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ON SEPARATION AXIOMS ($T_i, i = 0, 1, 2$) VIA FUZZY GPRW - OPEN SETS

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Abstract: In this paper we have introduced fuzzy gprw- closure, fuzzy gprw-interior and separation axioms via fuzzy gprw-open sets. Also we found out the relationship between fuzzy separation axioms, fuzzy gprw separation axioms and fuzzy pre separation axioms.

Keywords and Phrases: Fuzzy gprw- closure, $Fgprw - T_0$ spaces, $Fgprw - T_1$ spaces, $Fgprw - T_2$ spaces.

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1. Introduction

Soon after the introduction of fuzzy set theory by Lotfi A. Zadeh [8] in 1965, generalization of classical set theory starts taking place. Many fuzzy sets were introduced, studied and their properties were established in a timely manner. In the same framework fuzzy separation axioms were introduced and studied by M. H. Ghanim et. al in 1984 [1]. Similarly fuzzy pre separation axioms were introduced and many of their properties were established by M. K. Singal et. al in 1991 [7]. In 2011 Seok Jong Lee and Sang Min Yun introduced and studied fuzzy delta separation axioms [5] based on fuzzy δ-open sets. They investigated the relationship between fuzzy separation axioms and fuzzy δ-separation axioms and showed fuzzy δ-separation axioms are hereditary in fuzzy regular open subspaces. In 2018 Gayatri Paul et. al studied and introduced separation axioms ($T_i, i = 0, 1, 2$) in the light of fuzzy γ^*-open set [6] via quasi-coincidence, quasi-neighborhood and also established

relation between fuzzy separation axioms, fuzzy pre-separation axioms and fuzzy γ^*-separation axioms.
In this paper we have introduced fuzzy separation axioms via fuzzy gprw-open sets and find out there relation with fuzzy separation axioms and fuzzy pre separation axioms introduced earlier. We find out that every FT_i space [1] is $FgprwT_i$ space for $i = 0, 1, 2$ and every FPT_i space [7] is $FgprwT_i$ space for $i = 0, 1, 2$. But the converse is not true for both the cases, which we proved by counter examples.

2. Preliminaries

Throughout the paper (Y, τ) always mean fuzzy topological space on which no separation axioms are mentioned unless otherwise explicitly stated. A fuzzy set in topological space (X, τ) is called a *fuzzy point* iff it takes the value 0 for all $y \in X$ except one, say $x \in X$. If its value at x is λ $(0 < \lambda \leq 1)$ we denote this fuzzy point by x_λ, where the point x is called its *support* see [2]. From the previous literature, following definitions and remarks play a key role in establishing the main work of this paper.

Definition 2.1. [2]*A fuzzy set on X is called a fuzzy singleton if it takes the value zero (0) for all points x in X except one point. The point at which a fuzzy singleton takes the non-zero value is called the support and the corresponding element of* $(0, 1]$ *its value. A fuzzy singleton with value 1 is called a fuzzy crisp singleton.*

Definition 2.2. [1] *A fuzzy topological space is said to be FT_0 iff for every pair of fuzzy singletons P_1 and P_2 with different supports, there exists an open fuzzy set O such that $p_1 \leq O \leq cop_2$ or $p_2 \leq O \leq cop_1$.*

Definition 2.3. [1] *A fuzzy topological space is said to be FT_1 iff for every pair of fuzzy singletons p_1 and p_2 with different supports, there exists open fuzzy sets O_1 and O_2 such that $p_1 \leq O_1 \leq cop_2$ and $p_2 \leq O_2 \leq cop_1$.*

Definition 2.4. [1] *A fuzzy topological space is said to be FT_2 (F-Hausdorff) iff for every pair of fuzzy singletons p_1 and p_2 with different supports, there exists open fuzzy sets O_1 and O_2 such that $p_1 \leq O_1 \leq cop_2$, $p_2 \leq O_2 \leq cop_1$ and $O_1 \leq coO_2$.*

Definition 2.5. [7] *A fuzzy topological space is said to be fuzzy pre-T_0 or in short FPT_0 if for every pair of fuzzy singletons p_1 and p_2 with different supports, there exists a fuzzy pre-open set u such that either $p_1 \leq u \leq cop_2$ or $p_2 \leq u \leq cop_1$.*

Definition 2.6. [7] *A fuzzy topological space (X, τ) is said to be fuzzy pre-T_1 or in short FPT_1 if for every pair of fuzzy singletons p_1 and p_2 with different supports x_1 and x_2, ($x_1 \neq x_2$), there exists fuzzy pre-open sets u and v such that $p_1 \leq u \leq cop_2$ and $p_2 \leq v \leq cop_1$.*

Definition 2.7. [7] *A fuzzy topological space is said to be fuzzy pre-Hausdorff or in short FPT_2 iff for every pair of fuzzy singletons p_1 and p_2 with different supports, there exists two fuzzy pre-open sets u and v such that $p_1 \leq u \leq cop_2$, $p_2 \leq v \leq cop_1$ and $u \leq cov$.*

Definition 2.8. [3] *Suppose (Y, τ) is a fuzzy topological space. Then a subset λ of (Y, τ) is called fuzzy generalized pre regular weakly closed (briefly fuzzy gprw-closed) if pcl(λ) $\leq \mu$ whenever $\lambda \leq \mu$ and μ is a fuzzy regular semi open set in (Y, τ) . Complement of fuzzy generalized pre regular weakly closed set is called fuzzy generalized pre regular weakly open (briefly fuzzy gprw-open).*

Remark 2.9. [3] *Suppose (Y, τ) is a fuzzy topological space and $\lambda \leq Y$. Then we call λ fuzzy gprw-open if $(1 - \lambda)$ is fuzzy gprw closed in (Y, τ).*

Remark 2.10. [3] *In fuzzy topological space (Y, τ) every fuzzy pre-closed set is fuzzy gprw-closed.*

Remark 2.11. [3] *In fuzzy topological space (Y, τ), every fuzzy open set is fuzzy gprw-open.*

3. Fuzzy gprw-closure

Definition 3.1. *Suppose (Y, τ) is a fuzzy topological space and $\alpha \leq Y$. Then fuzzy gprw-closure (briefly Fgprw-cl) and fuzzy gprw-interior (briefly Fgprw-int) of α are respectively defined as,*

$$\text{fuzzy gprw-cl}(\alpha) = \wedge \{\mu : \alpha \leq \mu, \mu \text{ is fuzzy gprw-closed set in } Y\}$$
$$\text{fuzzy gprw-int}(\alpha) = \vee \{\mu : \alpha \geq \mu, \mu \text{ is fuzzy gprw-open set in } Y\}$$

Theorem 3.2. *Suppose λ is a fuzzy set in fuzzy space (Y, τ). Then $gprw - cl(1 - \lambda) = 1 - (gprw - int(\lambda))$ and $gprw - int(1 - \lambda) = 1 - (gprw - cl(\lambda))$.*

Proof. From *Remark 2.9*, a fuzzy gprw-open set $p \leq \lambda$ is the complement of fuzzy gprw-closed set $q \geq 1 - \lambda$. So

gprw-int(λ) =$\vee${1-q:q is fuzzy gprw closed and q $\geq$ 1-λ}
gprw-int(λ) =1-$\wedge${q:q is fuzzy gprw closed and $q \geq 1 - \lambda$}
gprw-int(λ)=1-gprw-cl(1-λ)
$\Longrightarrow$ gprw-cl(1-λ) = 1- gprw-int(λ)

Now, suppose r is a fuzzy gprw-open set so for fuzzy gprw-closed set s$\geq\lambda$, $r = 1 - s \leq 1 - \lambda$

gprw-cl(λ) = $\wedge${1-r:r is fuzzy gprw-open and r $\leq$ 1-λ }
gprw-cl(λ)=1-$\vee${ r:r is fuzzy gprw-open and r$\leq$1-λ}

gprw-cl(λ)=1-gprw-int(1-λ)
$\implies$ gprw-int(1-λ)= 1-gprw-cl(λ)

Theorem 3.3. *Suppose* (Y, τ) *is a fuzzy topological space and* α, μ *are fuzzy subsets of* Y. *Then*
(a) *fuzzy* $gprw - cl(1_Y) = 1_Y$ *and fuzzy* $gprw - cl(0_Y) = 0_Y$
(b) $\alpha \leq$ *fuzzy gprw-cl*(α).
(c) *suppose* $\mu \leq \alpha$ *where* α *is fuzzy gprw-closed set. Then fuzzy gprw-cl*(μ)$\leq\alpha$.
(d) *If* $\alpha \leq \mu$ *then fuzzy* $gprw - cl(\alpha) \leq$ *fuzzy* $gprw - cl(\mu)$.

Proof. (a) Since fuzzy $gprw - cl(1_Y)$ is the intersection i.e. minimum of all fuzzy gprw-closed sets in Y containing 1_Y and since 1_Y is the minimum fuzzy gprw-closed set containing 1_Y . So fuzzy $gprw - cl(1_Y) = 1_Y$ .Now fuzzy $gprw - cl(0_Y)$ is the intersection i.e. minimum of all fuzzy gprw-closed sets in Y containing 0_Y and since 0_Y is the minimum fuzzy gprw-closed set containing 0_Y, implying fuzzy $gprw - cl(0_Y) = 0_Y$.

Proof. (b) As fuzzy $gprw - cl(\alpha)$ is the intersection of all fuzzy gprw-closed sets containing α. So $\alpha \leq fuzzy\ gprw - cl(\alpha)$ is obvious.

Proof. (c) Suppose $\mu \leq \alpha$, where α is fuzzy gprw-closed set. Now,

$$\textit{fuzzy gprw-cl}(\mu) = \wedge \{\pi : \mu \leq \pi, \pi \textit{ is fuzzy gprw-closed set in } Y\}$$

I.e. fuzzy $gprw - cl(\mu)$ is contained in all fuzzy gprw-closed sets, so in particular fuzzy $gprw - cl(\mu) \leq \alpha$.

Proof. (d) Suppose $\alpha \leq \mu$, also

$$\textit{fuzzy gprw-cl}(\mu) = \wedge \{\pi : \mu \leq \pi, \pi \textit{ is fuzzy gprw-closed set in } Y\} \quad \rightarrow (d.1)$$

Now if$\mu \leq \pi$, where π is fuzzy gprw-closed in Y, then by (c) of this theorem, fuzzy $gprw - cl(\mu) \leq \pi$. Now by (b) of this theorem $\mu \leq fuzzy\ gprw - cl(\mu)$ implies $\alpha \leq \mu \leq \pi$ where π is fuzzy gprw-closed. So fuzzy $gprw - cl(\alpha) \leq \pi$ (by (c) of this theorem). Therefore

$$\textit{fuzzy gprw-cl}(\alpha) \leq \wedge \{\pi : \mu \leq \pi, \pi \textit{ is fuzzy gprw-closed set in } Y\}$$

$$\implies \qquad \textit{fuzzy gprw-cl}(\alpha) \leq \text{fuzzy} gprw - cl(\mu) \quad (using(d.1))$$

4. Separation Axioms via Fuzzy gprw-open Set

Definition 4.1. *A fuzzy topological space* (Y, τ) *is* $Fgprw - T_0$ *if for arbitrary fuzzy singletons* x^1_λ *and* x^2_μ , *their exists a fuzzy gprw-open set* Y *such that* $x^1_\lambda \leq Y \leq (1 - x^2_\mu)$ *or* $x^2_\mu \leq Y \leq (1 - x^1_\lambda)$.

Theorem 4.2. *A fuzzy topological space (Y, τ) is $Fgprw - T_0$ iff fuzzy-gprw closure of any two fuzzy crisp singletons with different supports is distinct.*

Proof. Suppose (Y, τ) is $Fgprw - T_0$ and x^1, x^2 are two fuzzy crisp singletons with different supports. Now (y, τ) being $Fgprw - T_0$ implies that $\exists$ a fuzzy-gprw open set Y such that $x^1 \leq Y \leq (1 - x^2)$, implying $x^2 \leq Fgprw - cl(x^2) \leq 1 - Y$. Since $x^1 \nleq 1 - Y$ so $x^1 \nleq Fgprw - cl(x^2)$, but $x^1 \leq Fgprw - cl(x^1)$ implies $Fgprw - cl(x^1) \neq Fgprw - cl(x^2)$. Conversely, suppose x^1 and x^2 be two fuzzy crisp singletons with different supports y_1 and y_2, respectively such that $x^1(y_1) = x^2(y_2) = 1$. Also let l_1 and l_2 be fuzzy singletons with different supports y_1 and y_2, so by hypothesis $1_Y - Fgprw - cl\{x^1\} \leq 1_Y - \{x^1\}$ and so $(1_Y - Fgprw - cl(x^1)) \leq (1_Y - \{l_1\})$. Now $(1_Y - Fgprw - cl(x^1))$ is a Fgprw-open set such that $l_2 \leq (1_Y - Fgprw - cl(x^1) \leq (1_Y - \{l_1\})$. Implying (Y, τ) is $Fgprw - T_0$.

Definition 4.3. *A fuzzy topological space (Y, τ) is $Fgprw - T_1$ if for arbitrary fuzzy singletons X^1_λ and x^2_μ, their exists fuzzy gprw open sets Y_1 & Y_2 such that $x^1_\lambda \leq Y_1 \leq (1 - x^2_\mu)$ and $x^2_\mu \leq Y_2 \leq (1 - x^1_\lambda)$.*
Obviously every $Fgprw - T_1$ space is a $Fgprw - T_0$ space.

Theorem 4.4. *A fuzzy topological space (Y, τ) is $Fgprw - T_1$ iff every fuzzy crisp singleton is fuzzy-gprw closed.*

Proof. Consider (Y, τ) is $Fgprw - T_1$ and l_1 is a fuzzy singleton with support y_1 such that $l_1(y_1) = 1$. So for any arbitrary fuzzy singleton l_2 with support $y_2 \neq y_1$, their exists fuzzy-gprw open sets α and β such that $l_1 \leq \alpha \leq 1_Y - l_2$ and $l_2 \leq \beta \leq 1_Y - l_1$. Now, as every fuzzy set can be written as the union of fuzzy singletons contained in it [4]. So $1_Y - l_1 = \vee_{l_2 \leq 1_Y - l_1} l_2$. From $1 - l_1(y_1) = 0$ it is clear that $1_Y - l_1 = \vee_{l_2 \leq 1_Y - l_1} \beta$, implying $1_Y - l_1$ is fuzzy-gprw open. Conversely suppose that l_1 and m_1 are fuzzy singletons with support y_1 such that $m_1(y_1) = 1$ and $l_1(y_1) \neq 1$ & l_2, m_2 are fuzzy singletons with support y_2 such that $m_2(y_2) = 1$ and $l_2(y_2) \neq 1$. Now the fuzzy sets $1_Y - m_1$ & $1_Y - m_2$ are fuzzy gprw open sets satisfying $l_1 \leq 1_Y - m_2 \leq 1_Y - l_2$ & $l_2 \leq 1_Y - m_1 \leq 1_Y - l_1$ implying (Y, τ) is $Fgprw - T_1$.

Definition 4.5. *A fuzzy topological space (Y, τ) is Fgprw-Hausdorff or $Fgprw - T_2$ if for arbitrary fuzzy singletons X^1_λ and x^2_μ, their exists fuzzy gprw-open sets Y_1 & Y_2 such that $x^1_\lambda \leq Y_1 \leq (1 - x^2_\mu)$, $x^2_\mu \leq Y_2 \leq (1 - x^1_\lambda)$ and $Y_1 \leq 1 - Y_2$.*
It is obvious that every $Fgprw - T_2$ space is $Fgprw - T_1$ space.

Definition 4.6. *A fuzzy topological space (Y, τ) is Fgprw-Uryshon or $Fgprw - T_{2\frac{1}{2}}$ if for arbitrary fuzzy singletons x^1_λ and x^2_μ, their exists fuzzy gprw-open sets Y_1 &*

Y_2 *such that* $x^1_\lambda \leq Y_1 \leq (1 - x^2_\mu)$, $x^2_\mu \leq Y_2 \leq (1 - x^1_\lambda)$ *and* $Fgprw - cl(Y_1) \leq 1 - (Fgprw - cl(Y_2))$.

Remark 4.7. *Every fuzzy pre-open set in fts* (Y, τ) *is a fuzzy gprw-open set in* (Y,τ).

Proof. Suppose α is a fuzzy pre-open set in (Y,τ), so $1 - \alpha$ is fuzzy pre-closed. Now by *Remark 2.10* every fuzzy pre-closed set is fuzzy gprw-closed, implying $1-\alpha$ is fuzzy gprw-closed & so α is a fuzzy gprw-open set in (Y, τ).

Theorem 4.8. *Every* FPT_0 *space is* $Fgprw - T_0$ *space.*

Proof. Suppose (Y, τ) is a FPT_0-space, so by [2] for fuzzy singletons l_1 & l_2 with supports y_1,y_2 ($y_1 \neq y_2$) their exists a fuzzy pre-open set ν such that $l_1 \leq \nu \leq 1_Y - l_2$ or $l_2 \leq \nu \leq 1_Y - l_1$. Now by Remark 4.7 ν is a fuzzy gprw-open set satisfying $l_1 \leq \nu \leq 1_Y - l_2$ or $l_2 \leq \nu \leq 1_Y - l_1$. Hence (Y,τ) is a $Fgprw - T_0$ space.

Remark 4.9. *The converse of the above theorem need not be true, for proof the following example is given.*

Example 4.10. If $Y = \{y_1, y_2, y_3, y_4\}$ is a space with fuzzy topology $\tau = \{0_Y, 1_Y, l, m, n, o\}$ where $l, m, n, o : Y \to [0, 1]$ are defined as

$$l(y) = \begin{cases} 1 & \text{if } y = y_1 \\ 0 & \text{otherwise} \end{cases}$$

$$m(y) = \begin{cases} 1 & \text{if } y = y_2 \\ 0 & \text{otherwise} \end{cases}$$

$$n(y) = \begin{cases} 1 & \text{if } y = y_1, y_2 \\ 0 & \text{otherwise} \end{cases}$$

$$o(y) = \begin{cases} 1 & \text{if } y = y_1, y_2, y_3 \\ 0 & \text{otherwise} \end{cases}$$

In this space Y with such kind of topology τ, the fuzzy set p defined below is Fgprw-open but not fuzzy pre-open, implying that the space (Y, τ) is $Fgprw - T_0$ but not FPT_0 .

$$p(y) = \begin{cases} 1 & \text{if } y = y_1, y_3, y_4 \\ 0 & \text{otherwise} \end{cases}$$

Theorem 4.11. *All* FPT_1 *spaces are* $Fgprw - T_1$ *spaces.*

Proof. Suppose (Y,τ) is a FPT_1 space, so by the definition of FPT_1 for arbitrary singletons l_1 and l_2 , $l_1 \leq \nu_1 \leq 1 - l_2$ & $l_2 \leq \nu_2 \leq 1 - l_1$ where ν_1 and ν_2 are fuzzy pre-open sets. Now by Remark 4.7 ν_1 and ν_2 are fuzzy gprw-open, concluding that (Y, τ) is a $Fgprw - T_1$ spaces.

Remark 4.12. *The converse of the above theorem may not be true as shown in the following example.*

Example 4.13. In the fuzzy topological space defined in Example 4.10, the fuzzy sets p & q defined below are Fgprw-open but not fuzzy pre-open, implying that the space (Y, τ) is $Fgprw - T_1$ but not FPT_1.

$$p(y) = \begin{cases} 1 & \text{if } y = y_1, y_3, y_4 \\ 0 & \text{otherwise} \end{cases}$$

$$q(y) = \begin{cases} 1 & \text{if } y = y_1, y_4 \\ 0 & \text{otherwise} \end{cases}$$

Theorem 4.14. *All FPT_2 spaces are $Fgprw - T_2$ spaces.*

Proof. From the definition of FPT_2 spaces in [7] and from Remark 4.7, the proof is obvious.

Remark 4.15. *The converse of the above theorem need not be true as shown in the given example.*

Example 4.16. In the fuzzy topological space defined in Example 4.10, the fuzzy sets r & s defined below are Fgprw-open but not fuzzy pre-open, implying that the space (Y, τ) is $Fgprw - T_2$ but not FPT_2.

$$r(y) = \begin{cases} 1 & \text{if } y = y_3 \\ 0 & \text{otherwise} \end{cases}$$

$$s(y) = \begin{cases} 1 & \text{if } y = y_4 \\ 0 & \text{otherwise} \end{cases}$$

Theorem 4.17. *Every FT_0 space is $Fgprw - T_0$ space.*

Proof. Suppose (Y, τ) is a FT_0 -space, so by [8] for fuzzy singletons l_1 & l_2 with different supports, their exists a fuzzy open set ν such that $l_1 \leq \nu \leq 1_Y - l_2$ or $l_2 \leq \nu \leq 1_Y - l_1$. Now from *Remark 2.11* every fuzzy open set is fuzzy gprw-open, implying that ν is a fuzzy gprw-open set satisfying $l_1 \leq \nu \leq 1_Y - l_2$ or $l_2 \leq \nu \leq 1_Y - l_1$. Hence (Y, τ) is a $Fgprw - T_0$.

Remark 4.18. *The converse of the above theorem need not be true as shown in the* *following example.*

Example 4.19. If $Y = \{y_1, y_2, y_3, y_4, y_5\}$ is a space with fuzzy topology $\tau = \{0_Y, 1_Y, \lambda_1, \lambda_2, \lambda_3\}$ where $\lambda_1, \lambda_2, \lambda_3 : Y \to [0, 1]$ are defined as

$$\lambda_1(y) = \begin{cases} 1 & \text{if } y = y_1, y_2 \\ 0 & \text{otherwise} \end{cases}$$

$$\lambda_2(y) = \begin{cases} 1 & \text{if } y = y_3, y_4 \\ 0 & \text{otherwise} \end{cases}$$

$$\lambda_3(y) = \begin{cases} 1 & \text{if } y = y_1, y_2, y_3, y_4 \\ 0 & \text{otherwise} \end{cases}$$

In this fuzzy topological space, the fuzzy set λ_4 defined below is a Fuzzy gprw-open set but not fuzzy open, implying that the space (Y, τ) is $Fgprw - T_0$ but not FT_0.

$$\lambda_4(y) = \begin{cases} 1 & \text{if } y = y_1, y_2, y_4, y_5 \\ 0 & \text{otherwise} \end{cases}$$

Theorem 4.20. *Every FT_1 space is $Fgprw - T_1$ space.*

Proof. The proof is trivial from the definitions of FT_1 and $Fgprw - T_1$ spaces and from the result that every fuzzy open set is fuzzy gprw-open [3].

Remark 4.21. *The converse that every $Fgprw - T_1$ space is a FT_1 space is not true, for proof the following example is given.*

Example 4.22. In fuzzy topological space (Y, τ) defined in Example 4.19, the fuzzy sets λ_4 and λ_5 defined below are Fuzzy gprw-open sets but not fuzzy open sets, implying the fuzzy space (Y, τ) is a $Fgprw - T_1$ space but not a FT_1 .

$$\lambda_4(y) = \begin{cases} 1 & \text{if } y = y_1, y_2, y_4, y_5 \\ 0 & \text{otherwise} \end{cases}$$

$$\lambda_5(y) = \begin{cases} 1 & \text{if } y = y_2, y_3, y_4, y_5 \\ 0 & \text{otherwise} \end{cases}$$

Theorem 4.23. *Every FT_2 space is $Fgprw - T_2$ space.*

Proof. The proof is straightforward.

Remark 4.24. *The converse of the above theorem need not be true as shown in* *the following example.*

Example 4.25. In fuzzy topological space (Y, τ) defined in Example 4.19, the fuzzy sets λ_4 and λ_5 defined below are Fuzzy gprw-open sets but not fuzzy open sets, implying the fuzzy space (Y, τ) is a $Fgprw - T_2$ space but not a FT_2.

$$\lambda_4(y) = \begin{cases} 1 & \text{if } y = y_1, y_2, y_4, y_5 \\ 0 & \text{otherwise} \end{cases}$$

$$\lambda_5(y) = \begin{cases} 1 & \text{if } y = y_2, y_3, y_4, y_5 \\ 0 & \text{otherwise} \end{cases}$$

From the above discussion, the following diagram of implications is given

$$\begin{array}{ccccc} FT_0 & \Leftarrow & FT_1 & \Leftarrow & FT_2 \\ \Downarrow \not\Uparrow & & \Downarrow \not\Uparrow & & \Downarrow \not\Uparrow \\ Fgprw-T_0 & \Leftarrow & Fgprw-T_1 & \Leftarrow & Fgprw-T_2 \\ \not\Downarrow \Uparrow & & \not\Downarrow \Uparrow & & \not\Downarrow \Uparrow \\ FPT_0 & \Leftarrow & FPT_1 & \Leftarrow & FPT_2 \end{array}$$

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