South East Asian J. of Mathematics and Mathematical Sciences
Vol. 16, No. 2 (2020), pp. 71-78

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

PROPERTIES OF FUZZY PERFECT INTRINSIC EDGE-MAGIC GRAPHS

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(**Received:** *Jan. 11, 2020* **Accepted:** *May. 31, 2020* **Published:** *Aug. 30, 2020)*

Abstract: In this paper, we have discussed the idea of fuzzy perfect intrinsic edge-magic labelling and perfect intrinsic edge-magic graphs. We have checked fuzzy path, cycle, paw graph, banner graph, star graph & friendship graph are perfect intrinsic edge-magic graphs with intrinsic super constant. The vital and competent condition also discussed for the fuzzy perfect intrinsic edge-magic graphs. Quasi perfect intrinsic edge-magic graphs are also introduced. Some theorems related to stated graphs have been presented.

Keywords and Phrases: Fuzzy perfect intrinsic edge-magic labelling, quasi perfect intrinsic edge-magic graph, intrinsic super constant, weak constant.

2010 Mathematics Subject Classification: 05C72, 05C78.

1. Introduction

Fuzzy set was firstly introduced by [13]. Then various researches added productive concepts to develop fuzzy sets theory like [10] and [3]. In 1987 Bhattacharya has succeeded to develop the connectivity notions between fuzzy bridge and fuzzy cut nodes [2]. A fuzzy graph contains many properties similar to crisp graph due to generalization of crisp graphs but it diverge at many places.

A crisp graph G is an order pair of vertex-set V and edge set E such that $E \subseteq V \times V$. In addition $v = |V|$ is said to order and $e = |E|$ is called size of the graph G respectively. In a crisp graph, a bijective function $\rho : V \cup E \to N$ that produced a unique positive integer (To each vertex and/or edge) is called a labelling [4]. Introduced the notion of magic graph that the labels vertices and edges are natural numbers from 1 to $|V| + |E|$ such that sum of the labels of vertices and the edge between them must be constant in entire graph [4]. Extended the concept of magic graph with added a property that vertices always get smaller labels than edges and named it super edge magic labelling. Numerous other authors have explored diverse types of different magic graphs [1], [5] & [11]. The subject of edge-magic labelling of graphs had its origin in the work of Kotzig and Rosa on what they called magic valuations of graphs [7]. These labelling are currently referred to as either edge-magic labelling or edge-magic total labelling.

Fuzzy graphs are generalization of graphs. In graphs two vertices are either related or not related to each other. Mathematically, the degree of relationship is either 0 or 1. In fuzzy graphs, the degree of relationship takes values from $[0, 1]$. A fuzzy graph has ability to solve uncertain problems in a wide range of fields. The first definition of a fuzzy graph was introduced by Kaufmann in 1973. Azriel Rosenfield in 1975 [10] developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts. In [8], Nagoor Gani et. al. introduced the concepts of fuzzy labelling graphs, fuzzy magic graphs. A fuzzy graph contains many properties similar to crisp graph due to generalization of crisp graphs but it diverge at many places. In this paper we have developed the concept of fuzzy perfect intrinsic edge magic graphs and also we introduced some general form of intrinsic super constant of above graphs. Throughout this paper we only focussed on undirected fuzzy graphs.

2. Preliminaries

Definition 2.1. *A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u)\Lambda\sigma(v)$.*

Definition 2.2. *A path P in a fuzzy graph is a sequence of distinct nodes v_1, v_2, v_3, $\ldots$, v_n such that $\mu(v_i, v_{i+1}) > 0; 1 \leq i \leq n$; here $n \geq 1$ is called the length of the path P. The consecutive pairs (v_i, v_{i+1}) are called the edge of the path.*

Definition 2.3. *A path P is called a cycle if $v_1 = v_n$ and $n \geq 3$ and a cycle is* *called a fuzzy cycle if it contains more than one weakest arc.*

Definition 2.4. *A bijection ω is a function from the set of all nodes and edges of to $[0, 1]$ which assign each nodes $\sigma^{\omega}(a)$, $\sigma^{\omega}(b)$ and edge $\mu^{\omega}(a, b)$ a membership value such that $\mu^{\omega}(a, b) \leq \sigma^{\omega}(a) \Lambda \sigma^{\omega}(b)$ for all $a, b \in V$ is called fuzzy labelling. A graph is said to be fuzzy labelling graph if it has a fuzzy labelling and it is denoted by G^{ω}.*

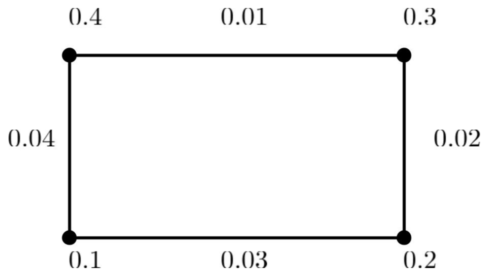

Figure 1: Fuzzy Labelling Graph

Definition 2.5. [6] *A fuzzy labelling graph G is said to be fuzzy intrinsic labelling if $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ is bijective such that the membership values of edges and vertices are $\{z, 2z, 3z, \ldots, Nz\}$ without any repetition where N is the total number of vertices and edges and let $z = 0.1$ for $N < 6$ & $z = 0.01$ for $N \geq 6$.*

Definition 2.6. [6] *A fuzzy intrinsic labelling graph is said to be a fuzzy intrinsic edge-magic labelling if it has an intrinsic constant $\lambda_c = \sigma(v_i) + \mu(v_i v_j) + \sigma(v_j)$ for all $v_i, v_j \in V$.*

Definition 2.7. [6] *A fuzzy graph G is said to be intrinsic edge-magic if it satisfies the intrinsic edge-magic labelling with intrinsic constant$'\lambda_c'$.*

Definition 2.8. [6] *An edge-magic constant in a fuzzy intrinsic edge-magic graph is said to be mock constant $'\lambda_m'$ if it is equal to $\sigma(v_i) + \mu(v_i v_j) + \sigma(v_j)$ for some $v_i, v_j \in V$ with $\lambda_c \neq \lambda_m$.*

Definition 2.9. *A fuzzy intrinsic edge-magic labelling graph is said to be fuzzy intrinsic edge-magic graph if it satisfies both vital and competent condition.*

Definition 2.10. *The friendship graph F can be constructed by joining n-copies of the cycle graph C_3 with a common vertex. The graph F_2 is isomorphic to the butterfly graph.*

Definition 2.11. *The Pan graph is the graph obtained by joining a cycle graph to a singleton graph with a bridge. The 3-Pan graph is sometimes known as the Paw graph.*

3. Fuzzy Perfect Intrinsic Edge-magic Graphs

Definition 3.1. *A fuzzy labelling graph G is said to be fuzzy perfect intrinsic labelling if $f : \sigma \to [0, 1]$ and $f : \mu \to [0, 1]$ is bijective such that the membership values of edges are$\{z, 2z, 3z, \ldots, \varepsilon z\}$ and vertices are $\{(\varepsilon + 1)z, (\varepsilon + 2)z, \ldots.(\varepsilon + \upsilon)z\}$where $\varepsilon + \upsilon = N$ is the total number of vertices and edges and let $z = 0.1$ for $N \leq 5\,\&\, z = 0.01$ for $N > 5$.*

Definition 3.2. *A fuzzy perfect intrinsic labelling graph is said to be a fuzzy perfect intrinsic edge-magic labelling if it has an intrinsic super constant $\lambda_s = \sigma(v_i) + \mu(v_iv_j) + \sigma(v_j)$ for all $v_i, v_j \in V$.*

Definition 3.3. *A fuzzy graph G is said to be perfect intrinsic edge-magic if it satisfies the perfect intrinsic edge-magic labelling with intrinsic super constant $'\lambda_s'$.*

Definition 3.4. *An edge-magic constant in a fuzzy perfect intrinsic edge-magic graph is said to be weak constant $'\lambda_w'$ if it is equal to $\sigma(v_i) + \mu(v_iv_j) + \sigma(v_j)$ for some $v_i, v_j \in V$ with $\lambda_s \neq \lambda_w$.*

Definition 3.5. *A fuzzy graph is said to be a quasi-intrinsic edge-magic graph if it contains atleast one weak constant $'\lambda_w'$which is denoted by $'G_q'$.*
Vital condition: For perfect intrinsic edge-magic, the vital condition is that the above mentioned graph satisfies only the perfect intrinsic edge magic labelling.
Competent condition: A competent condition for perfect intrinsic edge-magic is that if it has the intrinsic super constant for all edges.

Definition 3.6. *A fuzzy perfect intrinsic edge-magic labelling graph is said to be perfect intrinsic edge-magic if it satisfies both vital and competent condition.*

Theorem 3.7. *A path P_n is fuzzy perfect intrinsic edge-magic if $n \geq 2$ where n is length of P_n.*

Theorem 3.8. *A fuzzy cycle $C_n$is fuzzy perfect intrinsic edge-magic iff $n = 3$.*

Theorem 3.9. *A 3-pan graph (paw graph) is fuzzy perfect intrinsic edge-magic.*
Proof. Let G be a 3-Pan graph (Paw graph). Consider the fuzzy perfect intrinsic edge-magic labelling, for 3-pan graph,

$$\begin{aligned}
\sigma(v_{2i}) &= (2n + 2 - i)z \text{ for } 1 \leq i \leq n (i \text{ is odd })\\
\sigma(v_n) &= (2n + 2)z, \qquad \sigma(v_{n+1}) = 2nz\\
\mu(v_1v_n) &= (n - 2)z, \qquad \mu(v_iv_{i+1}) = (n + 2 - i) \text{ for } 1 \leq i \leq n - 1\\
\mu(v_nv_{n+1}) &= (n - 1)z
\end{aligned}$$

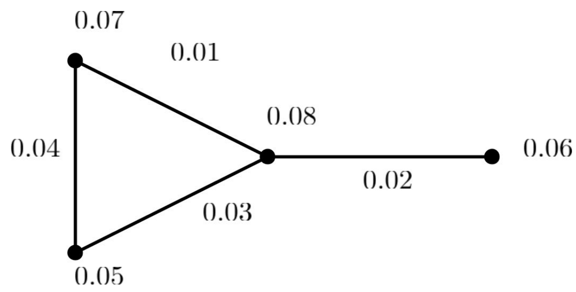

Figure 2:

Now , we consider the above labelling, we get

$$\begin{aligned}\lambda\,(3-\ \text{ pan graph }) &= \sigma\,(v_n) + \mu\,(v_n v_{n+1}) + \sigma\,(v_{n+1})\\ &= (2n+2)z + (n-1)z + 2nz\\ &= (5n+1)z\\ \lambda\,(3-pangraph) &= (5n+1)z\end{aligned}$$

Here, intrinsic super constant $\lambda_s = (5n+1)z$ (In general) i.e., $\lambda_s = 0.16$

In the above observation, the 3-pan graph satisfies both vital & competent condition for intrinsic perfect intrinsic edge-magic. We conclude that a 3-pan graph is fuzzy perfect intrinsic edge-magic.

Theorem 3.10. *A 4-pan graph (Banner graph) is a fuzzy quasi perfect intrinsic edge-magic graph with one weak constant.*

Proof. Let G be a 4-Pan graph with $n = 4$. Consider the fuzzy perfect intrinsic edge-magic labelling, we get the following graph.

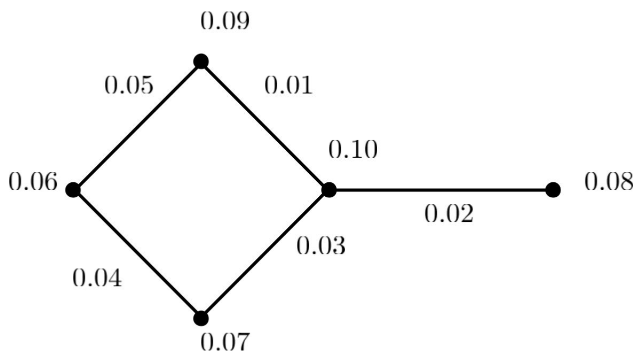

Figure 3: 4-pan graph (Banner Graph)

$$\begin{aligned}
\sigma(v_1)+\mu(v_1v_2)+\sigma(v_2) &= 0.09+0.06+0.05=0.20=\lambda_s\\
\sigma(v_2)+\mu(v_2v_3)+\sigma(v_3) &= 0.06+0.07+0.04=0.17=\lambda_w\\
\sigma(v_3)+\mu(v_3v_4)+\sigma(v_4) &= 0.07+0.10+0.03=0.20=\lambda_s\\
\sigma(v_4)+\mu(v_4v_5)+\sigma(v_5) &= 0.10+0.08+0.02=0.20=\lambda_s\\
\sigma(v_1)+\mu(v_1v_4)+\sigma(v_4) &= 0.09+0.10+0.01=0.20=\lambda_s
\end{aligned}$$

Here the 4-pan graph has a weak constant for some edge. But it obviously satisfies perfect intrinsic edge-magic labelling.

We conclude that the Banner graph is a Quasi perfect intrinsic edge-magic.

Theorem 3.11. *The friendship graph F_n is not a perfect intrinsic edge-magic graph for all $n > 1$. (i.e, The cycle C_3 always fuzzy perfect intrinsic edge-magic but the n-copies of C_3 need not be a fuzzy perfect intrinsic edge-magic).*

Theorem 3.12. *The star graph $K_{1,n}$ is a fuzzy perfect intrinsic edge-magic with intrinsic super constant $\lambda_s = (4n+2)z$ for all $n > 2$ and let $z = 0.1$ for $N < 6$ &$z = 0.01$ for $N \geq 6$.*

Proof. Let G be a star graph $K_{1,n}$ with n-vertices. We put n=3, 4, 5... it exhibits the respective graph is a fuzzy perfect intrinsic edge-magic with intrinsic super constant. Apply the perfect intrinsic edge-magic labelling,

$$\begin{aligned}
\sigma(v) &= (2n+1)z & \lambda(K_{1,n}) &= \sigma(v)+\mu(vv_i)+\sigma(v_i)\\
\sigma(v_i) &= (n+i)z,\ \text{for } 1\leq i\leq n & &= (2n+1+n+n+i)z\\
\mu(vv_i) &= nz & \lambda_s &= (4n+2)z
\end{aligned}$$

Case (i): Let $n=3$, we get $\lambda_s = 0.14$

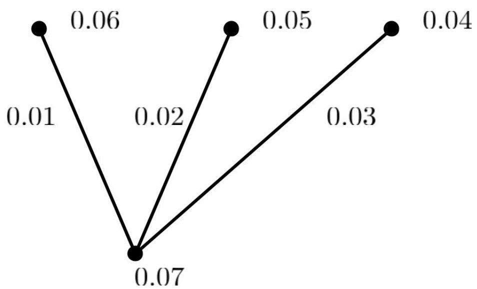

Figure 4:

Intrinsic super constant value for all edges is the following:

$$\sigma(v_1) + \mu(v_1v_2) + \sigma(v_2) = 0.07 + 0.01 + 0.06 = 0.14 = \lambda_s$$
$$\sigma(v_1) + \mu(v_1v_3) + \sigma(v_3) = 0.07 + 0.02 + 0.05 = 0.14 = \lambda_s$$
$$\sigma(v_1) + \mu(v_1v_4) + \sigma(v_4) = 0.07 + 0.03 + 0.04 = 0.14 = \lambda_s$$

Case (ii): Let $n = 4$, we get $\lambda_s = 0.18$

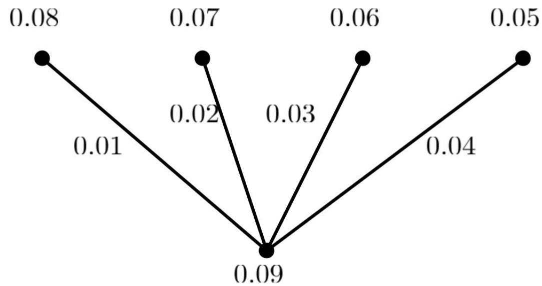

Figure 5:

The intrinsic super constant value for all edges are the following:

$$\sigma(v_1) + \mu(v_1v_2) + \sigma(v_2) = 0.09 + 0.01 + 0.08 = 0.18 = \lambda_s$$
$$\sigma(v_1) + \mu(v_1v_3) + \sigma(v_3) = 0.09 + 0.02 + 0.07 = 0.18 = \lambda_s$$
$$\sigma(v_1) + \mu(v_1v_4) + \sigma(v_4) = 0.09 + 0.03 + 0.06 = 0.18 = \lambda_s$$
$$\sigma(v_1) + \mu(v_1v_5) + \sigma(v_5) = 0.09 + 0.04 + 0.05 = 0.18 = \lambda_s$$

Case(iii): Let $n = 5$, we get $\lambda_s = 0.22$.
Case(iv): Let $n = 6$, we get $\lambda_s = 0.26$.
Continuing this process, we put different values of 'n', it gives intrinsic super constant. Hence the star graph $K_{1,n}$ is fuzzy perfect intrinsic edge-magic for all $n > 2$.

4. Conclusion

In this paper, we discussed the idea of fuzzy perfect intrinsic edge-magic graphs with intrinsic super constant and the fuzzy perfect intrinsic edge magic labelling graphs like fuzzy paths, fuzzy cycles and fuzzy stars are also discussed. We focussed some theorems on fuzzy perfect intrinsic edge-magic graphs. It ought to be note that the necessary and sufficient conditions are given for all the above mentioned graphs.

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