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STRONG COMMENSALISM WITH MONAD MODEL - A SCIENTIFIC STUDY

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Abstract: In this paper, Strong Commensalism with Monad model is discussed. This model is formed by a couple of first order non linear differential equations with Monad coefficient. The stability and the characteristic behaviors of two species are established with the impact of Monad Parameter. Phase plane analysis has been used to investigate this model.

Keywords and Phrases: Commensal species, Host Species, Equilibrium points, Stability and Phase Plane analysis.

2010 Mathematics Subject Classification: 92D25, 92D40.

1. Introduction and Preliminaries

Ecology is one of the scientific fields of population dynamics to determine the implications and to provide suitable solutions to the ecological models. In this connection , mathematical modeling plays wonderful role to fulfill our dreams to get the effective solutions in the complicated situations. Many scholars like Lotka [10], Volterra [13], Meyer [11], Cushing [6], Gause [7], Paul Colinvaux [12], Haberman

[8] and Kapur [9] did superlative work in mathematical modeling by achieving outstanding results. K. V. L. N. Acharyulu et.al [1-5] discussed exclusively different models of Ecological Ammensalism. In this paper, it is aimed to investigate a strong commensal Model with the influence of Monad parameter.

Notations Adopted.
x (t) : The population rate of Commensal Species(S1) at time t y (t) : The population rate of Host Species(S2) at time t α : The natural growth rate of Commensal Species(S1) χ: The natural growth rate of Host Species (S2) β :The inhibition coefficient of Commensal due to Host Species $\mu(y)$: Monad Coefficient which is defined with two parameters i.e $\mu(y) = \frac{\lambda y}{\eta+y}$ The state variables x and y as well as the model parameters $\alpha, \chi, \beta, \gamma$ are assumed to be non-negative constants.

2. Results

(i). Equation for the growth rate of Commensal Species (S1):
$\frac{dx}{dt} = \alpha x - \beta x^2 + x\ \mu(y)$ where $\mu(y)$ is Monad coefficient and defined by

$$\mu(y) = \frac{\lambda y}{\eta + y} \tag{1}$$

(ii). Equation for the growth rate of Enemy species (S2):

$$\frac{dy}{dt} = \chi\ y - \gamma\ y^2 \tag{2}$$

3. Equilibrium states

The system has the following four equilibrium states from $\frac{dx}{dt} = 0$ $\frac{dy}{dt} = 0$
(i). Fully washed out state:

$$\bar{x} = 0, \bar{y} = 0. \tag{3}$$

(ii). The state in which only the Host exists and the Commensal is washed out:

$$\bar{x} = 0, \bar{y} = \frac{\chi}{\gamma} \tag{4}$$

(iii). The state in which only the Commensal exists and the Host is washed out:

$$\bar{y} = 0, \bar{x} = \frac{\alpha}{\beta} \tag{5}$$

(iv). Co-existent state: (Both Commensal and Host survive)

$$\bar{y} = 0, \bar{x} = \frac{1}{\beta}\Big[\alpha - \frac{(\lambda\ \frac{\chi}{\gamma})}{\eta + \frac{\chi}{\gamma}}\Big]. \tag{6}$$

Case(i): When $\alpha = 0.76$, β=0.89, χ=0.76, η=0.05, γ=5.4 and λ=.08. The Null clines and Trajectories are shown in the Fig.(1) and Fig.(2) respectively. In this case, The Eigen values are -0.98 and -1.041 with the eigen vectors (0.19835, 0.98013) and (0,1) and the Jacobean matrix is $\begin{bmatrix} -1.041 & 0.012347 \\ 0 & -0.98 \end{bmatrix}$. The Equilibrium Point occurs at (1.3698, 1.6066).

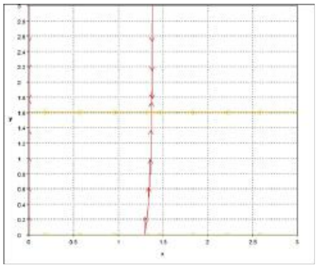


Figure 1

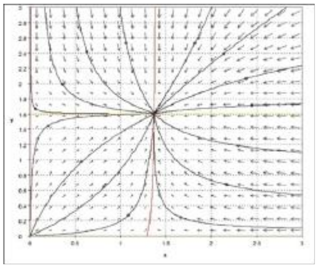


Figure 2

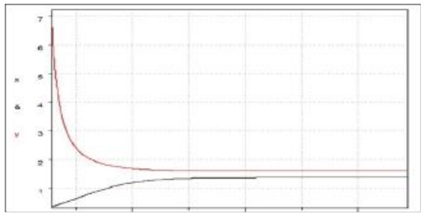


Figure 3

The interaction between the Commensal and Host Species is shown in Fig.(3).

Case(ii): When $\alpha = 0.76$, β=0.89, χ=0.76, η=0.05, γ=5.4 and λ=.18. The Null clines and Trajectories are shown in the Fig.(4) and Fig.(5) respectively. In this case, The Eigen values are -0.98 and -1.1173 with the eigen vectors (0.21225, 0.97722) and (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.1173 & 0.029816 \\ 0 & -0.98 \end{bmatrix}$ The Equilibrium Point occurs at (1.4701, 1.6066)

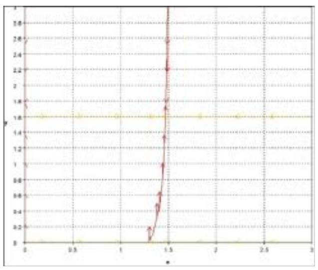


Figure 4

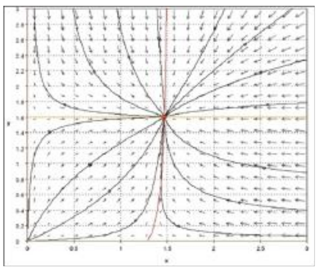


Figure 5

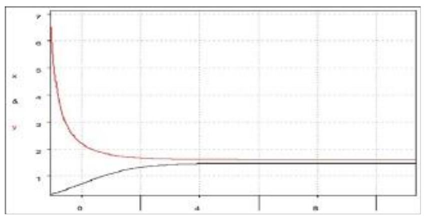


Figure 6

The interaction between the Commensal and Host Species is shown in Fig.(6).

Case(iii): When $\alpha = 0.76$, β=0.89, χ=0.76, η=0.05, γ=5.4 and λ=.28. The Null clines and Trajectories are shown in the Fig.(7) and Fig.(8) respectively. In this case, The Eigen values are -0.98 and -1.1935 with the eigen vectors (0.22602, 0.97412) and (0,1) and the Jacobean matrix is $\begin{bmatrix} 1.1935 & 0 \\ 0 & -0.98 \end{bmatrix}$ The Equilibrium Point occurs at (1.5704, 1.6066)

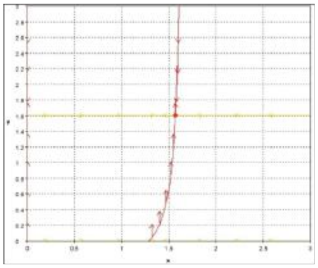


Figure 7

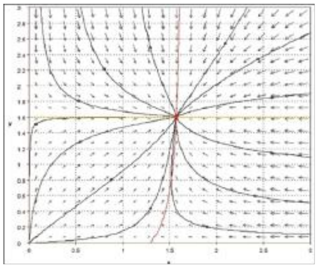


Figure 8

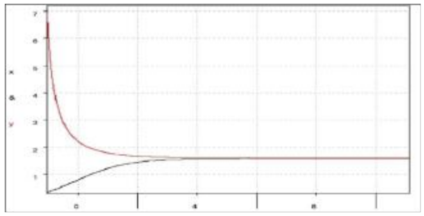


Figure 9

The interaction between the Commensal and Host Species is shown in Fig.(9).
Case(iv): When α = 0.76, β =0.89, χ=0.76, η=0.05, γ=5.4 and λ=.38. The Null clines and Trajectories are shown in the Fig.(10) and Fig.(11) respectively. In this case, The Eigen values are -0.98 and -1.2698 with the eigen vectors (0.23965, 0.97086) and (0,1) and the Jacobean matrix is $\begin{bmatrix} -1.2698 & 0.071537 \\ 0 & -0.98 \end{bmatrix}$

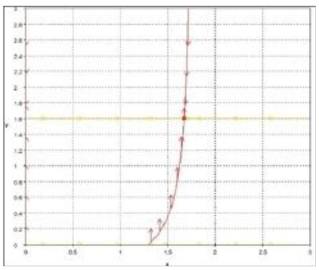


Figure 10

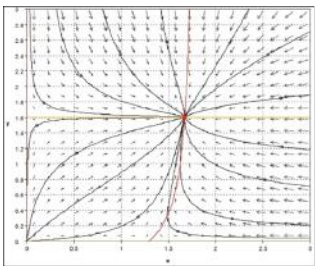


Figure 11

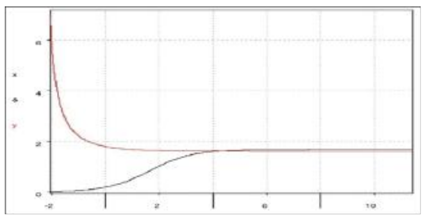


Figure 12

The interaction between the Commensal and Host Species is shown in Fig.(12).
Case(v): When α=0.76, β=0.89, χ=0.76, η=0.05, γ=5.4 and λ=.48. The Null clines and Trajectories are shown in the Fig.(13) and Fig.(14) respectively. In this case, The Eigen values are -0.98 and -1.3461 with the eigen vectors (0.25315, 0.96743) and (0,1) and the Jacobean matrix is $\begin{bmatrix} -1.3461 & 0.09579 \\ 0 & -0.98 \end{bmatrix}$ The Equilibrium Point occurs at (1.7711, 1.6066).

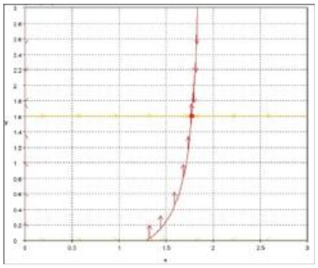


Figure 13

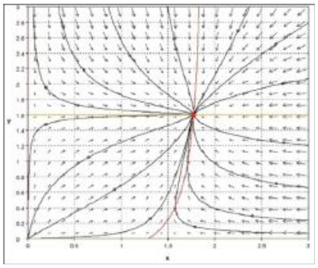


Figure 14

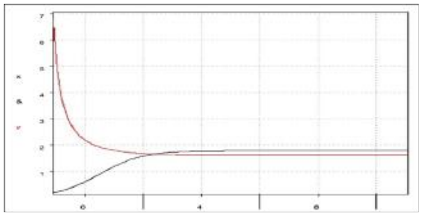
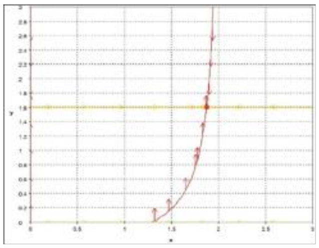


Figure 15

The interaction between the Commensal and Host Species is shown in Fig.(15).
Case(vi): When α =0.76, β=0.89, χ=0.76, η=0.05, γ=5.4 and λ=.58. The Null clines and Trajectories are shown in the Fig.(16) and Fig.(17) respectively. In this case, The Eigen values are -0.98 and -1.4223 with the eigen vectors (0.2665, 0.96384) and (0,1) and the Jacobean matrix is $\begin{bmatrix} -1.4223 & 0.1223 \\ 0 & -0.98 \end{bmatrix}$ The Equilibrium Point occurs at (1.8715, 1.6066)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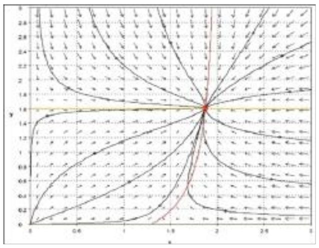


Figure 17

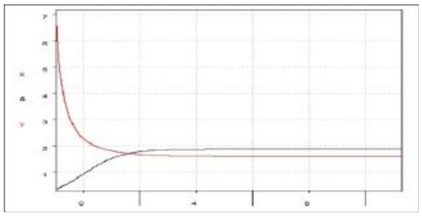


Figure 18

The interaction between the Commensal and Host Species is shown in Fig.(18).
Case(vii): When α=0.76, β=0.89, χ=0.76, η=0.05, γ=5.4 and λ=.68. The Null clines and Trajectories are shown in the Fig.(19) and Fig.(20) respectively. In this case, The Eigen values are -0.98 and -1.4986 with the eigen vectors (0.27969, 0.96009) and (0,1) and the Jacobean matrix is $\begin{bmatrix} -1.4986 & 0.15108 \\ 0 & -0.98 \end{bmatrix}$
The Equilibrium Point occurs at (1.9718, 1.6066).
The interaction between the Commensal and Host Species is shown in Fig.(21).
Case(viii): When α =0.76, β=0.89, χ=0.76, η=0.05, γ=5.4 and λ=.78. The Null clines and Trajectories are shown in the Fig.(22) and Fig.(23) respectively. In this case, The Eigen values are -0.98 and -1.5749 with the eigen vectors (0.29274,

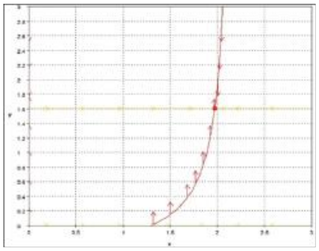


Figure 19

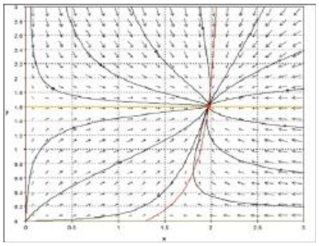


Figure 20

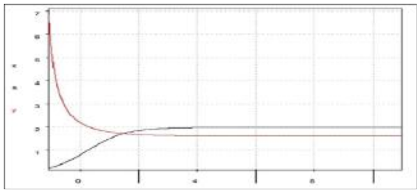
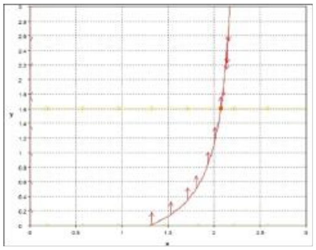


Figure 21

0.95619), (0,1) and the Jacobean matrix is $\begin{bmatrix} -1.5749 & 0.18212 \\ 0 & -0.98 \end{bmatrix}$ The Equilibrium Point occurs at (2.0722, 1.6066).



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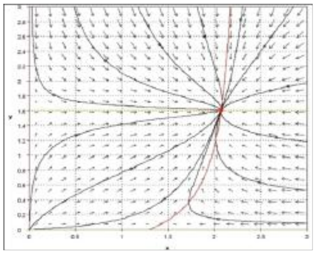


Figure 23

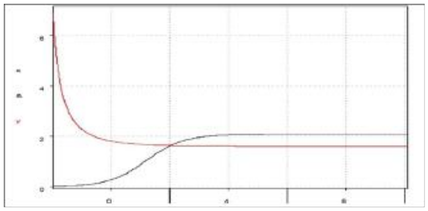


Figure 24

The interaction between the Commensal and Host Species is shown in Fig.(24).
Case(ix): When α =0.76, β=0.89, χ=0.76, η=0.05, γ=5.4 and λ=.88. The Null clines and Trajectories are shown in the Fig.(25) and Fig.(26) respectively. In this case, The Eigen values are -0.98 and -1.6511 with the eigen vectors (0.30562, 0.95216)), (0,1) and the Jacobean matrix is $\begin{bmatrix} -1.6511 & 0.221541 \\ 0 & -0.98 \end{bmatrix}$ The Equilibrium Point occurs at (2.1725, 1.6066).

The interaction between the Commensal and Host Species is shown in Fig.(27).
Case(x): When α=0.76, β=0.89, χ=0.76, η=0.05, γ=5.4 and λ=.98. The Null clines and Trajectories are shown in the Fig.(28) and Fig.(29) respectively. In this case, The Eigen values are -0.98 and -1.7274 with the eigen vectors (0.31833, 0.94798), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.17274 & 0.25097 \\ 0 & -0.98 \end{bmatrix}$ The Equilibrium Point occurs at (2.2729, 1.6066).

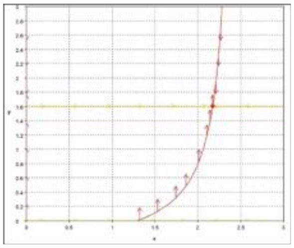


Figure 25

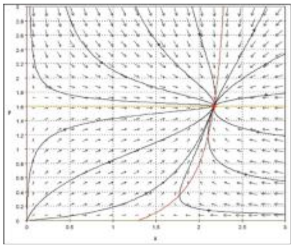
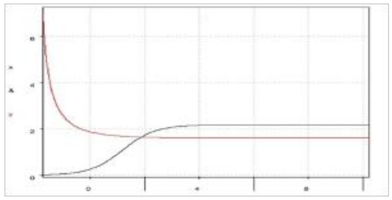


Figure 26



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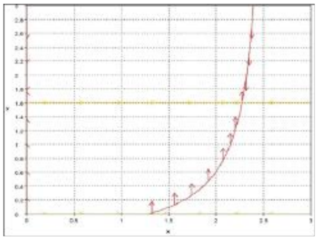

Figure 28

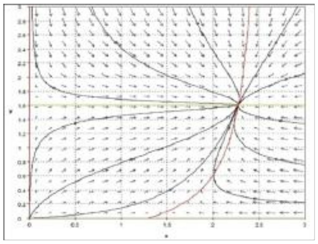

Figure 29

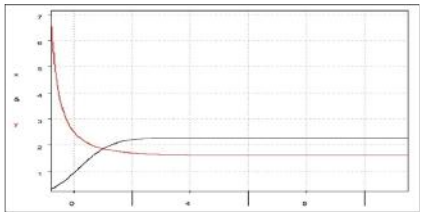

Figure 30

The interaction between the Commensal and Host Species is shown in Fig.(30).

4. Conclusion

It is observed that this model is stable at coexistence point. At the early stage, there is no direct interaction between commensal and host species. Later onwards, host species dominates over commensal species till the dominance reversal time and then commensal species dominates over the host species in throughout the interval. Commensal model becomes strong with the impact of the parameter ($\lambda > 0$) of Monad coefficient. Strong Commensalism is obtained by increasing the values of one of the parameters (λ).

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