

## FUZZY $gp^*$ -CLOSED SETS IN FUZZY TOPOLOGICAL SPACE

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**Abstract:** In this paper fuzzy  $gp^*$ - closed sets, fuzzy  $gp^*$  continuous functions, fuzzy  $gp^*$ -irresolute functions, fuzzy  $gp^*$ -connectedness and fuzzy  $T^*gp$ -space are introduced and also their relation with some other fuzzy sets and some of their properties are investigated.

**Keywords and Phrases:** Fuzzy topological spaces; fuzzy  $gp^*$ -closed sets; fuzzy  $gp^*$  continuous functions and fuzzy  $gp^*$ -irresolute functions; fuzzy  $gp^*$ -open sets; fuzzy  $T^*gp$ -space.

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### 1. Introduction

Fuzzy set theory as introduced by Lotfi A. Zadeh [1] in 1965 is the expansion of the classical set theory and it expanded the basic definition of the classical or crisp sets. So fuzzy mathematics is just a kind of mathematics developed in this framework and fuzzy topology introduced by C.L Chang [2] in 1968 is the generalization of ordinary topology in classical mathematics. Since the introduction of fuzzy sets and fuzzy topological spaces, work started taking place at a good rate in this field of mathematics and various types of fuzzy sets were introduced and studied by various researchers, Like S.S Benchalli and G.P.Siddapur introduced fuzzy  $g^*$  pre continuous maps[3], Hamid Reza Moradi and Anahid Kamali introduced fuzzy strongly  $g^*$  -closed sets and  $g^{**}$ -closed sets in 2015 [4], And almost all the mathematical, engineering, medicinal etc concepts have been redefined using fuzzy theory and it has further deepened the understanding of basic set theory.

In this paper fuzzy  $gp^*$ - closed sets is defined and its relation with other sets like fuzzy closed sets, fuzzy  $g^*$ -closed sets and  $g^*p$ -closed sets are found and also some other properties of these sets are investigated. Moreover fuzzy  $gp^*$ -open sets are introduced and their relation with other fuzzy sets are found. Fuzzy  $gp^*$ - continuous function and fuzzy  $gp^*$ -irresolute functions are defined and their relation with other fuzzy functions are investigated, also investigated some other properties of these functions. Fuzzy  $gp^*$ -connectedness and fuzzy  $T^*gp$ -spaces in fuzzy topological spaces are also introduced and some of their properties are investigated.

## 2. Preliminaries

**Definition 2.1.** [1] Let  $X$  be a space of objects, with a generic element of  $X$  denoted by  $x$ . Then a fuzzy set  $A$  in  $X$  is a set of ordered pairs  $\{(x, f(x))\}$  where  $f_A(x)$  is called the membership function which associates each point in  $X$  a real number in the interval  $[0,1]$ .

**Definition 2.2.** [2] A family  $\tau$  of fuzzy sets of  $X$  is called fuzzy topology on  $X$  if  $0$  and  $1$  belong to  $\tau$  and  $\tau$  is closed with respect to arbitrary union and finite intersection. The elements of  $\tau$  are called fuzzy open sets and their complements are called fuzzy closed sets. The space  $X$  with topology  $\tau$  is called fuzzy topological space denoted by  $(X, \tau)$ .

**Definition 2.3.** [2] For a fuzzy set  $\alpha$  of  $X$ , the closure  $cl \alpha$  and the interior  $int \alpha$  of  $\alpha$  are defined respectively, as  
 $cl\alpha = \wedge\{\mu : \mu \geq \alpha, 1 - \mu \in \tau\}$  and  
 $int\alpha = \vee\{\mu : \mu \leq \alpha, \mu \in \tau\}$

**Definition 2.4.** [5] A subset  $A$  of  $X$  is called fuzzy pre-closed (in short  $pcl$ ) set if  $A \leq cl(int(A))$  and fuzzy pre-open set if  $A \leq int(cl(A))$ .

**Definition 2.5.** [4] Let  $(X, \tau)$  be a fuzzy topological space. A fuzzy set  $A$  of  $(X, \tau)$  is called fuzzy strongly  $g^*$ -closed if  $cl(int(A)) \leq H$ , whenever  $A \leq H$  and  $H$  is fuzzy generalized -open in  $X$ .

**Definition 2.6.** [6] A fuzzy set  $A$  of a fuzzy topological space  $(X, \tau)$  is called a fuzzy generalized star closed or  $g^* -$ closed if  $cl(A) \leq O$  whenever  $A \leq O$  and  $O$  is fuzzy generalized-open or  $g$ -open.

**Definition 2.7.** [7] A fuzzy set  $A$  of a fuzzy topological space  $(X, \tau)$  is called fuzzy generalized closed or  $g$ -closed if  $cl(A) \leq G$  whenever  $A \leq G$  and  $G \in \tau$  and is called fuzzy generalized open or  $g$ -open if  $1 - A$  is fuzzy  $g$ -closed.

**Definition 2.8.** [8] A fuzzy set  $A$  of a fuzzy topological space  $(X, \tau)$  is called fuzzy

generalized pre -closed or  $gp$ -closed set if  $pcl(A) \leq U$  whenever  $A \leq U$  and  $U$  is a fuzzy open set in  $(X, \tau)$ . And complement of a Fuzzy  $gp$ -closed set is called fuzzy generalized pre-open or  $gp$ -open set.

**Definition 2.9.** [3] A fuzzy set  $A$  of a fuzzy topological space  $(X, \tau)$  is called a fuzzy generalized star pre-closed (briefly  $g^*p$ -closed) set if  $pcl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy  $g$ -open set in  $(X, \tau)$ .

**Definition 2.10.** [2] A function  $f$  from a fts  $(X, \tau)$  to a fts  $(Y, \delta)$  is fuzzy-continuous iff the inverse of each  $\delta$ -open fuzzy set in  $Y$  is  $\tau$ -open fuzzy set in  $X$ .

**Definition 2.11.** [9] A function  $f$  from a fts  $(X, \tau)$  to a fts  $(Y, \delta)$  is fuzzy  $g^*$ -continuous if  $f^{-1}(A)$  is fuzzy  $g^*$ -closed in  $X$  for every fuzzy closed set of  $Y$ .

**Definition 2.12.** [10] A fuzzy topological space  $X$  is said to be fuzzy connected if it has no proper fuzzy clopen set, (A fuzzy set  $\lambda$  in  $X$  is proper if  $\lambda \neq 0$  and  $\lambda \neq 1$ , clopen means closed-open).

**Definition 2.13.** [9] A fuzzy topological space  $(X, \tau)$  is called a fuzzy  $T_{1/2}^*$  space if every  $g^*$ -closed fuzzy set is a closed fuzzy set.

**Definition 2.14.** [3] A fts  $(X, \tau)$  is called a fuzzy- $T_p^*$  - space if every  $g^*p$  closed fuzzy set is closed fuzzy set.

**Theorem 1.** Every fuzzy generalized-closed set is fuzzy generalized pre-closed set.

**Proof.** Let  $\theta$  is a fuzzy  $g$ -closed set and  $\mu$  be a fuzzy open set such that  $\theta \leq \mu$ , then  $cl(\theta) \leq \mu$  and hence  $pcl(\theta) \leq cl(\theta) \leq \mu$  implies  $\theta$  is a fuzzy  $gp$ -closed set.

**Theorem 2.** All fuzzy generalized open sets are fuzzy generalized pre-open sets.

**Proof.** Consider  $\theta$  is a fuzzy generalized open set. Then  $(1 - \theta)$  is a fuzzy generalized closed set. Now by Theorem 1,  $(1 - \theta)$  is a fuzzy generalized pre closed set implying that  $\theta$  is a fuzzy generalized pre-open set.

### 3. Fuzzy $gp^*$ -closed sets

**Definition 3.1.** A fuzzy set  $\lambda$  of a fuzzy topological space (fts)  $(Y, \tau)$  is called fuzzy generalized pre star closed (briefly fuzzy  $gp^*$ -closed) if  $cl(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy generalized pre-open in  $Y$ .

**Example 3.2.** Let  $Y = \{y\}$  and  $\tau = \{0_Y, y_{2/3}, y_{3/4}, 1_Y\}$ . Then in this fuzzy topological space  $(Y, \tau)$ , fuzzy sets  $0_Y$ ,  $A = y_{1/3}$ ,  $B = y_{1/4}$  and  $1_Y$  satisfy the condition  $cl(\lambda) \leq \mu$  whenever  $\lambda \leq \mu$  and  $\mu$  is fuzzy generalized pre-open in  $Y$ . Implying  $0_Y$ ,  $A = y_{1/3}$ ,  $B = y_{1/4}$  and  $1_Y$  are fuzzy  $gp^*$ -closed sets in  $(Y, \tau)$ .

**Theorem 3.3.** *All fuzzy closed sets are fuzzy  $gp^*$  closed sets.*

**Proof.** Consider  $\theta$  is a fuzzy closed set in fuzzy topological space  $Y$  and  $\mu$  is a fuzzy generalized pre-open set in  $Y$  containing  $cl(\theta) \leq \theta = \mu$ . Implying that  $\theta$  is a fuzzy  $gp^*$ -closed set in  $Y$ .

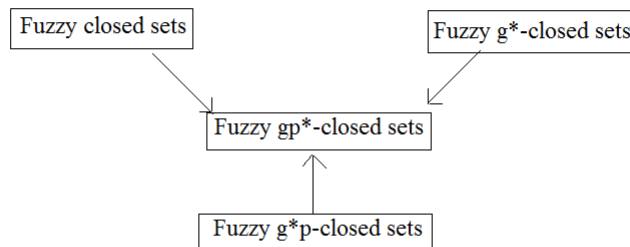
**Theorem 3.4.** *All fuzzy generalized star pre-closed sets are fuzzy  $gp^*$  closed.*

**Proof.** Consider  $\sigma$  is any arbitrary fuzzy generalized star pre closed set in fuzzy topological space  $(Y, \tau)$ . Let  $\sigma$  is contained in fuzzy generalized open  $\mu$ . Now as every fuzzy generalized open set is fuzzy generalized pre-open set (By Theorem 2) so  $pcl(\sigma) \leq cl(\sigma) \leq \mu$ . Implying  $cl(\sigma) \leq \mu$ , Which in turn implies that  $\sigma$  is fuzzy  $gp^*$ -closed.

**Theorem 3.5.** *All fuzzy  $g^*$ -closed sets are fuzzy  $gp^*$ -closed sets.*

**Proof.** Consider  $\theta$  is a fuzzy  $g^*$ -closed set in fuzzy topological space  $(Y, \tau)$  and  $\mu$  is any generalized-open set that contains  $\theta$ . Now as every fuzzy generalized-open set is fuzzy generalized pre-open set (By theorem 2), so  $cl(\theta) \leq \mu$ , where  $\mu$  is a fuzzy generalized pre-open set in  $Y$ . Implying that  $\theta$  is fuzzy  $gp^*$ -closed set.

**Remark 3.6.** *The following diagram depicts the relation of fuzzy  $gp^*$ -closed set and other fuzzy sets discussed above.*



**Theorem 3.7.** *The Union of two fuzzy  $gp^*$ -closed sets  $\Delta$  and  $\nabla$  in fuzzy topological space  $(Y, \tau)$  is also fuzzy  $gp^*$ -closed set in  $Y$ .*

**Proof.** Suppose that  $\Delta$  and  $\nabla$  are two fuzzy  $gp^*$ -closed sets in  $Y$ . Let  $\mu$  be a fuzzy generalized pre-open set that contains both  $\Delta$  and  $\nabla$ . so  $cl(\Delta) \leq \mu$  and

$cl(\nabla) \leq \mu$ . Now, as  $\Delta \leq \mu$  and  $\nabla \leq \mu$  implying that  $\Delta \cup \nabla \leq \mu$  which inturn implies  $cl(\Delta \cup \nabla) = cl(\Delta) \cup cl(\nabla) \leq \mu$ , which gives the required result i.e  $\Delta \cup \nabla$  is also a fuzzy  $gp^*$ -closed set in  $Y$ .

**Theorem 3.8.** *If  $\Delta$  and  $\nabla$  are two fuzzy  $gp^*$ -closed sets in fuzzy topological space  $(Y, \tau)$  then  $\Delta \cap \nabla$  is also fuzzy  $gp^*$ -closed in  $Y$ .*

**Proof.** Suppose  $\Delta$  &  $\nabla$  are two fuzzy  $gp^*$ -closed sets in fuzzy topological space  $Y$ , such that  $\Delta \leq \mu$  and  $\nabla \leq \mu$ , where  $\mu$  is a fuzzy generalized pre-open set in  $Y$ . Then  $cl(\Delta) \leq \mu$ ,  $cl(\nabla) \leq \mu$  therefore  $cl(\Delta \cap \nabla) \leq \mu$ , where  $\mu$  is fuzzy generalized pre-open set in  $Y$ . Which implies that  $\Delta \cap \nabla$  is also fuzzy  $gp^*$ -closed set in  $Y$ .

**Theorem 3.9.** *Suppose that  $\mu$  is fuzzy  $gp^*$ -closed set in fuzzy topological space  $\Delta$  such that  $\mu \leq \nabla \leq \Delta$ , then  $\mu$  is also fuzzy  $gp^*$ -closed relative to  $\nabla$ .*

**Proof.** Given that  $\mu \leq \nabla \leq \Delta$ , where  $\mu$  is fuzzy  $gp^*$ -closed in  $\Delta$ . Now suppose that  $\mu \leq \nabla \cap \theta$ , where  $\theta$  is fuzzy generalized pre-open in  $\Delta$ . As  $\mu$  is fuzzy  $gp^*$ -closed,  $\mu \leq \theta$  implies  $cl(\mu) \leq \theta$ . Which implies that  $\nabla \cap cl(\mu) \leq \nabla \cap \theta$  i.e  $\mu$  is also fuzzy  $gp^*$ -closed relative to  $\nabla$ .

#### 4. Fuzzy $gp^*$ -open sets

**Definition 4.1.** *Suppose a fuzzy set  $\lambda$  is fuzzy generalized pre star closed set in fts  $(Y, \tau)$ , Then its complement i.e  $1 - \lambda$  is called fuzzy generalized pre star open (briefly fuzzy  $gp^*$ -open) in  $(Y, \tau)$ .*

**Example 4.2.** In the fuzzy topological space  $(Y, \tau)$  defined in example 3.2, the complements of the fuzzy  $gp^*$ -closed sets  $0_Y$ ,  $A = y_{1/3}$ ,  $B = y_{1/4}$  and  $1_Y$  are respectively as  $1_Y$ ,  $C = y_{2/3}$ ,  $D = y_{3/4}$  and  $0_Y$ . Implying  $1_Y$ ,  $C = y_{2/3}$ ,  $D = y_{3/4}$  and  $0_Y$  are fuzzy  $gp^*$ -open sets in  $(Y, \tau)$ .

**Theorem 4.3.** *All fuzzy open sets are fuzzy  $gp^*$ -open.*

**Proof.** Consider  $\mu$  is a fuzzy open set in fuzzy topological space  $(Y, \tau)$ , implies  $1 - \mu$  is a fuzzy closed set. Now from the theorem 3.3 all fuzzy closed sets are fuzzy  $gp^*$ -closed sets. So  $1 - \mu$  is also a fuzzy  $gp^*$ -closed set implying that  $\mu$  is fuzzy  $gp^*$ -open in fuzzy topological space  $(Y, \tau)$ .

**Theorem 4.4.** *The intersection of two fuzzy  $gp^*$ -open sets  $\Delta$  and  $\nabla$  in fuzzy topological space  $(Y, \tau)$  is also a fuzzy  $gp^*$ -open set in  $(Y, \tau)$ .*

**Proof.** Suppose  $\Delta$  &  $\nabla$  are two fuzzy  $gp^*$ -open sets in fuzzy topological space  $(Y, \tau)$ . Which implies  $1 - \Delta$  and  $1 - \nabla$  are fuzzy  $gp^*$ -closed in  $(Y, \tau)$ . Now according to theorem 3.9  $(1 - \Delta) \cup (1 - \nabla)$  is also a fuzzy  $gp^*$ -closed in  $(Y, \tau)$ . So  $(1 - \Delta) \cup (1 - \nabla) = (1 - (\Delta \cap \nabla))$  is fuzzy  $gp^*$ -closed in  $(Y, \tau)$ . Implying that  $\Delta \cap \nabla$  is also a fuzzy  $gp^*$ -open set in  $(Y, \tau)$ .

## 5. Fuzzy $gp^*$ -continuous mappings

**Definition 5.1.** If  $G$  and  $H$  are two fuzzy topological spaces then a mapping  $g : G \rightarrow H$  is called fuzzy  $gp^*$ -continuous mapping if  $g^{-1}(\phi)$  is fuzzy  $gp^*$ -open set in  $G$ , for every fuzzy open  $\phi$  of  $H$ .

**Definition 5.2.** If  $G$  and  $H$  are two fuzzy topological spaces then a mapping  $g : G \rightarrow H$  is called fuzzy  $gp^*$ -irresolute mapping if  $g^{-1}(\phi)$  is fuzzy  $gp^*$ -closed set in  $G$ , for every fuzzy  $gp^*$ -closed set  $\phi$  of  $H$ .

**Theorem 5.3.** A function  $g : G \rightarrow H$  is fuzzy  $gp^*$ -continuous if & only if the inverse image of each fuzzy closed set in  $H$  is fuzzy  $gp^*$ -closed set in  $G$ .

**Proof.** Suppose that  $G$  and  $H$  are two fuzzy topological spaces and  $g : G \rightarrow H$  be a fuzzy  $gp^*$ -continuous function. Let  $\alpha$  be a fuzzy closed set in  $H$  implies that  $1 - \alpha$  is a fuzzy open set in  $H$ . Now as  $g$  is a fuzzy  $gp^*$ -continuous function implies  $g^{-1}(1 - \alpha) = 1 - g^{-1}(\alpha)$  is a fuzzy  $gp^*$ -open set in  $G$ , implying  $g^{-1}(\alpha)$  is a fuzzy  $gp^*$ -closed set in  $G$ . Conversely let's suppose that  $\alpha$  is a fuzzy closed set in  $H$  and  $g^{-1}(\alpha)$  is fuzzy  $gp^*$ -closed in  $G$ . Now  $1 - \alpha$  is a fuzzy open set in  $H$  and  $g^{-1}(1 - \alpha) = 1 - g^{-1}(\alpha)$  is fuzzy  $gp^*$ -open, which was the required proof.

**Theorem 5.4.** All fuzzy continuous functions are fuzzy  $gp^*$ -continuous.

**Proof.** Suppose that  $G$  and  $H$  are two fuzzy topological spaces and  $g : G \rightarrow H$  be a fuzzy continuous function. Now, suppose  $\alpha$  is a fuzzy open set in  $H$  & as  $g$  is fuzzy continuous function implies  $g^{-1}(\alpha)$  is fuzzy open set in  $G$ . So by theorem 4.3  $g^{-1}(\alpha)$  is fuzzy  $gp^*$ -open set in  $G$ , implying that  $g : G \rightarrow H$  is a fuzzy  $gp^*$ -continuous function.

**Theorem 5.5.** All fuzzy  $g^*$ -continuous functions are fuzzy  $gp^*$ -continuous function.

**Proof.** Let  $G$  and  $H$  are two fuzzy topological spaces and  $g : G \rightarrow H$  be a fuzzy  $g^*$ -continuous function. Now, suppose  $\alpha$  is a fuzzy closed set in  $H$  & as  $g$  is fuzzy  $g^*$ -continuous function implies  $g^{-1}(\alpha)$  is fuzzy generalized star-closed set in  $G$ . Now as by theorem 3.4 All fuzzy generalized star pre-closed sets are fuzzy  $gp^*$  closed implying that  $g^{-1}(\alpha)$  is also a fuzzy  $gp^*$ -closed set, means  $g$  is fuzzy  $gp^*$ -continuous.

**Theorem 5.6.** If  $G, H$  and  $I$  are fuzzy topological spaces and  $j : G \rightarrow H$  &  $k : H \rightarrow I$  are such that  $k$  is a fuzzy  $gp^*$ -continuous function and  $j$  is fuzzy  $gp^*$ -irresolute, then  $koj$  is a fuzzy  $gp^*$  continuous function.

**Proof.** Suppose  $\alpha$  is a fuzzy closed set in  $I$ . Also  $(koj)^{-1}(\alpha) = j^{-1}(k^{-1}(\alpha))$ . Now as  $k$  is fuzzy  $gp^*$ -continuous, so by its definition  $A = k^{-1}(\alpha)$  is a fuzzy  $gp^*$ -closed set in  $H$ . Now as  $j$  is a fuzzy  $gp^*$ -irresolute implies  $j^{-1}(A) = j^{-1}(k^{-1}(\alpha))$  is also fuzzy  $gp^*$ -closed set in  $G$ , implying that  $koj$  is a fuzzy  $gp^*$  continuous function.

**Theorem 5.7.** *Suppose  $j : G \rightarrow H$  &  $k : H \rightarrow I$  are such that  $k$  is a fuzzy continuous function and  $j$  is fuzzy  $gp^*$ -continuous, then  $koj$  is a fuzzy  $gp^*$  continuous function.*

**Proof.** Suppose  $\alpha \leq I$  be any fuzzy closed set in  $I$ . Also  $(koj)^{-1}(\alpha) = j^{-1}(k^{-1}(\alpha))$ . Now as  $k$  is a fuzzy continuous function, implies  $A = k^{-1}(\alpha)$  is a fuzzy closed set in  $H$ . Now  $j$  is a fuzzy  $gp^*$ -continuous function, implying that  $j^{-1}(A) = j^{-1}(k^{-1}(\alpha))$  is a fuzzy  $gp^*$ -closed set in  $G$ . Which shows that  $koj$  is a fuzzy  $gp^*$ -continuous function by theorem 5.3.

**Theorem 5.8.** *Suppose  $j : G \rightarrow H$  &  $k : H \rightarrow I$  are fuzzy  $gp^*$ -irresolute functions, then  $koj$  is also fuzzy  $gp^*$ -irresolute function.*

**Proof.** Let  $\alpha \leq I$  be any fuzzy  $gp^*$ -closed set in  $I$ . Also  $(koj)^{-1}(\alpha) = j^{-1}(k^{-1}(\alpha))$ . Now as  $k$  is a fuzzy  $gp^*$ -irresolute function, implies  $A = k^{-1}(\alpha)$  is a fuzzy  $gp^*$ -closed set in  $H$ . Now as  $j$  is also fuzzy  $gp^*$ -irresolute, implying that  $j^{-1}(A) = j^{-1}(k^{-1}(\alpha))$  is a fuzzy  $gp^*$ -closed set in  $G$ . So by definition 5.2  $koj$  is also a fuzzy  $gp^*$ -irresolute function.

## 6. Fuzzy $gp^*$ -connectedness

**Definition 6.1.** *A fuzzy  $gp^*$ -connected space is a fuzzy topological space  $(Y, \tau)$  that cannot be written as the union of two non-empty disjoint fuzzy  $gp^*$ -open sets in  $(Y, \tau)$ .*

**Theorem 6.2.** *If  $(Y, \tau)$  is a fts, then the following are equivalent;*

- (a)  $Y$  is a fuzzy  $gp^*$ -connected space.
- (b) The only subsets in  $Y$  which are both fuzzy  $gp^*$ -open and fuzzy  $gp^*$ -closed are  $0_Y$  &  $1_Y$ .

**Proof.** (a)  $\Rightarrow$  (b): Let  $Y$  is a fuzzy  $gp^*$ -connected space. Now, suppose  $\alpha < Y$  is both fuzzy  $gp^*$ -open & fuzzy  $gp^*$ -closed. Then  $1 - \alpha$  is also both fuzzy  $gp^*$ -closed & fuzzy  $gp^*$ -open. So  $Y = \alpha \vee (1 - \alpha)$  is the union of two disjoint non empty fuzzy  $gp^*$ -open sets, which contradicts (a). Implying  $\alpha = 0_Y$  or  $\alpha = 1_Y$ .

(b)  $\Rightarrow$  (a): Suppose  $\alpha$  &  $\beta$  are non-empty disjoint fuzzy  $gp^*$ -open sets such that  $Y = \alpha \vee \beta$ . Now  $\alpha = 1 - \beta$  &  $\beta = 1 - \alpha$  are fuzzy  $gp^*$ -open sets, which in turn implies  $\alpha$  &  $\beta$  are also fuzzy  $gp^*$ -closed sets. Now by (b)  $\alpha = 0_Y$  or  $\alpha = 1_Y$  implies  $Y$  is fuzzy  $gp^*$ -connected.

**Theorem 6.3.** *All fuzzy  $gp^*$ -connected spaces are fuzzy connected spaces.*

**Proof.** Let  $Y$  is a fuzzy  $gp^*$ -connected space and suppose that  $Y$  is not a connected space. Then by Definition 2.12 there exists a non-empty proper fuzzy clopen subset  $\lambda$  in  $Y$ . Now as every fuzzy closed set is fuzzy  $gp^*$ -closed implying that  $\lambda$  is also a non-empty proper subset of  $Y$ , which is both fuzzy  $gp^*$ -closed and fuzzy

$gp^*$ -open in  $Y$ . So by Theorem 6.2  $Y$  is not a fuzzy  $gp^*$ -connected space, which is a contradiction implying that  $Y$  is a connected space.

**Theorem 6.4.** *Suppose  $g : G \rightarrow H$  is an onto fuzzy  $gp^*$ -continuous map and  $G$  is a fuzzy  $gp^*$ -connected space then  $H$  is also a fuzzy connected space.*

**Proof.** Let's suppose that  $H$  is not a fuzzy connected space and suppose that  $H = M \vee N$ , where  $M$  &  $N$  are disjoint fuzzy non-empty open sets in  $H$ . Since  $g$  is fuzzy  $gp^*$ -continuous implies  $g^-(M)$  &  $g^-(N)$  are non-empty disjoint fuzzy  $gp^*$ -open sets in  $G$  and as  $g$  is onto also implies  $G = g^{-1}(M) \vee g^{-1}(N)$ , which contradicts fuzzy  $gp^*$ -connectedness of  $G$ . So  $H$  is a fuzzy connected space.

**Theorem 6.5.** *Suppose  $g : G \rightarrow H$  is an onto fuzzy  $gp^*$ -irresolute map and  $G$  is fuzzy  $gp^*$ -connected space then  $H$  is also a fuzzy  $gp^*$ -connected space.*

**Proof.** Let's suppose that  $H$  is not a fuzzy  $gp^*$ -connected space and let's suppose that  $H = M \vee N$  where  $M$  &  $N$  are non-empty fuzzy disjoint  $gp^*$ -open sets in  $H$ . Now, as  $g$  is fuzzy  $gp^*$ -irresolute function implies  $g^{-1}(M)$  &  $g^{-1}(N)$  are non-empty disjoint fuzzy  $gp^*$ -open sets in  $G$  and as  $g$  is onto also implies  $G = g^{-1}(M) \vee g^{-1}(N)$ , which contradicts fuzzy  $gp^*$ -connectedness of  $G$ . Implies  $H$  is a fuzzy connected space.

## 7. Fuzzy $T^*gp$ -Space

**Definition 7.1.** *A fts  $(Y, \tau)$  is called a fuzzy  $T^*gp$ -space if every fuzzy  $gp^*$ -closed set in  $(Y, \tau)$  is a fuzzy closed set in  $(Y, \tau)$ .*

**Theorem 7.2.** *Every fuzzy  $T^*gp$ -space is fuzzy  $T^*p$ -space.*

**Proof.** Let  $Y$  be a fuzzy  $T^*gp$ -space. Let  $A$  be a fuzzy  $g^*p$ -closed set in  $Y$ . Now by Theorem 3.4 as every fuzzy  $g^*p$ -closed set is fuzzy  $gp^*$ -closed set, implies  $A$  is fuzzy  $gp^*$ -closed set in  $Y$ . Since  $Y$  is a fuzzy  $T^*gp$ -space,  $A$  is a fuzzy closed set in  $Y$ . Hence  $Y$  is a fuzzy  $T^*p$ -space.

**Theorem 7.3.** *Every fuzzy  $T^*gp$ -space is fuzzy  $T_{1/2}^*$  space.*

**Proof.** Let  $Y$  be a fuzzy  $T^*gp$ -space. Let  $A$  be a fuzzy  $g^*$ -closed set in  $Y$ . Now by Theorem 3.6,  $A$  is fuzzy  $gp^*$ -closed set in  $Y$ . Since  $Y$  is a fuzzy- $T^*gp$ -space implies  $A$  is fuzzy closed set in  $Y$ . Hence  $Y$  is a  $T_{1/2}^*$  space.

**Theorem 7.4.** *If  $G$  is a fuzzy  $T^*gp$ -space then  $G$  is fuzzy connected iff it is fuzzy  $gp^*$ -connected.*

**Proof.** Let  $G$  is a fuzzy connected space & suppose that  $G$  is not fuzzy  $gp^*$ -connected. Then there exists two proper fuzzy  $gp^*$ -open sets  $M$  &  $N$  of  $G$  such that  $G = M \vee N$  &  $M \wedge N = \phi$ , which implies  $M = 1 - N$  &  $N = 1 - M$  are also fuzzy  $gp^*$ -closed sets and  $G$  is a fuzzy  $T^*gp$ -space implies  $M$  &  $N$  are fuzzy



closed sets (by Definition 7.1). So  $M = 1 - N$  &  $N = 1 - M$  implies  $M$  &  $N$  are fuzzy open sets &  $G = M \vee N$ ,  $M \wedge N = \phi$  contradicts the fuzzy connectedness of  $G$ . So  $G$  is a fuzzy  $gp^*$ -connected space. Conversely suppose that  $G$  is fuzzy  $gp^*$ -connected and let  $G$  is not fuzzy connected implies there exists two proper fuzzy open subsets  $M$  &  $N$  of  $G$  such that  $G = M \vee N$  &  $M \wedge N = \phi$ . Now, as every fuzzy open set is fuzzy  $gp^*$ -open, so  $G = M \vee N$  contradicts the fuzzy  $gp^*$ -connectedness of  $G$ . Implies  $G$  is a fuzzy connected space.

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