South East Asian J. of Mathematics and Mathematical Sciences
Vol. 16, No. 3 (2020), pp. 181-198

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

EDGE DOMINATION IN INTUITIONISTIC FUZZY GRAPHS

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(**Received:** *Nov. 23, 2019* **Accepted:** *Apr. 10, 2020* **Published:** *Dec. 30, 2020)*

Abstract: In this paper we introduce the concepts of edge domination and total edge domination in product intuitionistic fuzzy graphs. We determine the edge domination number $\gamma'(G)$ and the total edge domination number $\gamma'_t(G)$ for several classes product intuitionistic fuzzy graphs and obtain bounds for the same. We also obtain Nordhaus - Gaddum type results for this parameter.

Keywords and Phrases: Fuzzy graph, intuitionistic fuzzy graphs, product intuitionistic fuzzy graph, edge domination number, total edge domination number and independent edge domination number.

2010 Mathematics Subject Classification: Primary: 05C72 , 05C69. Secondary: 03E72, 02F55.

1. Introduction

In 1736, Euler first introduced the concept of graph theory. In the history of mathematics, the solution given by Euler of the well known Konigsberg bridge problem is considered to be the rest theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science. In 1965 [22], Zadeh published his seminal paper on "Fuzzy sets" which described fuzzy set theory and, consequently, fuzzy logic. The purpose of Zadeh's paper was to develop a theory which could deal with ambiguity and imprecision of certain classes of sets in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction. In (1999) [1], Atanassov introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Atanassov added a new component(which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the non-membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics.

Rosenfeld (1975) [14] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles, connectedness and etc. Zadeh (1987) [23] introduced the concept of fuzzy relations. The concept of complement of fuzzy graph was investigated by Sunitha and Vijayakumar (2002) [19]. The concept of domination in fuzzy graphs was investigated by Somasundaram (1998) [16]. Ramaswamy and Poornima (2009) [15] introduced the concept of product fuzzy graphs. The first definition of intuitionistic fuzzy relations and intuitionistic fuzzy graphs were introduced by Atanassov (1999) [1], and further studied in (2009) [11]. R. Parvathi and M. G. Karunambigai (2006, 2009) [11, 12] introduced the concept of intuitionistic fuzzy graph and analyzed its components. Parvathi and Thamizhendhi (2010) [13] introduced the concept of domination number in intuitionistic fuzzy graphs. N. Vinoth Kumar and G. Geetha Ramani (2011) [21] introduced the concept of Product Intuitionistic Fuzzy Graph. Mahioub Shubatah (2012) [8, 9] introduced the concepts of domination in product fuzzy graphs and domination in product intuitionistic fuzzy graphs. S. Velammal (2012) [20] introduced the concept of edge domination in intuitionistic fuzzy graphs. In this paper, we introduce the concepts of edge domination and total domination in product intuitionistic fuzzy graphs.
For graph theoretic terminology and fuzzy graph theoretic we refer to Harary 1969

[5] and Mordeson 2000 [10], for domination and edge domination in intuitionistic fuzzy graphs we refer to [13, 20].

2. Preliminaries

In this section, we review briefly some definitions in intuitionistic fuzzy graphs and product intuitionistic fuzzy graphs and introduce some new notations.

A crisp graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G, called edges. The vertex set and the edge set of G are denoted by $V(G)$ and $E(G)$ respectively. A fuzzy graph $G = (\mu, \rho)$ is a set with two function $\mu : V \to [0,1]$ and $\rho : E \to [0,1]$ such that $\rho(\{x,y\}) \leq \mu(x) \wedge \mu(y)$ for all $x, y \in V$. We write $\rho(x,y)$ for $\rho(\{x,y\})$. The order p and size q of a fuzzy graph $G = (\mu, \rho)$ are defined to be $p = \sum_{x \in V} \mu(x)$ and $q = \sum_{xy \in E} \rho(x,y)$.

A mapping $A = (\mu_1, \rho_1) : X \longrightarrow [0,1] \times [0,1]$ is called an intuitionistic fuzzy set in X if $\mu_1(x) + \rho_1(x) \leq 1$ for all $x \in X$, where the mappings $\mu_1 : X \longrightarrow [0,1]$ and $\rho_1 : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_1(x)$) and the degree of nonmembership (namely) $\rho_1(x)$) of each element $x \in X$ to A, respectively.

An intuitionistic fuzzy graph G with underlying set V is defined to be a pair $G = (V; E)$, where (i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \to [0,1]$, $\rho_1 : V \to [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively and $0 \leq \mu_1(v_i) + \rho_1(v_i) \leq 1$, for every $v_i \in V$, $i = 1, 2, ..., n$.
(ii) $E \subseteq V \times V$ where $\mu_2 : E \to [0,1]$, $\rho_2 : E \to [0,1]$ denote the degree of membership and nonmembership of the element $(v_i, v_j) \in E$, respectively such that $\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$, $\rho_2(v_i, v_j) \leq \rho_1(v_i) \vee \rho_1(v_j)$ and $0 \leq \mu_2(v_i, v_j) + \rho_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.
Here the triple $(v_i, \mu_{1i}, \rho_{1i})$ denotes the degree of membership and degree of nonmembership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \rho_{2ij})$ denotes the degree of membership and degree of nonmembership of the edge relation $e_{ij} = (v_i, v_j)$ on V.
In an intuitionistic fuzzy graph G, when $\mu_{2ij} = \rho_{2ij} = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j.
An intuitionistic fuzzy graph, $G = (V, E)$ is said to be a $semi - \mu$ strong intuitionistic fuzzy graph if $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ for every i and j. An intuitionistic fuzzy graph, $G = (V, E)$ is said to be a $semi - \rho$ strong intuitionistic fuzzy graph if $\rho_{2ij} = max(\rho_{1i}, \rho_{1j})$ for every i and j. An intuitionistic fuzzy graph, $G = (V, E)$ is said to be strong intuitionistic fuzzy graph if $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ for every i and j and $\rho_{2ij} = max(\rho_{1i}, \rho_{1j})$ for all $(v_i, v_j) \in E$.
An intuitionistic fuzzy graph, $H = (V', E')$ is said to be an intuitionistic fuzzy subgraph of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. That is, $\mu'_{1i} \leq \mu_{1i}$; $\rho'_{1i} \geq \rho_{1i}$ and

$\mu'_{2ij} \leq \mu_{2ij}$; $\rho'_{2ij} \geq \rho_{2ij}$ for every $i, j = 1, 2, ..., n$.
Let $G = (V, E)$ be an intuitionistic fuzzy graph. Then the cardinality of G is defined to be
$|G| = |\sum_{v_i \in V} \frac{1+\mu_1(v_i)-\rho_1(v_i)}{2} + \sum_{(v_i,v_j)\in E} \frac{1+\mu_2(v_i,v_j)-\rho_2(v_i,v_j)}{2}|$.
Let $G = (V, E)$ be an intitionistic fuzzy graph. Then the vertex cardinality of G is defined by
$|V| = \sum_{v_i \in V} \frac{1+\mu_1(v_i)-\rho_1(v_i)}{2}$ for all $v_i \in V$ is called the order of an intitionistic fuzzy graph $G = (V, E)$ and is denoted by $p(G)$. The edge cardinality of an intitionistic fuzzy graph G is defined by
$|E| = \sum_{(v_i,v_j)\in E} \frac{1+\mu_2(v_i,v_j)-\rho_2(v_i,v_j)}{2}$ for all $(v_i, v_j) \in E$ is called the size of an intitionistic fuzzy graph, $G = (V, E)$ and is denoted by $q(G)$.

An edge $e = (x, y)$ of an intuitionistic fuzzy graph is called an effective edge if $\mu_2(x, y) = min\{\mu_1(x), \mu_1(y)\}$ and $\rho_2(x, y) = max\{\rho_1(x), \rho_1(y)\}$.
The degree of a vertex can be generalized in different ways for an intuitionistic fuzzy graph $G = (V, E)$. The effective degree of a vertex v in an intuitionistic fuzzy graph, $G = (V, E)$ is defined to be sum of the weights of the effective edges incident at v and it is denoted by $d_E(v)$. The minimum effective degree of G is $\delta_E(G) = min\{d_E(v)|v \in V\}$. The maximum effective degree of G is $\Delta_E(G) = max\{d_E(v)|v \in V\}$.
The effective degree of an edge $e = (u, v)$ in an intuitionistic fuzzy graph, $G = (V, E)$ is defined to be

$$d_E(e) = \begin{cases} \{d_E(u) + d_E(v) - 1 \\ \text{if } e = uv \text{ is an effective edge}\} \\ \{d_E(u) + d_E(v) \\ \text{if } e = uv \text{ is not an effective edge}\} \end{cases}$$

The minimum edge effective degree of an intuitionistic fuzzy graph, $G = (V, E)$ is $\delta'_E(G) = min\{d_E(e)|e \in E\}$ The maximum edge effective degree of an intuitionistic fuzzy graph,
$G = (V, E)$ is $\Delta'_E(G) = max\{d_E(e)|e \in E\}$.

Two vertices v_i and v_j are said to be neighbors in an intuitionistic fuzzy graph $G = (V, E)$ if either one of the following conditions holds,
(1) $\mu_2(v_i, v_j) > 0, \rho_2(v_i, v_j) > 0$,
(2) $\mu_2(v_i, v_j) = 0, \rho_2(v_i, v_j) > 0$,
(3) $\mu_2(v_i, v_j) > 0, \rho_2(v_i, v_j) = 0, v_i, v_j \in V$.
Two vertices v_i and v_j are said to be strong neighbors in an intuitionistic fuzzy graph
$G = (V, E)$ if $\mu_2(v_i, v_j) = min\{\mu_1(v_i), \mu_1(v_j)\}$, $\rho_2(v_i, v_j) = max\{\rho_1(v_i), \rho_1(v_j)\}$.

An intuitionistic fuzzy graph $G = (V, E)$ is called strong intuitionistic fuzzy graph if $\mu_2(v_i, v_j) = min\{\mu_1(v_i), \mu_1(v_j)\}, \rho_2(v_i, v_j) = max\{\rho_1(v_i),$
$\rho_1(v_j)\}$ for all $(v_i, v_j) \in E$. A vertex subset $N(v) = \{u \in V : v \textit{ adjacent to } u\}$ is called the open neighborhood set of a vertex v and $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood set of v. The neighborhood degree of a vertex v in an intuitionistic fuzzy graph, $G = (V, E)$ is defined to be sum of the weights of the vertices adjacent to v, and it is denoted by $d_N(v)$, that is mean that $d_N(v) = |N(v)|$. The minimum neighborhood degree of G is $\delta_N(G) = min\{d_N(v)|v \in V\}$. The maximum neighborhood degree of G is $\Delta_N(G) = max\{d_N(v)|v \in V\}$. An intuitionistic fuzzy graph, $G = (V, E)$ is said to be complete intuitionistic fuzzy graph if $\mu_2(v_i, v_j) = min\{\mu_1(v_i), \mu_1(v_j)\}$, $\rho_2(v_i, v_j) = max\{\rho_1(v_i), \rho_1(v_j)\}$, for all $v_i, v_j \in V$ and is denoted by K_p.

The complement of an intuitionistic fuzzy graph, $G = (V, E)$ is an intitionistic fuzzy graph, $\overline{G} = (\overline{V}, \overline{E})$ where
(i) $\overline{V} = V$;
(ii) $\overline{\mu_{1i}} = \mu_{1i}$; $\overline{\rho_{1i}} = \rho_{1i}$ for all $i = 1, 2, 3, ..., n$;
(iii) $\overline{\mu_{2ij}} = min\{\mu_{1i}, \mu_{1j}\} - \mu_{2ij}$ and $\overline{\rho_{2ij}} = max\{\rho_{1i}, \rho_{1j}\} - \rho_{2ij}$ for all $i, j = 1, 2, 3, ..., n$.

An intuitionistic fuzzy graph, $G = (V, E)$ is said to bipartite if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that
(i) $\mu_2(v_i, v_j) = 0, \rho_2(v_i, v_j) = 0$ if $v_i, v_j \in V_1$ or $v_i, v_j \in V_2$;
(ii) $\mu_2(v_i, v_j) > 0, \rho_2(v_i, v_j) > 0$, if $v_i \in V_1$ and $v_j \in V_2$ for some i and j, (or);
$\mu_2(v_i, v_j) = 0, \rho_2(v_i, v_j) > 0$, if $v_i \in V_1$ and $v_j \in V_2$ for some i and j, (or);
$\mu_2(v_i, v_j) > 0, \rho_2(v_i, v_j) = 0$, if $v_i \in V_1$ and $v_j \in V_2$.

A bipartite intuitionistic fuzzy graph, $G = (V, E)$ is said to be complete bipartite intuitionistic fuzzy graph if $\mu_2(v_i, v_j) = min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\rho_2(v_i, v_j) = max\{\rho_1(v_i), \rho_1(v_j)\}$ for all $v_i \in V_1$ and $v_j \in V_2$. Its denoted by $K_{m,n}$, where $|V_1| = m,\ |V_2| = n$.

A vertex $u \in V$ of an intuitionistic fuzzy graph, $G = (V, E)$ is said to be an isolated vertex if $\mu_2(v, u) = 0$ and $\rho_2(v, u) = 0$ for all $v \in V$. That is $N(u) = \phi$. Thus, an isolated vertex does not dominate any other vertex in G.

Let $G = (V, E)$ be an intuitionistic fuzzy graph and $u, v \in V$, we say that u dominates v in G if there exists a strong arc between them.

A subset S of V is called a dominating set in G if for every $v \in V - S$, there exists $u \in S$ such that u dominates v.

A dominating set S of an intuitionistic fuzzy graph is said to be minimal dominating set if for each vertex $v \in S$, $V - \{v\}$ is not a dominating set.

Minimum cardinality among all minimal dominating set is called lower domination number of G, and is denoted by $\gamma(G)$. Maximum cardinality among all minimal dominating set is called upper domination number of G, and is denoted by $\Gamma(G)$.

Let $G = (V, E)$ be an intuitionistic fuzzy graph. If $\mu_2(x, y) \leq \mu_1(x) \times \mu_1(y)$ and $\rho_2(x, y) \leq \rho_1(x) \times \rho_1(y)$ the intuition fuzzy graph is called product partial intuitionistic fuzzy sub graph of G. A product Intuitionistic fuzzy graph $G = (V, E)$ is said to be complete if $\mu_2(x, y) = \mu_1(x) \times \mu_1(y)$ and $\rho_2(x, y) = \rho_1(x) \times \rho_1(y)$ for all $x, y \in V$.
The complement of a product intuitionistic fuzzy graph $G = (V, E)$ is $G^c = (V^c, E^c)$ where $V^c = (\mu_1^c, \rho_1^c)$ and $E^c = (\mu_2^c, \rho_2^c)$ such that $\mu_1^c = \mu_1$, $\rho_1^c = \rho_1$, $\mu_2^c(x, y) = \mu_1(x) \times \mu_1(y) - \mu_2(x, y)$ and $\rho_2^c(x, y) = \rho_1(x) \times \rho_1(y) - \rho_2(x, y)$. A product intuitionistic fuzzy graph $G = (V, E)$ is said to bipartite if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that
(i) $\mu_2(u, v) = 0, \rho_2(u, v) = 0$ if $u, v \in V_1$ or $u, v \in V_2$;
(ii) $\mu_2(u, v) > 0, \rho_2(u, v) > 0$, if $u \in V_1$ and $v \in V_2$ (or);
$\mu_2(u, v) = 0, \rho_2(u, v) > 0$, if $u \in V_1$ and $v \in V_2$ (or);
$\mu_2(u, v) > 0, \rho_2(u, v) = 0$, if $u \in V_1$ and $v \in V_2$.
A bipartite intuitionistic fuzzy graph $G = (V, E)$ is said to be complete bipartite product intuitionistic fuzzy graph if $\mu_2(u, v) = min\{\mu_1(u), \mu_1(v)\}$ and $\rho_2(u, v) = max\{\rho_1(u), \rho_1(v)\}$ for all $u \in V_1$ and $v \in V_2$. Its denoted by $K_{n,m}$, where $n = |V_1|$ and $m = |V_2|$.

Throughout this work, $PIFG$, $G = (V, E)$ means product intuitionistic fuzzy graph $G = (V, E)$.

3. The Edge Dominating Set

Definition 3.1. *Two edges x and y in a product intuitionistic fuzzy graph G are said to be adjacent if they are neighbors.*

Definition 3.2. *Two edges x and y are called independent if they are not adjacent.*

Definition 3.3. *An edge subset D of E in a product intuitionistic fuzzy graph $G = (V, E)$ is said to be independent if $\mu_2(u, v) < \mu_1(u) \times \mu_1(v)$ and $\rho_2(u, v) < \rho_1(u) \times \rho_1(v)$ for all $u, v \in D$. The maximum cardinality among all independent edge set in G is called the edge independence number and is denoted by $\beta_1(G)$ or simply β_1.*

Definition 3.4. *An independent edge set D in a $PIFG$, $G = (V, E)$ is said to be maximal independent edge set, if for every edge $e \in E - D$, the edge set $D \cup \{e\}$ is not independent. The maximum fuzzy cardinality among all maximal independent edge sets in G is called the upper edge independence number of G, and is denoted by $\beta_1(G)$. The minimum fuzzy cardinality among all maximal independent edge sets* *in G is called the lower edge independence number of G and is denoted by $i(G)$.*

Definition 3.5. *A vertex v and an edge e in a $(PIFG)$, $G = (V, E)$ are said to be cover each other if they are incident.*

Definition 3.6. *An edge subset S of E in a $(PIFG)$, $G = (V, E)$ which covers all vertices in G is called an edge covering set of G. The minimum fuzzy cardinality among all edge covering set is called the edge covering number of G and is denoted by $\alpha_1(G)$ or simply α_1.*

Theorem 3.1. *An edge subset $S \subseteq E$ in a $(PIFG)$, $G = (V, E)$ is an independent set in G if $E - S$ is an edge covering set of G.*
Proof. By definition , S is an independent set in G if and only if no two edges of S are adjacent, if and only if every edge of D is incident with at least one vertex of $E - S$ if and only if $E - S$ is an edge covering set of G.

An edge $e = (u, v)$ of a product intuitionistic fuzzy graph G is called a strong arc if, $\mu_2(u, v) \geq \mu_2^{\infty}(u, v)$ and $\rho_2(u, v) \geq \rho_2^{\infty}(u, v)$.
An edge $e = uv$ of a product intuitionistic fuzzy graph G is called an effective edge if, $\mu_2(u, v) = \mu_1(u) \times \mu_1(v)$ and $\rho_2(u, v) = \rho_1(u) \times \rho_1(v)$.
An edge e of a product intuitionistic fuzzy graph G is said to be an isolated edge if no effective edges incident with the vertices of e.
Thus an isolated edge does not dominate any other edge in G.

In [20] the author defined the open and closed neighborhood set of an edge x in an intuitionistic fuzzy graph G as follows. $N(x) = \{y \in E | \mu_2(x, y) = min\{\mu_1(x), \mu_1(y)\}$ and $\rho_2(x, y) = max\{\rho_1(x), \rho_1(y)\}$ is called the neighborhood set of x. $N[x] = N(x) \cup \{x\}$ is called the closed neighborhood set of x [20].
The above definition is not absolutely correct so we redefine the open and closed neighborhood set of any edge e in an intuitionistic fuzzy graph G and a product intuitionistic fuzzy graph G as follows.

Definition 3.7. *Let e be any edge in an intuitionistic fuzzy graph (a product intuitionistic fuzzy graph), $G = (E, V)$. Then*
$N(e) = \{x \in E : x\ is\ an\ effective\ edge\ incident\ with\ the\ vertices\ of\ e\}$ is called the open edge neighborhood set of e. That is $N(e)$ is the set of all effective edge incident with the vertices of e. $N[e] = N(e) \cup \{e\}$ is called the closed neighborhood set of e.

Definition 3.8. *Let e be any edge in a product intuitionistic fuzzy graph $G = (E, V)$. Then $d_N(e) = \sum_{x \in N(e)} |x|$ is called the edge neighborhood degree of e. The minimum edge neighborhood degree of a $(PIFG)$, $G = (V, E)$ is $\delta'_N(G) = min\{d_N(e) | e \in E\}$. The maximum edge neighborhood degree of a $(PIFG)$, $G =$*

(V, E) is $\Delta'_N(G) = max\{d_N(e)|e \in E\}$.

As a result in [16] for a fuzzy graph, we have the following in a product intuitionistic fuzzy graphs.

Theorem 3.2. *For any product intuitionistic fuzzy graph $G = (V, E)$ without isolated edges, $\alpha_1(G) + \beta_1(G) = q$.*
Proof. Let S be an edge independent set in G and K is an edge covering set in G such that $|S| = \beta_1(G)$ and $|K| = \alpha_1(G)$. Then by Theorem 3.1, $E - S$ is an edge covering set of G. Hence $|K| \leq |E - S| \Longrightarrow \alpha_1(G) \leq q - \beta_1(G)$, and $\alpha_1(G)+\beta_1(G) \leq q \longrightarrow (1)$. Also by Theorem 3.1, $E-K$ is an edge independent set in G. Hence $|S| \geq |E-K| \Longrightarrow \beta_1(G) \geq q-\alpha_1(G) \Longrightarrow \alpha_1(G)+\beta_1(G) \geq q \longrightarrow (2)$. From (1) *and* (2), we get $\alpha_1(G) + \beta_1(G) = q$.

Definition 3.9. *Let $G = (V, E)$ be a product intuitionistic fuzzy graph. Let $e, x \in E$. We say that e dominates x in G if they are adjacent and e is an effective edge.*

Definition 3.10. *An edge subset S of E in a $(PIFG)$, $G = (V, E)$ is called an edge dominating set in G if for every edge $e \in V - S$, there exists an effective edge $x \in S$ such that x dominates e. The minimum fuzzy cardinality among all edge dominating set in G is called the edge domination number of G and is denoted by $\gamma'(G)$ or simply γ'.*
The maximum fuzzy cardinality among all edge dominating set in G is called the upper edge domination number of G and is denoted by $\Gamma'(G)$ or simply Γ'.

The above definition of edge domination in product intuitionistic fuzzy graph is motivated by the following situation. Let G be a graph which represents the road network of a city. Let the vertices denote the junctions and the edges denote the connecting junctions. From the statistical data that represents the number of vehicles passing through various junctions and the number of vehicles passing through various roads during a peak hour, the membership functions (μ_1, ρ_1) and (μ_2, ρ_2) on the vertex set and edge set of G can be constructed by using the standard technique given in (Bobroewicz etal., 1990; Reha Civanlar and Joel Trussel, 1986). In this product intuitionistic fuzzy graph an edge dominating set S can be interpreted as a set of roads which are busy in the sense that every road not in S is connected to a member in S by having a common junction in which the traffic flow is full.

Example 3.1. Consider the product intuitionistic fuzzy graph $G = (V, E)$ given in FIGURE 3.1, where $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ such that $\mu_2(u, v) = \mu_1(u) \times \mu_1(v)$ for all $(u, v) \in E$ and $\rho_2(u, v) = \rho_1(u) \times \rho_1(v)$ for all $(u, v) \in E$. (i.e.

all edge in G are effective).

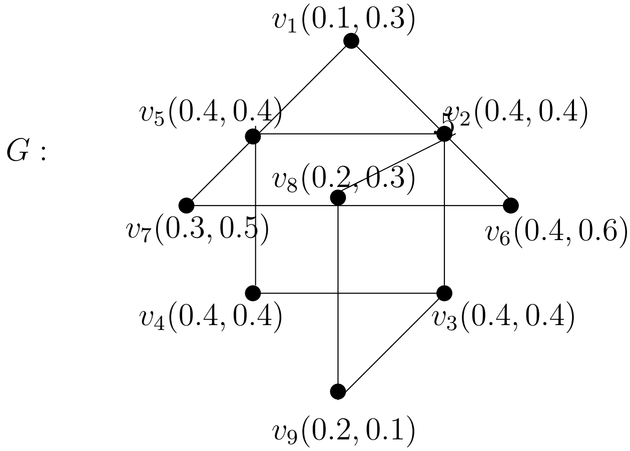

FIGURE 3.1. Intuitionistic Fuzzy Graph

We see that an edge subsets $S_1 = \{(v_2, v_5), (v_3, v_9), (v_6, v_7)\}$, $S_2 = \{(v_2, v_3), (v_4, v_5), (v_6, v_7), (v_8, v_9)\}$, $S_3 = \{(v_2, v_6), (v_4, v_5), (v_8, v_9)\}$ and $S_4 = \{(v_2, v_6), (v_4, v_5), (v_3, v_9)\}$ are edge dominating sets in G and hence $\gamma'(G) = min\{|S_1|\ ; |S_2|; |S_3|; |S_4|\} = \min\{1.43, 1.915, 1.465, 1.48\} = 1.43$.

Definition 3.11. *An edge dominating set S of a $(PIFG)$, $G = (V, E)$ is said to be minimal edge dominating set if for each edge $e \in S$, $S - \{e\}$ is not edge dominating set of G.*

By using the notion of minimal edge dominating set, we define the edge domination number of a product intuitionistic fuzzy graph G which equivalent to definition 3.9 as follows.

Definition 3.12. *The minimum fuzzy cardinality among all minimal edge dominating set in G is called the edge domination number of G and is denoted by $\gamma'(G)$ or simply γ'. The maximum fuzzy cardinality among all minimal edge dominating set in G is called the upper edge domination number of G and is denoted by $\Gamma'(G)$ or simply Γ'.*

Example 3.2. Consider the product intuitionistic fuzzy graph G given in Example 3.1, we see that an edge subsets S_1, S_2, S_3, S_4 are minimal edge dominating sets in G. Hence $\gamma'(G) = min\{|S_1|; |S_2|; |S_3|; |S_4|\} = min\{1.43, 1.915, 1.465, 1.48\} = 1.43$.

Remark 3.1. 1. *Note that for any effective edges $e, x \in E$, if e dominates x then x dominates e. Then the edge domination is symmetric relation on E.*

2. *For any* $x \in E$, $N(x)$ *is precisely the set of all edges in* E *which are dominated by* x.
3. $\gamma'(G) \leq q$. *If* $0 < \mu_2(u,v) < \mu_1(u) \times \mu_1(v)$ *and* $0 < \rho_2(u,v) < \rho_1(u) \times \rho_1(v)$ *for all* $(u,v) \in E$, *then the only edge dominating set of* G *is* E, *so* $\gamma'(G) = q$.
4. *Since all edges in complete product intuitionistic fuzzy graph* G *are effective edges, then* $\gamma'(K_p) = min\{|S| : S \text{ is an independent edge set in } G\}$.
5. *If* $G = K_{n,m}$ *is a complete bipartite product intuitionistic fuzzy graph, then* $\gamma'(G) = min\{|S| : S \text{ is an independent edge set in } G\}$.

Example 3.3.A [20] Let $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be any intuitionistic fuzzy graph. Then, $\gamma'(G) = \frac{p(p-1)}{2}$ if $0 < \mu_1(u,v) < \sigma_1(u) \wedge \sigma_1(v)$ and $0 < \mu_2(u,v) < \sigma_2(u) \vee \sigma_2(v)$ for all $u, v \in V$. In particular $\gamma'(\overline{G}) = \frac{p(p-1)}{2}$.

Theorem 3.2.B [20] *For any intuitionistic fuzzy graph* G, $\gamma'(G) + \gamma'(\overline{G}) \leq p(p-1)$ *and equality holds if* $0 < \mu_1(u,v) < \sigma_1(u) \wedge \sigma_1(v)$ *and* $0 < \mu_2(u,v) < \sigma_2(u) \vee \sigma_2(v)$ *for all* $u, v \in V$.

Now, in the following we show an example to explain that the result in Example 3.3.A in [20] and then theorem 3.2.B in [20] are not true in general.

Example 3.3. For the intuitionistic fuzzy graph G given in FIGURE 3.2, We see that $0 < \mu_1(u,v) < \sigma_1(u) \wedge \sigma_1(v)$ and $0 < \mu_2(u,v) < \sigma_2(u) \vee \sigma_2(v)$ for all $u, v \in V$.

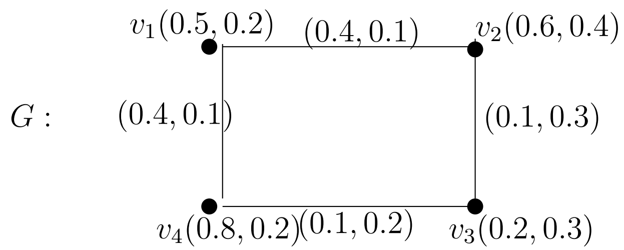

FIGURE 3.2. Intuitionistic Fuzzy Graph

We can verify that $\frac{p(p-1)}{2} = 3.52$ but $\gamma'(G) = q = 1.7 \lneq \frac{p(p-1)}{2} = 3.52$ if we compute the order p and the size q as follows:
$p = \sum_{v \in V}[\sigma_1(v) + \sigma_2(v)]$ and $q = \sum_{uv \in E}[\mu_1(uv) + \mu_2(uv)]$, here $p = 3.2$ and $q = 1.7$. Also, the result is not correct if we consider the vertex cardinality and edge cardinality of G as follows. $|V| = \sum_{v_i \in V} \frac{1+\mu_1(v_i)-\rho_1(v_i)}{2}$ for all $v_i \in V$ and $|E| = \sum_{(v_i,v_j) \in E} \frac{1+\mu_2(v_i,v_j)-\rho_2(v_i,v_j)}{2}$ for all $(v_i, v_j) \in E$, here $p = |V| = 2.5$ and $q = |E| = 2.15$.

For the edge domination number $\gamma'(G)$ the following theorem gives a Nordhaus - Gaddum type result.

Theorem 3.3. *For any product intuitionistic fuzzy graph $G = (V, E)$, $\gamma'(G) + \gamma'(\overline{G}) \leq 2q$. Further equality holds if and only if $0 < \mu_2(u, v) < \mu_1(u) \times \mu_1(v)$ and $0 < \rho_2(u, v) < \rho_1(u) \times \rho_1(v)$ for all $(u, v) \in E$.*
Proof. The inequality is trivial. Further $\gamma'(G) = q$ if and only if $\mu_2(u, v) < \mu_1(u) \times \mu_1(v)$ and $\rho_2(u, v) < \rho_1(u) \times \rho_1(v)$ for all $(u, v) \in E$. $\gamma'(\overline{G}) = q$ if and only if $\mu_2^c(u, v) = \mu_1(u) \times \mu_1(v) - \mu_2(u, v) < \mu_1(u) \times \mu_1(v)$ and $\rho_2^c(u, v) = \rho_1(u) \times \rho_1(v) - \rho_2(u, v) < \rho_1(u) \times \rho_1(v)$ for all $(u, v) \in E$ which is equivalent to $\mu_2(u, v) > 0$ and $\rho_2(u, v) > 0$. Hence $\gamma'(G) + \gamma'(\overline{G}) = 2q$ if and only if $0 < \mu_2(u, v) < \mu_1(u) \times \mu_1(v)$ and $0 < \rho_2(u, v) < \rho_1(u) \times \rho_1(v)$ for all $(u, v) \in E$.

The following theorem gives a characterization of minimal edge dominating sets in a product intuitionistic fuzzy graph which is analogous to the result of Velammal (2012) in an intuitionistic fuzzy graph.

Theorem 3.4. *An edge dominating set S in a product intuitionistic fuzzy graph G is a minimal edge dominating set if and only if for each edge $e \in S$, one of the following two conditions holds*
(a) $N(e) \cap S = \phi$;
(b) There exists an edge $x \in E - S$ such that $N(x) \cap S = \{e\}$ and x is an effective edge.
Proof. Let S be a minimal edge dominating set and $e \in S$. Then $S_e = S - \{e\}$ is not an edge dominating set and hence there exists $x \in E - S_e$ such that x is not dominated by any element of S_e. If $x = e$ we get (a) and if $x \neq e$ we get (b). Conversely, assume that S is an edge dominating set and for each edge $e \in S$, one of the two conditions holds. Suppose S is not a minimal edge dominating set, then there exists an edge $e \in S$, $S - \{e\}$ is an edge dominating set. Hence e is a strong neighbor to at least one edge in $S - \{e\}$, the condition one does not hold. If $S - \{e\}$ is an edge dominating set then every edge in $E - S$ is a strong neighbor to at least one edge in $S - \{e\}$, the second condition does not hold which contradicts our assumption that at least one of this conditions holds. So S is a minimal edge dominating set.

Theorem 3.5. *Let $G = (V, E)$ be any product intuitionistic fuzzy graph without isolated edges. Then for every minimal edge dominating set S, $E - S$ is also an edge domination set.*
Proof. Let x be any edge in S. Since G has no isolated edges, there is an edge $y \in N(x)$. It follows from Theorem 3.4 that $y \in E - S$. Thus every element of S is dominated by some element of $E - S$. Hence $E - S$ is an edge dominating set in G.

Corollary 3.1. *For any product intuitionistic fuzzy graph without isolated vertices,*

$\gamma'(G) \leq \frac{q}{2}$.

Proof. Any product intuitionistic fuzzy graph without isolated vertices has two disjoint edge dominating sets and hence the result follows.

Corollary 3.2. *Let G be a product intuitionistic fuzzy graph such that both G and G have no isolated vertices. Then $\gamma'(G) + \gamma'(\overline{G}) \leq q$. Further equality holds if and only if $\gamma'(G) = \gamma'(\overline{G}) = \frac{q}{2}$.*

Example 3.4. For the product intuitionistic fuzzy graph G given in FIGURE 3.3 , $\gamma'(G) = \gamma'(\overline{G}) = 1 = q/2$

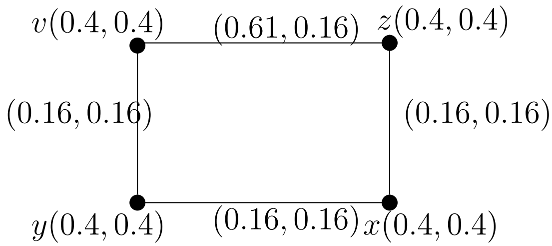

FIGURE 3.3. Intuitionistic Fuzzy Graph

Definition 3.13. *An edge dominating set S of a product intuitionistic fuzzy graph, $G = (V, E)$ is said to be independent edge dominating set in G if $\mu_2(u, v) < \mu_1(u) \times \mu_1(v)$ and $\rho_2(u, v) < \rho_1(u) \times \rho_1(v)$ for all $(u, v) \in S$. The minimum fuzzy cardinality among all independent edge dominating set in G is called independence edge domination number and is denoted by $\gamma'_i(G)$ or simply γ'_i.*

Example 3.5. For the product intuitionistic fuzzy graph G given in example 3.4, an edge dominating sets $S_1 = \{(v, x), (y, z)\}$ and $S_2 = \{(v, z), (y, x)\}$ are independent edge dominating sets in G. Hence $\gamma'_i(G) = min\{|S_1|, |S_2|\} = min\{0.5, 0.5\} = 0.5$.

The following Theorem gives a characterization of independent edge dominating sets.

Theorem 3.6. *If D is an independent edge dominating set in a product intuitionistic fuzzy graph G then D is both a minimal edge dominating set and a maximal edge independent set. Conversely any maximal independent edge set D in G is an independent edge dominating set in G.*

Proof. If D is an independent edge dominating set in G, then $D - \{e\}$ is not an edge dominating set for every $e \in D$ and $D \cup \{e\}$ is not independent for every $e \notin D$ so that D is a minimal edge dominating set and a maximal edge independent set. Conversely let D be a maximal independent edge set in G. Then for every edge

$e \in E - D$, $D \cup \{e\}$ is not independent and hence e is dominated by some element of D. Thus D is an independent edge dominating set of G.

Theorem 3.7. *An edge independent set D of a product intuitionistic fuzzy graph, $G = (V, E)$ is a maximal edge independent set of G, if and only if it is an edge independent and edge dominating set.*

Proof. Let D be a maximal edge independent set in a product intuitionistic fuzzy graph, $G = (V, E)$ and hence for every edge $e \in E - D$, the set $D \cup \{e\}$ is not independent. For every edge $e \in E - D$, there is an effective edge $x \in D$ such that x dominates e. Thus D is an edge dominating set. Hence D is both edge dominating and edge independent set. Conversely, assume D is both independent and edge dominating set. Suppose D is not maximal edge independent, then there exists an edge $e \in E - D$, the set $D \cup \{e\}$ is independent. If $D \cup \{e\}$ is independent then no effective edge in D is strong neighbor to e. Hence D can not be an edge dominating set, which is a contradiction. Hence D is a maximal edge independent set.

Theorem 3.8. *Every maximal edge independent set D in a product intuitionistic fuzzy graph, $G = (V, E)$ is a minimal edge dominating set.*

Proof. Let D be a maximal independent edge set in a product intuitionistic fuzzy graph G. By Theorem 3.7, D is an edge dominating set. Suppose D is not a minimal edge dominating set, then there exists at least one edge $e \in D$ for which $D - \{e\}$ is an edge dominating set. But if $D - \{e\}$ dominates $E - \{D - \{e\}\}$, then at least one edge in $D - \{e\}$ must be strong neighbor to e. This contradicts the fact that D is an edge independent set in G. Therefore, D must be a minimal edge dominating set.

The following Theorem gives an upper bound for the edge domination number of a product intuitionistic fuzzy graph.

Theorem 3.9. *For any product intuitionistic fuzzy graph G, $\gamma'(G) \leq q - \Delta'_E(G)$.*

Proof. Let e be an edge of maximum effective degree $d_E(e) = \Delta'_E(G)$. Let E be the edge set of G, then $|E| = q$. Clearly $E - N(e)$ is an edge dominating set of G so that $\gamma'(G) \leq |E - N(e)| = q - \Delta'_E(G)$.

Corollary 3.3. *For any product intuitionistic fuzzy graph G, $\gamma'(G) \leq q - \delta'_N(G) \leq q - \delta'_E(G)$.*

Theorem 3.10. *For any product intuitionistic fuzzy graph $G = (V, E)$ without isolated edges, $\gamma'(G) \leq q - \alpha_1(G)$ where $\alpha_1(G)$ is an edge covering number of G.*

Proof. Let D be a minimal edge covering set in G. Since G has not isolated edges, then by Theorem 3.2, $E - D$ is a maximal independent edge set in G and hence by Theorem 3.8 is a minimal edge dominating set of G. Then $\gamma'(G) \leq |E - D| = q - \alpha_1(G)$.

Corollary 3.4. *For any product intuitionistic fuzzy graph $G = (V, E)$, $\gamma'(G) \leq \beta_1(G)$. where $\beta_1(G)$ is an edge independence number of G.*

Theorem 3.11. *For any product intuitionistic fuzzy graph $G = (V, E)$, $\gamma'(G) \leq q - \beta_1(G)$ where $\beta_1(G)$ is an edge independence number of G.*

Proof. Let D be a maximal edge independent set in G. Then by Theorem 3.1, $E - D$ is an edge cover of G and hence an edge dominating set of G. Then $\gamma'(G) \leq |E - D| = q - \beta_1(G)$.

Corollary 3.5. *For any product intuitionistic fuzzy graph $G = (V, E)$, $\gamma'(G) \leq \alpha_1(G)$.*

An edge dominating set S of a product intuitionistic fuzzy graph $G = (V, E)$ is said to be minimum edge dominating set if $|S| = \gamma'(G)$ and is denoted by $\gamma' - set$.

Theorem 3.12. *For any product intuitionistic fuzzy graph $G = (V, E)$ without isolated edges, $\gamma'(G) = \alpha_1(G)$, if and only if there exists a minimum edge dominating set D of G such that $E - D$ is an edge independent.*

Proof. Let $E - D$ be an edge independent set, then D is an edge cover of G and hence $\alpha_1(G) \leq |D| = \gamma'(G)$. Now, by Corollary 3.5, $\gamma'(G) \leq \alpha_1(G)$. Hence $\gamma(G)' = \alpha_1(G)$.

Conversely, let $\gamma'(G) = \alpha_1(G)$ and let S be any edge covering set of G such that $|S| = \alpha_1(G)$. Then $E - S$ is an edge independent and this implies that S is an edge dominating set of G. Also, since $|S| = \alpha_1(G) = \gamma'(G)$, then S is a minimum edge dominating set of G.

Theorem 3.13. *For any product intuitionistic fuzzy graph $G = (V, E)$, $\gamma'(G) = \gamma'_i(G)$.*

Definition 3.14. *Let $G = (V, E)$ be a product intuitionistic fuzzy graph without isolated edges. An edge subset D of E is said to be total edge dominating set of G if for every edge $e \in E$ there exists an edge $x \in D$, $x \neq e$, such that x dominates e.*

Definition 3.15. *The minimum fuzzy cardinality among all total edge dominating set is called the total edge domination number of G and is denoted by $\gamma'_t(G)$.*

Example 3.6. Consider a product intuitionistic fuzzy graph $G = (V, E)$ given in the FIGURE 3.4, such that $V = \{u, v, x, y, z\}$, $E = \{(u, z), (u, y), (v, x), (v, z), (x, z), (z, y)\}$

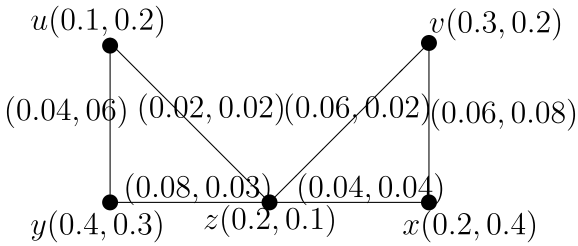

FIGURE 3.4 Intuitionistic Fuzzy Graph

Here, an edge subsets $S_1 = \{(u, z), (z, x)\}$, $S_2 = \{(v, z), (z, y)\}$, $S_3 = \{(x, z), (y, z)\}$, $S_4 = \{(v, x), (x, z), (y, z)\}$, $S_5 = \{(u, y), (x, z), (y, z)\}$, $S_6 = \{(v, z), (u, z)\}$, $S_7 = \{(v, z), (u, z), (u, y)\}$, $S_8 = \{(v, z), (u, z), (v, x)\}$, $S_9 = \{(v, z), (z, y), (y, u)\}$ and $S_{10} = \{(u, z), (z, x), (x, v)\}$ are total edge dominating sets of G and the total edge domination number of G is
$\gamma'_t(G) = min\{|S_1|, |S_2|, |S_3|, |S_4|, |S_5|, |S_6|, |S_7|, |S_8|, |S_9|, |S_{10}|\}$
$= min\{1, 1.045, 1.025, 1.515, 1.48, 1.02, 1.51, 1.51, 1.535, 1.98\} = |S_1| = 1.$

Theorem 3.14. *For any product intuitionistic fuzzy graph $G = (V, E)$, $\gamma'(G) \leq \gamma'_t(G)$.*

Theorem 3.15. *For any product intuitionistic fuzzy graph $G = (V, E)$, $\gamma'(G) \leq q$; and equality holds if and only if every edge of G has a unique neighbor.*

Proof. If every edge of G has a unique neighbor then E is the only total edge dominating set of G so that $\gamma'(G) = q$. Conversely, suppose $\gamma'(G) = q$. If there exists an edge with neighbors x and y then $E - \{x\}$ is a total edge dominating set of G. So that $\gamma'(G) < q$ which is a contradiction.

Corollary 3.6. *If $\gamma'_t(G) = q$ then the number of edges in a product intuitionistic fuzzy graph $G = (V, E)$ is even.*

Theorem 3.16. *Let $G = (V, E)$ be a product intuitionistic fuzzy graph without isolated edges. Then $\gamma'_t(G) + \gamma'_t(\overline{G}) \leq 2q$; and equality holds if and only if*
1. the number of edges in G is even, say $2m$.
2. there is a set S_1 of m mutually disjoint P'_3s (P_n denotes the path on n vertices) in G,
3. there is a set S_2 of m mutually disjoint P'_3s in $\overline{G}$, and
4. for any edge $(u, v) \notin S_1 \cup S_2$, $0 < \mu_2(u, v) < \mu_1(u) \times \mu_1(v)$ and $0 < \rho_2(u, v) < \rho_1(u) \times \rho_1(v)$.

Proof. Since $\gamma'_t(G) \leq q$ and $\gamma'_t(\overline{G}) \leq q$, the inequality follows. Further, $\gamma'_t(G) + \gamma'_t(\overline{G}) = 2q$ if and only if $\gamma'_t(G) = \gamma'_t(\overline{G}) = q$ and hence by Corollary 3.6, the number of edges in G is even, say $2m$. Since $\gamma'_t(G) = q$, there is a set S_1 of m disjoint P'_3s

in G. Similarly there is a set S_2 of m disjoint $P_3's$ in $\overline{G}$. Further if $(u, v) \notin S_1 \cup S_2$, then $0 < \mu_2(u, v) < \mu_1(u) \times \mu_1(v)$ and $0 < \rho_2(u, v) < \rho_1(u) \times \rho_1(v)$. The converse is obvious.

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