedness property of a suitable subspace gives rise to the closed range property.

Corollary 2.5. *Let (X,p) be a partial metric space and F, G, S and T be four self-mappings on X satisfying in the following conditions:*
(1) The pair (F,G) and (S,T) share $(E.A)$ property such that $T(X)$ (or $G(X)$) is closed subspace of X;
(2) $\int_0^{p(Fx,Sy)} \varphi(t)dt \le \psi\big(\int_0^{C^1_{F,G,S,T(x,y)}} \varphi(t)dt\big)$ $\forall x,y \in X$, *where* $(\varphi,\psi) \in \Phi \times \Psi$.
If the pairs (F,G) and (S,T) are weakly compatible, then F, G, S and T have a unique common fixed point in X.

Corollary 2.6. *Let (X,p) be a partial metric space and F, G, S and T be four self-mappings on X satisfying in the following conditions:*
(1) The pair (F,G) and (S,T) share $(E.A)$ property such that $T(X)$ (or $G(X)$) is closed subspace of X;
(2) $\int_0^{p(Fx,Sy)} \varphi(t)dt \le \psi\big(\int_0^{C^2_{F,G,S,T(x,y)}} \varphi(t)dt\big)$ $\forall x,y \in X$, *where* $(\varphi,\psi) \in \Phi \times \Psi$ *and*

$$C^2_{F,G,S,T(x,y)} = \max\Big\{p(Fx,Sy), p(Fx,Gx), p(Gy,Ty), \frac{1}{2}[p(Fx,Ty)+p(Sy,Gx)] \frac{p^*(Sy,Ty)p(Fx,Gx)}{1+p(Gx,Ty)}, \frac{p^*(Fx,Sy)p(Fx,Ty)}{1+p(Gx,Fx)}\Big\}.$$

If the pairs (F,G) and (S,T) are weakly compatible, then F, G, S and T have a unique common fixed point in X.

One can obtained other consequences from theorem 2.4 and corollaries 2.5 and 2.6 in a similar way as obtained from theorem 2.1.
If we replace $C^1_{F,G,S,T(x,y)}$ by $C^3_{F,G,S,T(x,y)}$, then Theorem 2.1 and ??? are still valid. Similarly, theorem 2.4 and Corollary 2.6 are still valid, if we replace $C^2_{F,G,S,T(x,y)}$ by $C^4_{F,G,S,T(x,y)}$. i. e.

$$C^3_{F,G,S,T(x,y)} = \max\Big\{p(Fx,Sy), p(Fx,Gx), p(Gy,Ty), \frac{1}{2}[p(Fx,Ty)+p(Sy,Gx)] \\ \frac{p^*(Sy,Ty)p(Fx,Gx)}{1+p(Gx,Ty)}, \frac{p^*(Fx,Sy)p(Fx,Ty)}{1+p(Gx,Fx)}\Big\},$$

and

$$C^4_{F,G,S,T(x,y)} = \max\Big\{p(Fx,Sy), p(Fx,Gx), p(Gy,Ty), \frac{1}{2}[p(Fx,Ty)+p(Sy,Gx)] \\ \frac{p^*(Sy,Ty)p(Fx,Gx)}{1+p(Gx,Ty)}, \frac{p^*(Fx,Sy)p(Fx,Ty)}{1+p(Gx,Fx)}\Big\}.$$

3. Example

In this section, we apply our main result to prove the following:

Example 3.1. Suppose $X = \mathbb{R}^+$ and $p(x,y) = \max\{x,y\}$; then (X, p) is a PMS. Define four self mappings F, S, T and G on X by

$$F(x) = \frac{x^2}{2} + \frac{1}{2}, G(x) = x^2, S(x) = x, T(x) = \frac{2}{x^2+1}.$$

Let $x_n = \{1 + \frac{1}{n}\}_{n\in N}$ and $y_n = \{\frac{n}{n+1}\}_{n\in N}$ be two sequences, so we have

$$\lim n \to \infty F(x_n) = \lim n \to \infty G(x_n) = \lim n \to \infty S(y_n) = \lim n \to \infty T(y_n) = 1.$$

Also $1 \in T(X) \cap G(X) = (0,2] \cap \mathbb{R}^+$,

Hence (F,G) and (S,T) satisfy CLR_{GT} property. It is easy to check that the pair (F,G) and (F,G) is weakly compatible at $x = 1$ as a coincidence point. To verify condition (2) of theorem 2.1, let us define $\phi, \psi : \mathbb{R}^+ \to \mathbb{R}^+$ by $\phi(t) = t$ and $\psi(t) = \frac{t^3}{3}$.
So

$$F(2) = \frac{5}{2}, G(2) = 4, S(\frac{1}{2}) = \frac{1}{2}, T(\frac{1}{2}) = \frac{8}{5}, \text{and } \ C_1(\frac{5}{2}, \frac{1}{2}) = 4.$$

Thus we obtain

$$\int_0^{p(F(2),S(\frac{1}{2}))} \phi(t)dt = \int_0^2 tdt = 2.$$

On the other hand

$$\psi\Big(\int_0^{C_1(2,\frac{1}{2})}\varphi(t)dt\Big)=\psi\Big(\int_0^4 tdt\Big)=\psi(8)=\frac{512}{3}.$$

Hence

$$\int_0^{p(F(2),S(\frac{1}{2}))}\phi(t)dt\leq\psi\Big(\int_0^{C_1(2,\frac{1}{2})}\varphi(t)dt\Big).$$

Thus all the conditions of Theorem 2.1, so we get 0 as common fixed point of have mappings F, G, S and T.

Remark 3.2. *Replacing the partial metric p in (X,p) by metric d we can get the similar results given in [19].*

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