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RADIATION ABSORPTION AND SORET EFFECTS ON MHD CONDUCTING FLUID FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE

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Abstract: In this article an investigation is done on hydromagnetic effects on electrically conducting fluid past an exponentially accelerated infinite vertical plate with exponentially varying temperature and concentration. The influence of thermal diffusion and radiation absorption is considered in this analysis. The problem is governed by coupled non-linear partial differential equations which are solved by finite difference method. The plate temperature is increasing linearly with time and the concentration level near the plate is increased. Among the effects of various physical parameters on the velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are broadly discussed with the help of graphs and table.

Keywords and Phrases: Accelerated vertical plate, Soret effect, finite difference method, radiation absorption and electrically conducting fluid.
2010 Mathematics Subject Classification: 35Q35, 65L12, 76W05, 76M55, 76D05.

1. Introduction

Free convection flows are of a great attention in a number of industrial applications like as fibre and granular insulation; geothermal systems etc. convection in porous media has application on geothermal energy recovery, oil extraction, thermal energy storage and flow throughout filtering devices. The occurrences of mass transfer are also very general in theory of stellar structure and remarkable erects are detectable, at least on the solar surface. The study of influence of magnetic field on free convection flow is main in liquid metal, electrolytes and ionized gases. Hayat et al. [8] investigated MHD flow and heat transfer over permeable stretching sheet with slip conditions. Cortell et al. [7] discussed MHD flow and mass transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet with chemically reactive species. Khan et al. [9] investigated MHD boundary layer flow of nanofluids containing gyro tactic microorganisms past a vertical plate with navier slip. Pal et al. [13] analysed hall current and MHD effects on heat transfer over an unsteady stretching permeable surface with thermal radiation. Prakash et al. [14] discussed diffusion- thermo and radiation effects on unsteady MHD flow through porous medium past an impulsively started infinite vertical plate with variable temperature and mass diffusion. Chandra Reddy et al. [1-3] analysed magnetohydrodynamic convective double diffusive laminar boundary layer flow past an accelerated vertical plate as well as Soret and Dufour effects on MHD free convection flow of Rivlin-Ericksen fluid past a semi-infinite vertical plate. Further Chandra Reddy et al. [4] studied the properties of free convective magneto-nanofluid flow past a moving vertical plate in the presence of radiation and thermal diffusion. Madhumohana Raju et al. [10-12] explored about the Casson fluid flow through rotating cone with non-linear convection and unsteady state with porous medium. Recently Chandra Reddy et al. [5, 6] examined the properties of MHD natural convective heat generating/ absorbing and buoyancy effects on chemically reactive magneto-nanofluid past a moving vertical plate. Motivated by the above studies we considered and analysed radiation absorption and Soret effects on MHD conducting fluid flow past an exponentially accelerated vertical plate with variable temperature and concentration.

2. Mathematical formulation

The unsteady MHD free convective fluid flow in the presence of thermal diffusion and radiation absorption with variable temperature and concentration has been considered. The physical model and coordinate system of the fluid flow is presented in Figure 1. The flow is assumed to be in x^*-direction which is taken along the vertical plate in the uphill direction. The y^*-axis is taken be normal to the plate. Initially, it is assumed that both the fluid and the plate are at rest and maintained at same temperature and concentration T^*_∞ and C^*_∞ respectively. At any time $t^* > 0$ the temperature and concentration of the plate $y^* = 0$ are raised to $T^*_\infty + (T^*_w - T^*_\infty)e^{a^*t^*}$ and $C^*_\infty + (C^*_w - C^*_\infty)e^{a^*t^*}$ with time t and thereafter remains constant and that of $y^* \to \infty$ is lowered to T^*_∞ and C^*_∞ respectively. A transverse magnetic field of uniform strength is assumed to be applied perpendicular to the plate. The induced magnetic field and viscous dissipation is assumed to be insignificant as the magnetic Reynolds number of the flow is taken to be very small. The polarization effects are assumed to be small and hence the electric field is also negligible. Based on the above assumption with usual Boussineq's approximation, the governing equations and related boundary conditions of the problem are given by

$$\frac{\partial u^*}{\partial t^*} = v\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta^*(C^* - C^*_\infty) + g\beta(T^* - T^*_\infty) - \frac{\sigma B_0^2}{\rho}u^* \tag{1}$$

$$\rho C_p\frac{\partial T^*}{\partial t^*} = k\frac{\partial^2 T^*}{\partial y^{*2}} + Q_1(C^* - C^*_\infty) \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} + D_1\frac{\partial^2 T^*}{\partial y^{*2}} \tag{3}$$

The boundary conditions related with the problem are $u^* = 0$, $C^* = C^*_\infty$, $T^* = T^*_\infty$ for all $y^*, t^* \leq 0$
$t^* > 0 : u^* = u_0e^{a^*t^*}$, $C^* = C^*_\infty + (C^*_w - C^*_\infty)e^{a^*t^*}$

$$T^* = T^*_\infty + (T^*_w - T^*_\infty)e^{a^*t^*} \quad \text{at} \quad y^* = 0 \tag{4}$$

$u^* \to 0$, $C^* \to C_\infty$, $T^* \to T_\infty$ as $y^* \to \infty$.
Where u^* is the velocity of the fluid in x^* directions, T^* is the temperature and C^* is the concentration of the fluid respectively, g is the acceleration due to gravity, C^*_∞ is the concentration in the fluid far away from the plate, C^*_w is the concentration of the plate, y^* is coordinate axis normal the plate, B_0 is the external magnetic field, Q is the radiation absorption parameter, C_p is specific heat at constant pressure,

T^*_∞ is the temperature of the fluid far away from the plate, T^*_w is the temperature of the plate, M is the magnetic parameter, is the acceleration parameter, D_1 is the thermal diffusivity, u_0 is the velocity of the plate, K Porous permeability, D is the chemical molecular diffusivity, K is the permeability parameter, ρ is the density, ν is kinematic viscosity and t^* is the corresponding time, β is the volumetric coefficient of thermal expansion and β^* is the volumetric coefficient of expansion with concentration respectively.

Here $A = \frac{u_0^2}{\nu}$, since the solutions of the governing equations under the boundary conditions will be based on the finite difference method so it is necessary to make the equation dimensionless. For this reason, now we introduce the following dimensionless quantities.

$$U = \frac{u^*}{u_0}, \quad y = \frac{y^* u_0}{\nu}, \quad \theta = \frac{T^* - T^*_\infty}{T^*_w - T^*_\infty}, \quad C = \frac{C^* - C^*_\infty}{C^*_w - C^*_\infty}, \quad Gm = \frac{\nu g \beta^* (C^*_w - C^*_\infty)}{u_0^3},$$

$$Gr = \frac{\nu g \beta (T^*_w - T^*_\infty)}{u_0^3}, \quad a = \frac{a^* \nu}{u_0^2}, \quad t\frac{t^* u_0^2}{\nu}, \quad P_r = \frac{\mu C_p}{k}, \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2},$$

$$Sc = \frac{\nu}{D}, \quad So = \frac{D_1}{\nu} \frac{(T^*_w - T^*_\infty)}{(C^*_w - C^*_\infty)}, \quad Q = \frac{\nu (C^*_w - C^*_\infty) Q_1}{u_0^2 (T^*_w - T^*_\infty) \rho C_p} \tag{5}$$

Then equation (1)-(3) and boundary conditions (4) leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GmC + \frac{\partial^2 U}{\partial y^2} - MU \tag{6}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + S_0 \frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + QC \tag{8}$$

With the initial and boundary conditions $t \leq 0: \ u = 0, \ \theta = 0, \ C = 0$ for all y

$$t > 0: \ u = e^{at}, \ \theta = e^{at}, \ C = e^{at} \text{ for all } y = 0 \tag{9}$$

3. Method of Solution

The governing equations of the problem contain a system of partial differential equations which are transformed by normal transformations into a non-dimensional system of non-linear coupled partial differential equations with initial and boundary conditions. Hence the solution of the problem would be based on advance numerical methods. The finite difference method formula is used for solving our obtained non-similar coupled partial differential equations. From the concept of the above discussion, for simplicity the explicit finite difference method has been used to solve from equations (6) to (8) subject to the boundary condition by (9). To obtain the difference equations the region of the flow is separated into a grid or mesh of lines horizontally and vertical lines are taken along the plate. Here, the suffix i refer to y and j to time. The mesh system is divided by taking $\Delta y = 0.1$. From the initial condition in (9), we have the following equivalent:

$$U(i,0) = 0,\ \theta(i,0) = 0\ C(i,0) = 0 \ \text{ for all } i \tag{10}$$

The boundary conditions from (9) are expressed in finite-difference form as follows

$$\left.\begin{array}{ll} U(0,j) = e^{a(j-1)\Delta t},\ \theta(0,j) = e^{a(j-1)\Delta t}, C(0,j) = e^{a(j-1)\Delta t} & \forall j \\ U(i_{\max},j) = 0,\ \theta(i_{\max},j) = 0,\ C(i_{\max},j) = 0 & \forall\ j \end{array}\right\} \tag{11}$$

(Here $i_{\max}$ was taken as 200).
Using the explicit finite difference approximation, the following appropriate set of finite difference equations are obtained as;

$$\theta_{i,j+1} - \theta_{i,j} = \frac{\Delta t}{Pr}\left(\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta y)^2}\right) + \Delta t Q C(i,j) \tag{13}$$

$$C_{i,j+1} - C_{i,j} = \frac{\Delta t}{Sc}\left(\frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j}}{(\Delta y)^2}\right) + \Delta t(S_0) \tag{14}$$

$$U_{i,j+1} - U_{i,j} = \Delta t Gr\theta(i,j) + \Delta t Gm C(i,j) + \Delta t\left(\frac{U_{i-1,j} - 2U_{i,j} + U_{i+1,j}}{(\Delta y)^2}\right) - \Delta t M U(i,j) \tag{15}$$

First $\theta(i, j+1)$ is computed from (13) and $C(i, j+1)$ is computed from (14). Then the velocity at the end of time step viz, $U(i, j+1)$ ($i = 1$ to 200) is computed from (15) in terms of velocity, temperature and concentration at points on the earlier time-step. The procedure is repeated until $t = 0.5$ (i.e. $j = 500$). During computation Δt was chosen as 0.001.

Also, the non-dimensional forms of skin friction (τ), heat transfer rate (Nu) and mass transfer rate (Sh) are given as follows:

$$\tau = \left(\frac{\partial U}{\partial y}\right)_{y=0} \quad Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} \quad Sh = \left(\frac{\partial C}{\partial y}\right)_{y=0}$$

4. Results and Discussion

Firstly, the validation of the results obtained by finite difference method has done by considering the published paper of Chandra Reddy et al. [3]. In order to get a physical perception into the research work, broad reckonings have been executed to analyse the effects of various persuading parameters on the dimensionless velocity (U), temperature (θ) and concentration (C) profiles and also on the Skin-friction (τ), Nusselt number (Nu) and Sherwood number (Sh). The effects of various physical parameters viz., the Schmidt number (Sc), Soret number (S_0), radiation absorption parameter (Q), Grashof number (Gr), the modified Grashof number (Gm), magnetic parameter (M), Prandtl number (Pr), Porous permeability (K) are exhibited in the figures 1-6 and the table 1.

The effect of radiation absorption parameter (Q) on velocity and temperature is plotted in Figs. 1-2. Increase in Q causes increase in u and θ. Also when it decreases (negative values) the velocity comes down. The increase in temperature is due to the rise of kinetic energy and thermal energy of the fluid. The momentum and thermal boundary layers get thinner in case of radiation absorbing fluids.

The influence of Prandtl number (Pr) on temperature profile is shown in Fig. 3. Rising values of Prandtl number causes fall in temperature. Fig. 4 is plotted to show the variation of Schmidt (Sc) number on concentration. It depreciates due to the enhancement of Schmidt number. Concentration effect is due to the dimensionless Sherwood number defined as the ratio of momentum diffusivity and mass diffusivity and simultaneous momentum and mass diffusion convection process.

As we expected the rising values of Soret number improves the concentration in the flow which is displayed in Fig. 5. Fig. 6 is portrayed to discuss the influence modified Grashof number (Gm) on velocity. The improving (descending) values of Gm lead to growth (decay) in the velocity profile. The modified Grashof number physically improves the distance between the molecules and hence the velocity grows.

We also extended the present work in analyzing the effects of special parameters like skin friction, Nusselt number and Sherwood number on radiation absorption parameter, Prandtl number, Schmidt number, Soret number, modified Grashof number and magnetic parameter with the help of tabular values.

Table 1 shows skin friction decreases with the increase of radiation absorption parameter, Soret number and modified Grashof number and it increases with the increase of magnetic parameter. Nusselt number fall down with the rise of radiation absorption parameter and grows up with the enhancement of Prandtl number, whereas Sherwood number escalates with the growth of Schmidt number and decelerates with the decreasing values of Soret number.

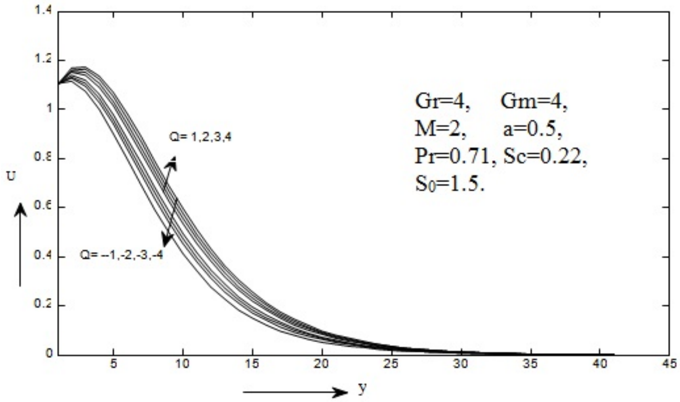

Fig. 1: Effect of radiation absorption on velocity

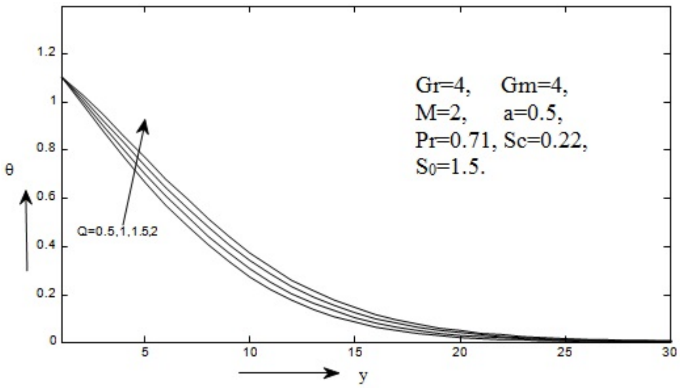

Fig. 2: Effect of radiation absorption on temperature

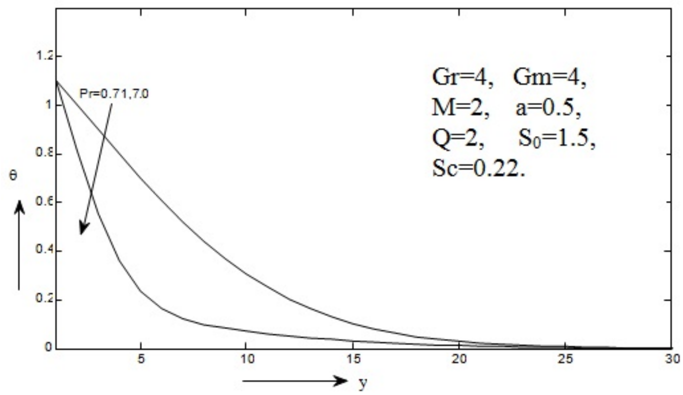

Fig. 3: Effect of Prandtl number on temperature

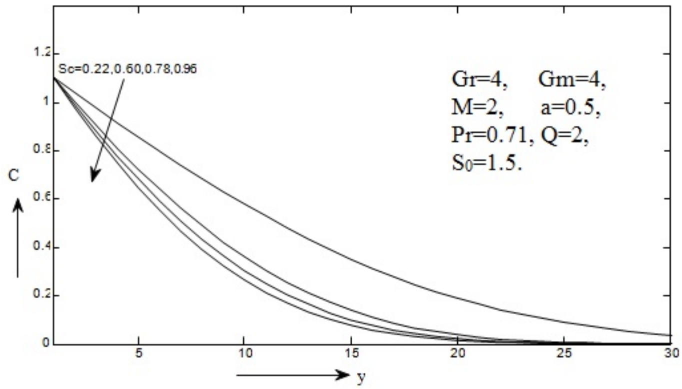

Fig. 4: Effect of Schmidt number on concentration

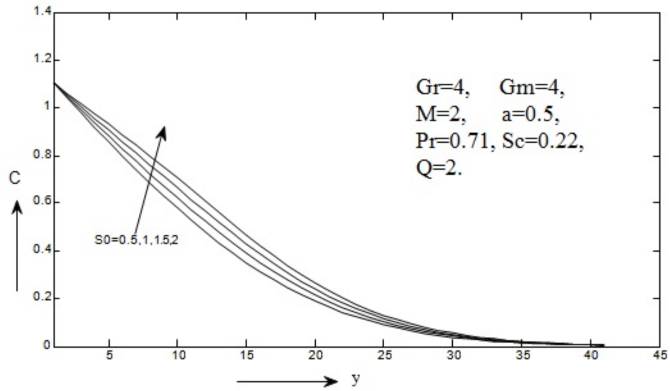

Fig. 5: Effect of Soret number on concentration

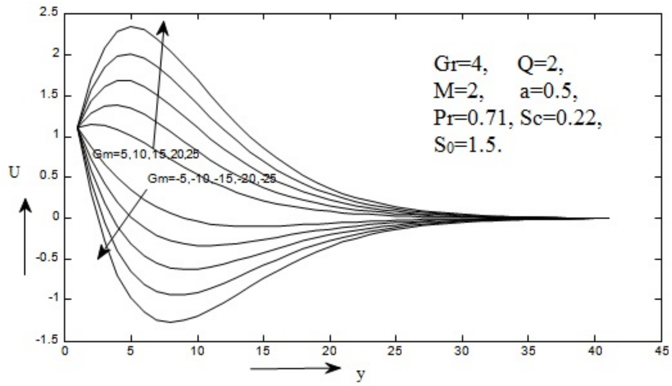

Fig. 6: Effect of modified Grashof number on velocity

Table: 1

Q	Pr	Sc	So	Gm	M	τ	Nu	Sh
1	0.71	0.22	0.5	0.4	5	**0.0866**	**0.1502**	1.1203
1.5	0.71	0.22	0.5	0.4	5	**0.0846**	**0.1408**	1.1203
2	0.71	0.22	0.5	0.4	5	**0.0826**	**0.1312**	1.1203
2.5	0.71	0.22	0.5	0.4	5	**0.0806**	**0.1136**	1.1203
1	**0.71**	0.22	0.5	0.4	5	0.0866	**0.1203**	1.1203
1	**2**	0.22	0.5	0.4	5	0.0866	**0.1304**	1.1203
1	**5**	0.22	0.5	0.4	5	0.0866	**0.1412**	1.1203
1	**7.1**	0.22	0.5	0.4	5	0.0866	**0.1535**	1.1203
1	0.71	**0.22**	0.5	0.4	5	0.0866	0.1502	**1.1203**
1	0.71	**0.60**	0.5	0.4	5	0.0866	0.1502	**1.1404**
1	0.71	**0.78**	0.5	0.4	5	0.0866	0.1502	**1.1912**
1	0.71	**0.96**	0.5	0.4	5	0.0866	0.1502	**1.2235**
1	0.71	0.22	**0.5**	0.4	5	**0.3042**	0.1502	**2.1242**
1	0.71	0.22	**1**	0.4	5	**0.2055**	0.1502	**2.1344**
1	0.71	0.22	**1.5**	0.4	5	**0.1429**	0.1502	**2.1466**
1	0.71	0.22	**2**	0.4	5	**0.0789**	0.1502	**2.1522**
1	0.71	0.22	0.5	**0.4**	5	**0.1083**	0.1502	1.1203
1	0.71	0.22	0.5	**0.6**	5	**0.1076**	0.1502	1.1203
1	0.71	0.22	0.5	**0.8**	5	**0.1073**	0.1502	1.1203
1	0.71	0.22	0.5	**1.2**	5	**0.1069**	0.1502	1.1203
1	0.71	0.22	0.5	0.4	**5**	**0.1080**	0.1502	1.1203
1	0.71	0.22	0.5	0.4	**10**	**0.1096**	0.1502	1.1203
1	0.71	0.22	0.5	0.4	**15**	**0.1109**	0.1502	1.1203
1	0.71	0.22	0.5	0.4	**20**	**0.1123**	0.1502	1.1203

5. Conclusion

The investigation on this research work is summarized as follows:

- Velocity is increasing with the increase of radiation absorption parameter and modified Grashof number.
- Temperature increases and decreases respectively with the increase of radiation absorption and Prandtl number.
- Concentration decreases and increases respectively with the increase of Schmidt number and Soret number.
- Skin friction decreases with the increase of radiation absorption parameter, Soret number and modified Grashof number and it increases with the increase of magnetic parameter.
- Nusselt number decreases with the increase of radiation absorption parameter, where as it increases with the increase of Prandtl number.

- Sherwood number increases with the increase of Schmidt number where as it decreases with the increase of Soret number.

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