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APPLY ON COMPLEMENTARY NEIGHBOURHOOD VIA NANO TOPOLOGY

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Abstract: This paper introduce a new notion of complementary neighbourhood and complementary covering based on nano topological space. Further we characterize the concept is equipped with the arbitrary binary relation. Finally, we discuss the complement of covering interior and closure and its properties and examples are given.

Keywords and Phrases: Neighbourhood, Right Cover, Left Cover, Binary relation, Covering interior, Covering Closure.

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1. Introduction

In 1982, Pawlak introduced the concept of rough set which is defined in terms of approximation based on equivalence relation [9]. A mathematical tool is an data mining,information system and vagueness and granularity for dealing with the concept of covering based rough set theory [3, 8, 10, 11, 12, 13, 14, 15]. Lellis Thivagar et al [5] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region of a set. However the lower and upper approximations and boundary region are based on equivalence relations distinct on it are known to be interchangeable notions, but it has been extended to arbitrary binary relation [6, 7]. Many authors has studied the concepts of topology and covering its extended equivalence relation into binary relation [1, 2, 4]. In this paper presents the study of nano topology induced by complementary neighbourhood and complement of covering. Further, we discuss the complement of covering and its properties are studied. Also, we define characterization of the complement of covering in nano topological space. Finally, we discuss the properties of complement of covering interior and closure as well as examples are studied.

2. Preliminaries

The following definitions are necessitated in the sequel of our work.

Definition 2.1. [5] *Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and R be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.*

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup\limits_{x \in \mathcal{U}} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup\limits_{x \in \mathcal{U}} \{R(x) : R(x) \cap X \neq \phi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [5] *Let $\mathcal{U}$ be an universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$. $\tau_R(X)$ satisfies the following axioms:*

(i) $\mathcal{U}$ and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on $\mathcal{U}$ called the nano topology on $\mathcal{U}$ with respect to X. We call $(\mathcal{U}, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets.

Definition 2.3. [2, 4] *Let $\mathcal{U}$ be a non empty finite set, $\mathcal{C} = \{C_k | k \in K\}$ a family of subsets of $\mathcal{U}$. If none subsets in $\mathcal{C}$ is empty and $\bigcup_{k \in K} C_k = \mathcal{U}$, then $\mathcal{C}$ is called covering of $\mathcal{U}$. The pair $(\mathcal{U}, \mathcal{C})$ is called covering approximation space, if $\mathcal{C}$ is a covering of $\mathcal{U}$.*

Definition 2.4. $[1, 2, 13]$ *For the pair of approximation space $(\mathcal{U}, R)$ where $\mathcal{U}$ is the non-empty finite set of objects called the universe, R be an binary relation on $\mathcal{U}$. Then the after set (or) successor of $x \in \mathcal{U}$ denoted by xR (or)$R_s(x)$, where $R_s(x)(or)(x)R = \{y \in \mathcal{U} | xRy\}$ and the fore set (or) predecessor of $x \in \mathcal{U}$ denoted by Rx (or)$R_p(x)$,where Rx (or)=$\{y \in \mathcal{U} | yRx\}$.*

Definition 2.5. [2, 12] *Let $\mathcal{U}$ be a non empty finite set and R be any binary relation on $\mathcal{U}$ and $(\mathcal{U}, R)$ be approximation space. Then, two different coverings for $\mathcal{U}$ by using the concept of after set and the fore set as follows:*

- *Right Covering (briefly, RC): $\mathcal{C}_r = \{xR : \forall x \in \mathcal{U}\}$ and $\mathcal{U} = \bigcup_{x \in \mathcal{U}} xR$.*
- *Left Covering (briefly, LC): $\mathcal{C}_l = \{Rx : \forall x \in \mathcal{U}\}$ and $\mathcal{U} = \bigcup_{x \in \mathcal{U}} Rx$.*

3. Complementary Neighbourhood via Nano Topology

In this section we define a new notion of complementary neighbourhood and presented some results. For any $X \subseteq \mathcal{U}$,we denote -X as the complement of X in $\mathcal{U}$.

Definition 3.1. *Let $(\mathcal{U}, R, C_n)$ be generalized covering approximation space,where $n \in \{RC, LC\}$ and*

(i) $N_{RC}(x) = \bigcap\{K \in RC : x \in K\}$ is called the neighbourhood of x with respect to right covering.

(ii) $N_{LC}(x) = \bigcap\{K \in LC : x \in K\}$ is called the neighbourhood of x with respect to left covering.

Remark 3.2. *From the above definition 3.1, we see that the neighbourhood of any element contains the element itself. Therefore,for a right and left covering of a universe $\mathcal{U}$. Also, $\{N_{RC}(x) : x \in \mathcal{U}\}$ and $\{N_{LC}(x) : x \in \mathcal{U}\}$ is also a covering of $\mathcal{U}$.*

Example 3.3. Let $\mathcal{U} = \{a, b, c, d\}$ and R be a binary relation on $\mathcal{U}$, where $R = \{(a, a), (a, b), (b, c), (b, d), (c, a), (d, a)\}$.Then $aR = \{a, b\}$;$bR = \{c, d\}$;$cR = \{a\}$;$dR = \{a\}$.Also $Ra = \{a, c, d\}$;$Rb = \{a\}$;$Rc = \{b\}$;$Rd = \{b\}$. Hence the neighbourhood of x with respect to right covering $N_{RC}(a) = \{a\}$;$N_{RC}(b) = \{a, b\}$; $N_{RC}(c) = \{c, d\}$; $N_{RC}(d) = \{c, d\}$. Also $N_{LC}(a) = \{a\}$; $N_{LC}(b) = \{b\}$; $N_{LC}(c) = \{a, c, d\}$; $N_{LC}(d) = \{a, c, d\}$ is the neighbourhood of x with respect to left covering.

Definition 3.4. *Let $(\mathcal{U}, R, C_n)$ be generalized covering approximation space. For any $x \in \mathcal{U}$ and*

(i) $M_{RC}(x) = \{y \in \mathcal{U} : x \in N_{RC}(y)\}$ is called the complementary neighbourhood of x with respect to right covering.

(ii) $M_{LC}(x) = \{y \in \mathcal{U} : x \in N_{LC}(y)\}$ is called the complementary neighbourhood of x with respect to left covering.

Definition 3.5. *Let $\mathcal{U}$ be a non empty finite set of objects called the universe and R be arbitrary binary relation on $\mathcal{U}$. The triple pair $(\mathcal{U}, R, C_n)$ is said to be generalized covering approximation space. Let $X \subseteq \mathcal{U}$ and, for each $i = RC, LC$ are defined respectively as follows:*

(i) The complementary neighbourhood of lower $(L_{M_i}(X))$ approximation of X is defined by

$$L_{M_i}(X) = \bigcup_{x \in \mathcal{U}} \{M_i(x) : M_i(x) \subseteq X\}.$$

(ii) The complementary neighbourhood of upper $(U_{M_i}(X))$ approximations of X is defined by

$$U_{M_i}(X) = \bigcup_{x \in \mathcal{U}} \{M_i(x) : M_i(x) \bigcap X \neq \emptyset\}.$$

(iii) The complementary neighbourhood of boundary region $(B_{M_i}(X))$ of X is defined by $B_{M_i}(X) = U_{M_i}(X)$ - $L_{M_i}(X)$.

Definition 3.6. *Let $\mathcal{U}$ be universe, $M_i(x)$ be two types of complementary neighbourhoods where $i = \{RC, LC\}$ and $\tau_{M_i}(X) = \{\mathcal{U}, \emptyset, L_{M_i}(X), U_{M_i}(X), B_{M_i}(X)\}$ forms a nano topology on $\mathcal{U}$ with respect to X. We call $\{\mathcal{U}, \tau_{M_i}(X)\}$ as the nano topology induced by complementary neighbourhoods.*

Example 3.7. Let $\mathcal{U} = \{a, b, c, d\}$ and $R = \{(a, b), (a, d), (b, b), (b, c), (c, d), (d, a)\}$. Let $X = \{a\} \subseteq \mathcal{U}$. Then $aR = \{b, d\}; bR = \{b, c\}; cR = \{d\}; dR = \{a\}$. Also $Ra = \{d\}; Rb = \{a, b\}; Rc = \{b\}; Rd = \{a, c\}$. Since $N_{RC}(a) = \{a\}, N_{RC}(b) = \{b\}, N_{RC}(c) = \{b, c\}, N_{RC}(d) = \{d\}$ and $N_{LC}(a) = \{a\}, N_{LC}(b) = \{b\}, N_{LC}(c) = \{a, c\}, N_{LC}(d) = \{d\}$. Therefore $M_{RC}(a) = \{a\}, M_{RC}(b) = \{b, c\}, M_{RC}(c) = \{c\} M_{RC}(d) = \{d\}$ and $M_{LC}(a) = \{a, c\}, M_{LC}(b) = \{b\}, M_{LC}(c) = \{c\} M_{LC}(d) = \{d\}$. Hence $\tau_{M_{RC}}(X) = \{\mathcal{U}, \emptyset, \{a\}\}, \tau_{M_{LC}}(X) = \{\mathcal{U}, \emptyset, \{a, c\}\}$.

Theorem 3.8. *Let $(\mathcal{U}, \tau_{M_i}(X))$ be nano topological space induced by complementary neighbourhoods on $\mathcal{U}$. Let $X, Y \subseteq \mathcal{U}$. Then*

(1) $L_{M_i}(X) \subseteq X \subseteq U_{M_i}(X)$.

(2) $L_{M_i}(X) = U_{M_i}(X) = X$.

(3) $L_{M_i}(\emptyset) = U_{M_i}(\emptyset) = \emptyset$.

(4) *If* $X \subseteq Y$ *then* $L_{M_i}(X) \subseteq L_{M_i}(Y)$, *and* $U_{M_i}(X) \subseteq U_{M_i}(Y)$.

(5) $L_{M_i}(X) = -[U_{M_i}(-X))]$,*where* $-X$ *is the complement of* X.

(6) *If* $U_{M_i}(X) = -[L_{M_i}(-X))]$,*where* $-X$ *is the complement of* X.

(7) $L_{M_i}(L_{M_i})(X) = L_{M_i}(X)$

Proposition 3.9. *Let* $(\mathcal{U}, \tau_{M_i}(X))$ *be nano topological space induced by complementary neighbourhoods on* $\mathcal{U}$ *with respect to* $X \subseteq \mathcal{U}$. *Let* $X, Y \subseteq \mathcal{U}$. *Then*

(1) $L_{M_i}(X \bigcap Y) = L_{M_i}(X) \bigcap L_{M_i}(Y)$.

(2) $U_{M_i}(X \bigcup Y) = U_{M_i}(X) \bigcup U_{M_i}(Y)$.

(3) $L_{M_i}(X) \bigcup L_{M_i}(Y) \subseteq L_{M_i}(X \bigcup Y)$.

(4) $U_{M_i}(X \bigcap Y) \subseteq U_{M_i}(X) \bigcap L_{M_i}(Y)$.

4. Complement of Covering in Nano Topology

In this section, we define the complement of covering based on generalized covering approximation space through the concept of nano topology and their properties are discussed.

Definition 4.1. *Let* (U, R, C_n) *be a generalized covering approximation space. We call*

(1) $RC^c = \{-K : K \in RC\}$ *the complement of right covering.*

(2) $LC^c = \{-K : K \in LC\}$ *the complement of left covering.*

Definition 4.2. *Let* $\mathcal{U}$ *be a non empty finite set of objects called the universe and* R *be arbitrary binary relation on* $\mathcal{U}$. *The triple pair* $(\mathcal{U}, R, C_n)$ *is said to be generalized covering approximation space. Let* $X \subseteq \mathcal{U}$ *and complement of covering lower* ($L_{N_i}(X)$), *complement of covering upper approximations* ($U_{N_i}(X)$) *and complement of covering boundary region* ($B_{N_i}(X)$) *of* X *and for each* $i = RC, LC$ *are defined respectively as follows:*

(1) $L_{N_i}(X) = \bigcup_{x \in \mathcal{U}}\{-N_i(x) : N_i(x) \subseteq X\}$.

(2) $U_{N_i}(X) = \bigcup_{x \in \mathcal{U}} \{-N_i(x) : N_i(x) \bigcap X \neq \emptyset\}$.

(3) $B_{N_i}(X) = U_{N_i}(X)$ - $L_{N_i}(X)$.

Definition 4.3. *Let $\mathcal{U}$ be universe, $N_i(x)$ be two types of complement of covering where $i = \{RC, LC\}$ and $\tau_{N_i}(X) = \{\mathcal{U}, \emptyset, L_{N_i}(X), U_{N_i}(X), B_{N_i}(X)\}$ forms a nano topology on $\mathcal{U}$ with respect to X. We call $\{\mathcal{U}, \tau_{N_i}(X)\}$ as the nano topology induced by complement of right and left covering.*

Example 4.4. Let us consider the example 3.10 $N_{RC}(a) = \{a\}, N_{RC}(b) = \{a, b\}$, $N_{RC}(c) = \{b, c\}, N_{RC}(d) = \{d\}$ and $N_{LC}(a) = \{a\}, N_{LC}(b) = \{b\}, N_{LC}(c) = \{a, c\}, N_{LC}(d) = \{d\}$ and $X = \{a\}$. Then -$N_{RC}(a) = \{b, c, d\}, -N_{RC}(b) = \{c, d\}, -N_{RC}(c) = \{a, d\}, -N_{RC}(d) = \{a, b, c\}$ and $-N_{LC}(a) = \{b, c, d\}, -N_{LC}(b) = \{a, c, d\}, -N_{LC}(c) = \{b, d\}, -N_{LC}(d) = \{a, b, c\}$. Hence $\tau_{N_{RC}}(X) = \{\mathcal{U}, \emptyset, \{b, c, d\}\}$, and $\tau_{N_{LC}}(X) = \{\mathcal{U}, \emptyset, \{b, c, d\}\}$.

Definition 4.5. *Let $(\mathcal{U}, R, C_n)$ be a generalized covering approximation space and $X \subseteq \mathcal{U}$. We define the characterization of five basic types of complement of covering induced by nano topological space as follows as:*

(i) If $L_{N_i}(X) = \phi$ and $U_{N_i}(X) = \mathcal{U}$, then $\tau_{N_i}(X) = \{\mathcal{U}, \phi\}$ is called as indiscrete nano topology induced by complement of covering in $\mathcal{U}$.

(ii) If $L_{N_i}(X) = U_{N_i}(X) = \mathcal{U}$, then the nano topology induced by complement of covering ,$\tau_{N_i}(X) = \{\mathcal{U}, \phi, L_{N_i}(X)\}$.

(iii) If $L_{N_i}(X) = \phi$ and $U_{N_i}(X) \neq \mathcal{U}$, then nano topology induced by complement of covering ,$\tau_{N_i}(X) = \{\mathcal{U}, \phi, U_{N_i}(X)\}$.

(iv) If $L_{N_i}(X) \neq \phi$ and $U_{N_i}(X) = \mathcal{U}$, then nano topology induced by complement of covering $\tau_{N_i}(X) = \{\mathcal{U}, \phi, L_{N_i}(X), B_{N_i}(X)\}$.

(v) If $L_{N_i}(X) \neq U_{N_i}(X)$, where $L_{N_i}(X) \neq \phi$ and $U_{N_i}(X) \neq \mathcal{U}$, then discrete nano topology induced by complement of covering in $\tau_{N_i}(X) = \{\mathcal{U}, \emptyset, L_{N_i}(X), U_{N_i}(X), B_{N_i}(X)\}$.

Proposition 4.6. *Let (U, R, C_n) be a generalized covering approximation space. Then C^c is a covering if and only if $\bigcap C = \emptyset$ and $U \notin C$.*

Proof. Let C^c is a covering if and only if $\bigcup_{K \in C^c} K = \mathcal{U}$ and $\phi \notin C^c$ if and only if $\bigcup_{-K \in C} K = \mathcal{U}$ and $\mathcal{U} \notin C$ if $\bigcap_{-K \in C} -K = \phi$ and $\mathcal{U} \notin C$ if and only if $\bigcap C = \phi$ and $\mathcal{U} \notin C$.

Example 4.7. Let $\mathcal{U} = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2), (2, 1), (2, 3), (3, 3)\}$ is a covering of $\mathcal{U}$ and $1R = \{1, 2\}, 2R = \{1, 3\}, 3R = \{3\}$. Then $RC = \{\{1, 2\}, \{1, 3\}, \{3\}\}$.

Therefore $\bigcap RC = \phi$ and $\{1,2,3\} \notin RC$. Then $RC^c = \{\{3\},\{2\},\{1,2\}\}$ is a covering of U.

Proposition 4.8. *Let $(\mathcal{U}, R, C_n)$ be a generalized covering approximation space. For any $x \in \mathcal{U}$, $N_i(x) = N_{i\bigcup\{\mathcal{U}\}}(x)$ and for each $i = RC, LC$.*

Proof. $N_{i\bigcup\{\mathcal{U}\}}(x) = \bigcap\{K \in RC \bigcup\{\mathcal{U}\} : x \in K\} = \bigcap\{K \in RC : x \in K\} \bigcap \mathcal{U} = \bigcap\{K \in RC : x \in K\} = N_i(x)$.Hence, similarly proved by left covering.

Definition 4.9. *Let $\{\mathcal{U}, \tau_{N_i}(X)\}$ be a nano topology induced by complement of covering over $\mathcal{U}$.Then nano topology induced by complement of covering interior $A \subseteq \mathcal{U}$ is denoted by A°.Thus A° is the largest nano topology induced by complement of covering open set contained in $\mathcal{U}$ and is defined as the union of all nano topology induced by complement of covering open set contained in X.*

Example 4.10. From Example 4.4 Let $A = \{b, c, d\}$, then nano topology induced by complement of covering interior is $A^\circ = \{b, c, d\}$.

Theorem 4.11. *Let $\{\mathcal{U}, \tau_{N_i}(X)\}$ be a nano topology induced by complement of covering over $\mathcal{U}$ and $A \subseteq \mathcal{U}$ and A is an nano topology induced by complement of covering open set if and only if $A = A^\circ$.*

Proof. If A is an nano topology induced by complement of covering open set,then the largest nano topology induced by complement of covering open set that is contained by A is equal to A°.Therefore $A = A^\circ$.Conversely,It is know that A° is a nano topology induced by complement of covering open set,and if $A^\circ = A$,then A is an nano topology induced by complement of covering open set.

Theorem 4.12. *Let $\{\mathcal{U}, \tau_{N_i}(X)\}$ be a nano topology induced by complement of covering and $A, B \subseteq \mathcal{U}$. Then*

(a) $[A^\circ]^\circ = A^\circ$.

(b) $A \subseteq B \Rightarrow A^\circ \subseteq B^\circ$.

(c) $A^\circ \cap B^\circ = [A \cap B]^\circ$.

(d) $A^\circ \cup B^\circ \subseteq [A \cap B]^\circ$.

Proof.

(a) Let $A^\circ = C$.Then $C \in \tau_{N_i}(X)$ if and only if $A = C^\circ$. Therefore,$[A^\circ]^\circ = A^\circ$.

(b) $A \subseteq B$,from a definition 4.9 of a nano topology induced by complement of covering interior,$A^\circ \subseteq A$, $B^\circ \subseteq B$,B° is the largest nano topology induced by complement of covering open set that is contained by I_D.Hence $A \subseteq B \Rightarrow A^\circ \subseteq B^\circ$.

(c) By the definition of a nano topology induced by complement of covering interior,$A^\circ \subseteq A$ and $B^\circ \subseteq B$.Then $A^\circ \cap B^\circ \subseteq A \cap B$,$[A \cap B]^\circ$is the largest nano topology induced by complement of covering open set that is contained by $A \cap B$.Hence $A^\circ \cap B^\circ \subseteq [A \cap B]^\circ$.Conversely, $A \cap B \subseteq A$,$A \cap B \subseteq B$.Then $[A \cap B]^\circ \subseteq A^\circ$ and $[A \cap B]^\circ \subseteq B^\circ$.Therefore $[A \cap B]^\circ \subseteq A^\circ \cap B^\circ$.

(d) Now,we consider $A^\circ \subseteq A$,$B^\circ \subseteq B$.Then $A^\circ \cup B^\circ \subseteq A \cup B$.Then $A^\circ \cup B^\circ$ is the largest nano topology induced by complement of covering open set that is contained by $A \cup B$.Hence $A^\circ \cup B^\circ \subseteq [A \cap B]^\circ$.

Definition 4.13. *Let $\{\mathcal{U}, \tau_{N_i}(X)\}$ be a nano topology induced by complement of covering over $\mathcal{U}$. Then nano topology induced by complement of covering $A \subseteq \mathcal{U}$ is denoted by $\overline{A}$.Thus $\overline{A}$ is the smallest nano topology induced by complement of covering closed set which containing A and is defined as the intersection of all nano topology induced by complement of covering closed supersets of A.*

Example 4.14. By example 4.4, $A = \{a\}$ and the complement of nano topology induced by complement of covering $[\tau_{N_i(X)}]^c = \{\tilde{\mathcal{U}}, \tilde{\phi}, \{a\}\}$ is a nano topology induced by complement of covering closed set ,then nano topology induced by complement of covering closure $\overline{A} = \{a\} = A$.

Theorem 4.15. *Let $\{\mathcal{U}, \tau_{N_i}(X)\}$ be a nano topology induced by complement of covering and $A, B \subseteq \mathcal{U}$. Then*

(a) $\overline{\phi} = \phi$ *and* $\overline{\mathcal{U}} = \mathcal{U}$.

(b) $A \subseteq \overline{A}$.

(c) *A is a nano topology induced by complement of covering closed set if and only if $A = \overline{A}$.*

(d) $\overline{\overline{A}} = \overline{A}$.

(e) $B \subseteq A \Rightarrow \overline{B} \subseteq \overline{A}$.

(f) $\overline{A} \cap \overline{B} \subseteq \overline{[A \cap B]}$.

(g) $\overline{A} \cup \overline{B} = \overline{[A \cup B]}$.

Proof:

(a) The proof (a) and (b) are obvious.

(c) If A is a nano topology induced by complement of covering closed set over $\mathcal{U}$ then A is itself a nano topology induced by complement of covering closed set over $\mathcal{U}$ which contains A. So A is the smallest nano topology induced by complement of covering closed set containing A and $A = \overline{A}$.Conversely suppose that $A = \overline{A}$. Since $\overline{A}$ is a nano topology induced by complement of covering closed set,so A is a nano topology induced by complement of covering closed set over $\mathcal{U}$.

(d) Let $A = H_D$. Then, H_D is a nano topology induced by complement of covering closed set. Therefore,H_D and $\overline{H}_D$ are equal.Hence $\overline{\overline{A}} = \overline{A}$.

(e) Let $A \subseteq B$,by the definition of nano topology induced by complement of covering closure,$A \subseteq \overline{A}$ and $B \subseteq \overline{B}$. $\overline{B}$ is the smallest nano topology induced by complement of covering closed set that containing by B.Then $\overline{A} \subseteq \overline{B}$.

(f) Let $A$and B are nano topology induced by complement of covering closed sets. So $\overline{A} \cap \overline{B}$ is a nano topology induced by complement of covering closed set. Since $A \cap B \subseteq \overline{A} \cap \overline{B}$ and $\overline{A \cap B}$ is the smallest nano topology induced by complement of covering closed set that containing $A \cap B$.Hence $\overline{A} \cap \overline{B} \subseteq \overline{[A \cap B]}$.

(g) Let $A \subseteq \overline{A}$ and $B \subseteq \overline{B}$. Then $A \cup B \subseteq \overline{A} \cup \overline{B}$.Since $\overline{(A \cup B)}$ is the smallest nano topology induced by complement of covering closed set that containing $A \cap B$ and $\overline{(A \cup B)} \subseteq \overline{A} \cup \overline{B}$.Conversely,$B \subseteq \overline{B} \subseteq \overline{(A \cup B)}$ and $A \subseteq \overline{A} \subseteq \overline{(A \cup B)}$.Therefore,$\overline{A} \cup \overline{B} \subseteq \overline{(A \cup B)}$.Hence, $\overline{A} \cup \overline{B} = \overline{[A \cup B]}$.

5. Conclusion

This paper, newly defined a nano topology induced by complementary neighbourhood and complement of covering for computing general binary relation. We discuss characterization for the complement of covering and its properties are defined. Finally, we presented a properties of complement of covering interior and closure are discussed.

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