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# SEPARATION AXIOMS ON BIPOLAR FUZZY ROUGH TOPOLOGICAL SPACES

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**Abstract:** In this paper we introduce separation axioms on bipolar fuzzy rough (BFR) topological spaces using the concept of bipolar fuzzy rough points and establish some of their properties.

**Keywords and Phrases:** Bipolar fuzzy rough open and closed sets, bipolar fuzzy rough point, bipolar fuzzy rough subspaces, bipolar fuzzy rough closure, bipolar fuzzy rough separation axioms.

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## 1. Introduction

The theory of rough sets was proposed by Pawlak [8, 9]. It is an extension of set theory for the study of systems characterized by insufficient and incomplete informations. A key notion in Pawlak rough set model is equivalence relation. The equivalence classes are the building blocks for the construction of lower and upper approximations. By replacing the equivalence relation by an arbitrary binary relation, different kind of generalization in Pawlak rough set models were obtained. In [3, 4, 7], the concept of fuzzy rough sets were studied by replacing crisp binary relations with fuzzy relations on the universe. Yong Chan Kim [10] introduced the separation axioms of fuzzy topological spaces using R-fuzzy semi-open (closed) sets. Mathew and John [2] developed general topological structures on rough sets.

Lellis Thivagar et al. [5] introduced a new topology called rough topology in terms of rough sets and showed that rough topology can be used to analyze many real life problems.

Zhang [11] introduced the concept of bipolar fuzzy sets as an extension of fuzzy sets. Muthuraj [6] defined bipolar fuzzy set and established some properties of bipolar fuzzy and anti fuzzy subgroup. Anita shanthi et al. [1] introduced the concept of separation axioms on fuzzy rough topological spaces.

In this paper we introduce separation axioms on bipolar fuzzy rough topological spaces using the concept of bipolar fuzzy rough points and study some of their properties.

## 2. Bipolar fuzzy rough separation axioms

In this section we define BFR $T_0, T_1$, Hausdorff, regular, normal spaces using the concept of BFR points and study some of their properties. We have provided several examples to substantiate the results established.

**Definition 2.1.** *Let $U$ be the universe $A \subseteq U$ and $BF\mathbb{R}$ be an equivalence relation on $U$ and $\tau = \{\phi, U, BF\underline{R}(A), BF\overline{R}(A)\}$.*
*(i) $\phi, U \in \tau$.*
*(ii) Union of collection of elements of $\tau$ is in $\tau$.*
*(iii) Intersection of collection of elements of $\tau$ is in $\tau$.*
*$\tau$ forms a topology called as the BFR topology on $U$ with respect to $A$ and we call $(U, BFR(A), \tau)$ as the BFR topological space.*

**Definition 2.2.** *Two BFR sets $BFR(A)$ and $BFR(B)$ are said to be quasi coincident denoted by $BFR(A)qBFR(B)$ if there exists $x \in U \ni \mu_{BFR^n(A)}(x) + \mu_{BFR^n(B)}(x) < -1,\ \mu_{BFR^p(A)}(x) + \mu_{BFR^p(B)}(x) > 1$ Otherwise $BFR(A)\overline{q}BFR(B)$, in which case $\mu_{BFR^n(A)}(x) + \mu_{BFR(B)^n}(x) \geq -1,\ \mu_{BFR^p(A)}(x) + \mu_{BFR(B)^p}(x) \leq 1$ $\forall\ x \in U$.*

**Definition 2.3.** *Let $(U, BFR(A), \tau)$ be a BFR topological space and $BFR(B_s) \subset BFR(A)$. Then the BFR topology $\tau_{BFR(B_s)} = \{BFR(B_s) \cap BFR(O) | BFR(O) \in \tau\}$ is called BFR subspace topology and $(BFR(B_s), \tau_{BFR(B_s)})$ is called BFR subspace of $(U, BFR(A), \tau)$.*

**Definition 2.4.** *A BFR topological space $(U, BFR(A), BFR(B), \tau)$ is said to be a BFR $T_0$ space if for every pair of distinct BFR points $xBFR(P), yBFR(Q)$, there exists a BFR open set $BFR(A) \in \tau \ni xBFR(P) \in BFR(A), yBFR(Q) \notin BFR(A)$ or there exists a BFR open set $BFR(B) \in \tau \ni yBFR(Q) \in BFR(B)$, $xBFR(P) \notin BFR(B)$.*

**Theorem 2.5.** *A BFR subspace of a BFR $T_0$ space is a BFR $T_0$ space.*
**Proof.** Let $(BFR(B_s), \tau_{(BFR(B_s))})$ be a BFR subspace of a $BFR\ T_0$ space $(U, BFR(A), \tau)$ and $xBFR(P)$, $yBFR(Q)$ be two distinct BFR points of $BFR(B_s)$. Then these BFR points are also in $BFR(A)$.
$\Rightarrow \exists$ a BFR open set $BFR(C) \in \tau$ containing one BFR point but not the other.

$\Rightarrow BFR(B_s) \cap BFR(C)$ is a BFR open set in $\tau_{BFR(B_s)}$ containing one BFR point but not the other. Hence $(BFR(B_s), \tau_{(BFR(B_s))})$ is a BFR $T_0$ space.

**Definition 2.6.** *A BFR topological space $(U, BFR(A), BFR(B), \tau)$ is said to be a BFR $T_1$ space if for every pair of BFR points $xBFR(P), yBFR(Q)$ such that $xBFR(P) \neq yBFR(Q)$, there exist BFR open sets $BFR(A)$ and $BFR(B)$ such that $xBFR(P) \in BFR(A)$, $xBFR(P) \notin BFR(B)$ and $yBFR(Q) \in BFR(B)$, $yBFR(Q) \notin BFR(A)$.*

**Theorem 2.7.** *A BFR subspace of a BFR $T_1$ space is a BFR $T_1$ space.*
**Proof.** Let $(BFR(B_s), \tau_{BFR(B_s)})$ be a BFR subspace of a BFR $T_1$ space $(U$, $BFR(X)$, $\tau)$, where $BFR(X) = \{BFR(A), BFR(B)\}$ and $xBFR(P)$, $yBFR(Q)$ be two distinct BFR points of $BFR(B_s.)$ Then these BFR points are also in $BFR(X)$.
$\Rightarrow \exists$ BFR open sets $BFR(C)$ and $BFR(D)$ in $\tau$, such that $xBFR(P) \in BFR(C)$, $xBFR(P) \notin BFR(D)$ and $yBFR(Q) \in BFR(D)$, $yBFR(Q) \notin BFR(C)$. It follows that there exists $BFR(B_s) \cap BFR(C)$, $BFR(B_s) \cap BFR(D) \in \tau_{BFR(B_s)}$. Also, $xBFR(P) \in BFR(B_s) \cap BFR(C)$, $xBFR(P) \notin BFR(B_s) \cap BFR(D)$, and $yBFR(Q) \in BFR(B_s) \cap BFR(D)$, $yBFR(Q) \notin BFR(B_s) \cap BFR(C)$. Hence $(BFR(B_s), \tau_{BFR(B_s)})$ is a BFR $T_1$ space.

**Definition 2.8.** *A BFR topological space $(U, BFR(A), BFR(B), \tau)$ is said to be a BFR Hausdorff if for every pair of distinct BFR points $xBFR(P), yBFR(Q)$ there exists $BFR(A), BFR(B) \in \tau$ such that,*
*$xBFR(P) \in BFR(A), yBFR(Q) \in BFR(B)$ and $BFR(A)\overline{q}BFR(B)$.*

**Example 2.9.** Consider a BFR point,
$U = \{x_1, x_2, x_3\}$,
$P = \{x_1/(-0.11, 0.15), x_2/(-1, 0), x_3/(-1, 0)\}$ and

$$BF\mathbb{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} (0,1) & (-1,0) & (-1,0) \\ (-1,0) & (0,1) & (-1,0) \\ (-1,0) & (-1,0) & (0,1) \end{pmatrix} \end{array}$$

$x_1BFR(P) = \{\{x_1/(-0.11, 0.15), x_2/(-1, 0), x_3/(-1, 0)\}$,
$\{x_1/(-0.11, 0.15), x_2/(-1, 0), x_3/(-1, 0)\}\}$
Again,$U = \{x_1, x_2, x_3\}$,
$Q = \{x_1/(-1, 0), x_2/(-1, 0), x_3/(-0.35, 0.2)\}$ and

$$BF\mathbb{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} (0,1) & (-1,0) & (-1,0) \\ (-1,0) & (0,1) & (-1,0) \\ (-1,0) & (-1,0) & (0,1) \end{pmatrix} \end{array}$$

$x_3BFR(Q) = \{\{x_1/(-1, 0), x_2/(-1, 0), x_3/(-0.35, 0.2)\}$,
$\{x_1/(-1, 0), x_2/(-1, 0), x_3/(-0.35, 0.2)\}\}$

Again, let $U = \{x_1, x_2, x_3\}$,
$A = \{x_1/(-0.1, 0.2), x_2/(-0.07, 0.22), x_3/(-0.09, 0.19)\}$ and

$$BF\mathbb{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} (-1,1) & (-0.11,0.3) & (-0.11,0.22) \\ (-0.11,0.3) & (-1,1) & (-0.2,0.22) \\ (-0.11,0.22) & (-0.2,0.22) & (-1,1) \end{pmatrix} \end{array}$$

$BFR(A) = \{\{x_1/(-0.09, 0.2), x_2/(-0.1, 0.22), x_3/(-0.1, 0.19)\},$
$\{x_1/(-0.11, 0.22), x_2/(-0.11, 0.22), x_3/(-0.11, 0.22)\}\}$
Again, consider $U = \{x_1, x_2, x_3\}$,
$B = \{x_1/(-0.2, 0.4), x_2/(-0.18, 0.38), x_3/(-0.25, 0.3)\}$ and

$$BF\mathbb{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} (-1,1) & (-0.32,0.42) & (-0.32,0.3) \\ (-0.32,0.42) & (-1,1) & (-0.4,0.3) \\ (-0.32,0.3) & (-0.4,0.3) & (-1,1) \end{pmatrix} \end{array}$$

$BFR(B) = \{\{x_1/(-0.25, 0.4), x_2/(-0.25, 0.38), x_3/(-0.2, 0.3)\},$
$\{x_1/(-0.32, 0.4), x_2/(-0.32, 0.4), x_3/(-0.32, 0.3)\}\}$
Hence $x_1BFR(P) \in BFR(A)$ but $x_1BFR(P) \notin BFR(B)$ and
$x_3BFR(Q) \in BFR(B)$ but $x_3BFR(Q) \notin BFR(A)$.
Also, $\mu_{BF\underline{R}^n(A)}(x_1) + \mu_{BF\underline{R}^n(B)}(x_1) = -0.34 \geq -1$
$\mu_{BF\underline{R}^p(A)}(x_1) + \mu_{BF\underline{R}^p(B)}(x_1) = 0.6 \leq 1.$
Similarly, the values are calculated.
Therefore $BFR(A)\overline{q}BFR(B)$.
Hence $(U, BFR(A), BFR(B), \tau)$ is a BFR Hausdorff space.

**Theorem 2.10.** *A BFR subspace of a BFR Hausdorff space is a BFR Hausdorff space.*

**Proof.** Let $(BFR(B_s), \tau_{BFR(B_s)})$ be a BFR subspace of a BFR Hausdorff space $(U, BFR(X), \tau)$ and $xBFR(P), yBFR(Q)$ be two distinct BFR points of $BFR(B_s)$. $(U, BFR(X), \tau)$ is a BFR Hausdorff space. Hence there exists $BFR(C), BFR(D) \in \tau$ such that $xBFR(P) \in BFR(C), yBFR(Q) \in BFR(D)$ and $BFR(C)\overline{q}BFR(D)$. It follows that, there exist $BFR(B_s) \cap BFR(C), BFR(B_s) \cap BFR(D) \in \tau_{BFR(B_s)}$ such that $xBFR(P) \in BFR(B_s) \cap BFR(C), yBFR(Q) \in BFR(B_s) \cap BFR(D)$ and $(BFR(B_s) \cap BFR(C)) \cap (BFR(B_s) \cap BFR(D))$
$= BFR(B_s)\overline{q}(BFR(C)\overline{q}BFR(D))$.
Hence $(BFR(B_s), \tau_{BFR(B_s)})$ is a BFR Hausdorff space.

**Definition 2.11.** *A BFR topological space $(U, BFR(A), BFR(B), \tau)$ is said to be BFR regular, if for each pair consisting of a BFR point $xBFR(P)$ and a BFR closed set $[BFR(E)]^c \ni xBFR(P) \notin [BFR(E)]^c$, there exists BFR open sets $BFR(A), BFR(B) \in \tau$ such that*
*$xBFR(P) \in BFR(A), [BFR(E)]^c \subseteq BFR(B)$ and $BFR(A)\overline{q}BFR(B)$.*

**Example 2.12.** Consider $U = \{x_1, x_2, x_3\}$,

$P = \{x_1/(-0.2, 0.3), x_2/(-1, 0), x_3/(-1, 0)\}$ and

$$BF\mathbb{R} = \begin{matrix} & x_1 & x_2 & x_3 \\ x_1 & (0,1) & (-1,0) & (-1,0) \\ x_2 & (-1,0) & (0,1) & (-1,0) \\ x_3 & (-1,0) & (-1,0) & (0,1) \end{matrix}$$

$x_1BFR(P) = \{\{x_1/(-0.2, 0.3), x_2/(-1, 0), x_3/(-1, 0)\},$
$\{x_1/(-0.2, 0.3), x_2/(-1, 0), x_3/(-1, 0)\}\}$
Consider $U = \{x_1, x_2, x_3\}$,
$E = \{x_1/(-0.02, 0.5), x_2/(-0.05, 0.6), x_3/(-0.04, 0.3)\}$ and

$$BF\mathbb{R} = \begin{matrix} & x_1 & x_2 & x_3 \\ x_1 & (-1,1) & (-0.06,0.6) & (-0.06,0.5) \\ x_2 & (-0.06,0.6) & (-1,1) & (-0.12,0.5) \\ x_3 & (-0.06,0.5) & (-0.12,0.5) & (-1,1) \end{matrix}$$

$BFR(E) = \{\{x_1/(-0.05, 0.5), x_2/(-0.04, 0.5), x_3/(-0.04, 0.3)\},$
$\{x_1/(-0.06, 0.6), x_2/(-0.06, 0.6), x_3/(-0.06, 0.5)\}\}$
$[BFR(E)]^c = \{\{x_1/(-0.95, 0.5), x_2/(-0.96, 0.5), x_3/(-0.96, 0.5)\},$
$\{x_1/(-0.94, 0.4), x_2/(-0.94, 0.4), x_3/(-0.94, 0.5)\}\}$
Consider $U = \{x_1, x_2, x_3\}$,
$A = \{x_1/(-0.17, 0.3), x_2/(-0.13, 0.3), x_3/(-0.15, 0.26)\}$ and

$$BF\mathbb{R} = \begin{matrix} & x_1 & x_2 & x_3 \\ x_1 & (-1,1) & (-0.18,0.7) & (-0.18,0.5) \\ x_2 & (-0.18,0.7) & (-1,1) & (-0.2,0.5) \\ x_3 & (-0.18,0.5) & (-0.2,0.5) & (-1,1) \end{matrix}$$

$BFR(A) = \{\{x_1/(-0.15, 0.3), x_2/(-0.17, 0.3), x_3/(-0.17, 0.26)\},$
$\{x_1/(-0.18, 0.3), x_2/(-0.18, 0.3), x_3/(-0.18, 0.3)\}\}$
and also consider $U = \{x_1, x_2, x_3\}$,
$B = \{x_1/(-0.2, 0.4), x_2/(-0.15, 0.45), x_3/(-0.13, 0.5)\}$ and

$$BF\mathbb{R} = \begin{matrix} & x_1 & x_2 & x_3 \\ x_1 & (-1,1) & (-0.21,0.68) & (-0.21,0.54) \\ x_2 & (-0.21,0.68) & (-1,1) & (-0.31,0.54) \\ x_3 & (-0.21,0.54) & (-0.31,0.54) & (-1,1) \end{matrix}$$

$BFR(B) = \{\{x_1/(-0.15, 0.4), x_2/(-0.2, 0.4), x_3/(-0.2, 0.46)\},$
$\{x_1/(-0.21, 0.5), x_2/(-0.21, 0.5), x_3/(-0.21, 0.5)\}\}$
Thus, $x_1BFR(P) \notin [BFR(E)]^c$, $x_1BFR(P) \in BFR(A)$, $[BFRE]^c \subseteq BFR(B)$
$\mu_{BF\underline{R}^n(A)}(x_1) + \mu_{BF\underline{R}^n(B)}(x_1) = -0.3 \geq -1$
$\mu_{BF\underline{R}^p(A)}(x_1) + \mu_{BF\underline{R}^p(B)}(x_1) = 0.7 \leq 1$

Similarly, the values are calculated.
Therefore $BFR(A)\overline{q}BFR(B)$.
Hence $(U, BFR(A), BFR(B), \tau)$ is a BFR regular space.

**Theorem 2.13.** *A BFR subspace of a BFR regular space is a BFR regular.*
**Proof.** Let $(BFR(B_s), \tau_{BFR(B_s)})$ be a BFR subspace of a BFR regular space $(U, BFR(X), \tau)$. Let $xBFR(P), [BFR(E)]^c$ be BFR point and bipolar fuzzy rough closed set respectively, in $\tau_{BFR(B_s)}$ such that $xBFR(P) \notin [BFR(E)]^c$. $BFR(B_s) \subseteq BFR(X)$. $(U, BFR(X), \tau)$ is a BFR regular space. So there exists $BFR(C)$, $BFR(D) \in \tau$ such that $xBFR(P) \in BFR(C)$, $[BFR(E)]^c \subseteq BFR(D)$ and $BFR(C)\ \overline{q}BFR(D)$. It follows that there exists $BFR(B_s) \cap BFR(C)$, $BFR(B_s) \cap BFR\ (D) \in \tau_{BFR(B_s)}$ such that $xBFR(P) \in BFR(B_s) \cap BFR(C)$, $[BFR(E)]^c \subseteq BFR(B_s) \cap BFR\ (D)$ and $(BFR(B_s) \cap BFR(C)) \cap (BFR(B_s) \cap BFR(D)) = BFR(B_s)\overline{q}\ (BFR(C)\ \overline{q}BFR(D))$. Hence $(BFR(B_s), \tau_{BFR(B_s)})$ is a BFR regular space.

**Theorem 2.14.** *A BFR topological space $(U, BFR(A), BFR(B), \tau)$ is a BFR regular if and only if for every BFR open set $BFR(E)$ and $x\overline{BFR(P)} \in BFR(E)$ there exists $BFR(A) \in \tau$ such that $xBFR(P) \in BFR(A) \subseteq \overline{BFR(A)} \subseteq BFR(E)$.*
**Proof.** Let $(U, BFR(A), BFR(B), \tau)$ be a BFR regular space and $BFR(E)$ be a BFR open set, $xBFR(P) \in BFR(E)$. Now $[BFR(E)]^c$ is a BFR closed set, $xBFR(P) \notin [BFR(E)]^c$. $(U, BFR(A), BFR(B), \tau)$ is a BFR regular space. Hence there exists $BFR(A), BFR(B) \in \tau$ such that $xBFR(P) \in BFR(A), [BFR(E)]^c \subseteq BFR(B)$ and $BFR(A)\overline{q}BFR(B)$. Now we have $xBFR(P) \in BFR(A) \subseteq \overline{BFR(A)} \subseteq [BFR(B)]^c \subseteq BFR(E)$. Hence there exists $BFR(A) \in \tau$, such that $xBFR(P) \in BFR(A) \subseteq \overline{BFR(A)} \subseteq BFR(E)$.
Conversely, let $[BFR(E)]^c$ be a BFR closed set, $xBFR(P)$ be a BFR point such that $xBFR(P) \notin [BFR(E)]^c$. $BFR(E)$ is an BFR open set, $x\overline{BFR(P)} \in BFR(E)$. Then there exists $BFR(A) \in \tau, \ni\ xBFR(P) \in BFR(A) \subseteq \overline{BFR(A)} \subseteq BFR(E)$. Since $\overline{BFR(A)} \subseteq BFR(E)$ and $[BFR(E)]^c \subseteq [\overline{BFR(A)}]^c$ and $[\overline{BFR(A)}]^c \in \tau$. Therefore there exists $BFR(A)$, $[\overline{BFR(A)}]^c \in \tau$ such that $xBFR(P) \in BFR(A)$, $[BFR(E)]^c \subseteq [\overline{BFR(A)}]^c$ and $BFR(A)\overline{q}[\overline{BFR(A)}]^c$. Thus $(U, BFR(A), BFR(B), \tau)$ is a BFR regular space.

**Definition 2.15.** *A BFR topological space $(U, BFR(A), BFR(B), \tau)$ is said to be a BFR normal if for every pair of disjoint BFR closed sets $[BFR(E)]^c$, $[BFR(F)]^c$, there exist BFR open sets $BFR(A), BFR(B) \in \tau$ such that $[BFR(E)]^c \subseteq BFR(A)$, $[BFR(F)]^c \subseteq BFR(B)$ and $BFR(A)\overline{q}BFR(B)$.*

**Theorem 2.16.** *Every BFR subspace of a BFR normal space is BFR normal.*

**Proof.** Let $(U, BFR(X), \tau)$ be a BFR normal space and $(BFR(B_s), \tau_{BFR(B_s)})$ be a BFR subspace of $(U, BFR(X), \tau)$. To prove that $(BFR(B_s), \tau_{BFR(B_s)})$ is a BFR normal space. Consider two disjoint BFR closed sets $[BFR(B_E)]^c$, $[BFR(B_G)]^c$ of $\tau_{BFR(B_s)}$. Since $[BFR(B_E)]^c$, $[BFR(B_G)]^c \in \tau_{[BFR(B_s)]}$, from Definition, there exists $[BFR(E)]^c$, $[BFR(G)]^c \in \tau^c \ni [BFR(B_E)] = [BFR(B_s)]^c \cap [BFR(E)]^c$ and $[BFR(B_G)]^c = [BFR(B_s)]^c \cap [BFR(G)]^c$. Since $[BFR(B_s)]^c \in \tau_{[BFR(B_s)]}$ and $[BFR(E)]^c, [BFR(G)]^c \in \tau$, we have $[BFR(B_E)]^c, [BFR(B_G)]^c \in \tau_{[BFR(B_s)]}$. Now, $[BFR(B_E)]^c, [BFR(B_G)]^c$ are two disjoint BFR closed subsets of $\tau_{BFR(B_s)}$, and since $(U, BFR(X), \tau)$ is a BFR normal space,$[BFR(B_E)]^c \subseteq BFR(A)$, $[BFR(B_G)]^c \subseteq BFR(B)$ and $BFR(A)\ \overline{q}\ BFR(B)$. It follows that, there exists $BFR(B_s) \cap BFR(A)$, $BFR(B_s) \cap BFR(B) \in \tau_{BFR(B_s)}$ such that $[BFR(B_E)]^c \subseteq BFR(B_s) \cap BFR(A)$, $[BFR(B_G)]^c \subseteq BFR(B_s) \cap BFR(B)$ and $(BFR(B_s) \cap BFR(A)) \cap (BFR(B_s) \cap BFR(B)) = BFR(B_s)\overline{q}\ (BFR(A)\ \overline{q}\ BFR(B))$. Therefore $(BFR(B_s), \tau_{BFR(B_s)})$ is a BFR normal space.

**Theorem 2.17.** *A BFR topological space $(U, BFR(A), BFR(B), \tau)$ is BFR normal if and only if for any BFR closed set $[BFR(E)]^c$ and BFR open set $BFR(D)$ containing $[BFR(E)]^c$, there exists a BFR open set $BFR(A) \in \tau$ such that $[BFR(E)]^c \subset BFR(A)$ and $\overline{BFR(A)} \subset BFR(D)$.*

**Proof.** Let $[BFR(E)]^c$ and $[BFR(D)]^c$ be disjoint BFR closed sets.
Let $(U, BFR(A), BFR(B), \tau)$ is BFR normal space.
$\Leftrightarrow \exists$ disjoint BFR open sets $BFR(A), BFR(B)$ such that $[BFR(E)]^c \subset BFR(A)$ and $[BFR(D)]^c \subset BFR(B)$.
Now $BFR(A) \subset [BFR(B)]^c$
$\Rightarrow \overline{BFR(A)} \subset \overline{[BFR(B)]^c} = [\underline{BFR(B)}]^c$. Also, $[BFR(D)]^c \subset BFR(B)$
$\Rightarrow [BFR(B)]^c \subset BFR(D) \Rightarrow \overline{BFR(A)} \subset BFR(D)$.

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