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SELECTION OF THREE STAGE CHAIN SAMPLING PLAN OF TYPE (0, 1, 2) WITH REPETITIVE GROUP SAMPLING PLAN USING TRIGONOMETRIC RATIO

V. Sangeetha and S. Nandhini

Department of Statistics,
PSG College of Arts and Science
Coimbatore - 641014, Tamil Nadu, INDIA

E-mail : Sangeetha_v@psgcas.ac.in, psm.nandhini@gmail.com

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Abstract: Acceptance Sampling plans are the practical tools for quality assurance applications involving product quality control. Using Trigonometric ratio, one can get a better plan which has an OC curve similar to ideal OC curve. The approach of trigonometric ratio method by considering the tangent of the angle between the lines joining the points $(AQL, 1-\alpha)(LQL, \beta)$. This paper introduces a procedure and tables for the selection of Three Stage Chain Sampling Plan $(0, 1, 2)$ with Repetitive group sampling plan using Trigonometric ratio, involving producers and Consumers quality levels. A table and methods are given for the construction of plans indexed by using trigonometric ratio.

Keywords and Phrases: Acceptable Quality Level, Indifference Quality Level, Limiting Quality Level and Three Stage Chain Sampling Plan, Minimum Angle method, Repetitive Group Sampling plan.

2010 Mathematics Subject Classification: 62P30, 62D05.

1. Introduction

Acceptance sampling is a statistical tool used to make decisions concerning whether or not a lot of products should be released for consumer use. An acceptance sampling plan is a statement regarding the required sample size for product inspection and the associated acceptance or rejection criteria for sentencing individual lots. The criteria used for measuring the performance of an acceptance sampling plan, is usually based on the operating characteristic (OC) curve which quantifies the risks for producers and consumers. The OC curve plots the probability of accepting the lot versus the lot fraction nonconforming, which displays the discriminatory power of the sampling plan. The basic acceptance sampling plan called the single-sampling plan is widely used in industry to inspect items due to its easiness of implementation. A single sampling attribute inspection plan calls for acceptance of a lot under consideration. If the number of nonconforming units found in a random sample of size n is less than or equal to the acceptance number Ac.

Whenever a sampling plan for costly or destructive testing is required, it is common to force the OC curve to pass through a point, say, (LQL, β). Unfortunately, the $Ac = 0$ plan has the following disadvantages.

- The OC curve of the $Ac = 0$ plan has no point of inflection and hence it starts to drop rapidly even for the smallest increases in the fraction nonconforming p.
- The producer dislikes an $AC = 0$ plan since a single occasional nonconformity will call for the rejection of the lot. The chain sampling plan Chsp-1 by Dodge is an answer to the question of whether anything can be done to improve the pathological shape of the OC curve of a zero-acceptance –number plan.

Review of Literature: Dodge (1955) treats this problem using a procedure, called chain sampling plan (ChSP – 1). These plans make use of the cumulative inspection results from several results, from one or more samples along with the results from the current sample, in making a decision regarding acceptance or rejection of the current lot. The chain sampling plans are applicable for both small and large samples. Dodge and Stephens (1966) extended the concept of chain sampling plans and presented a set of two stage chain sampling plans based on the concept of ChSP – 1 developed by Dodge (1955). They presented expressions for OC curves of certain two – stage chain sampling plans and made comparison with single and double sampling attributes inspection plans. The three-stage chain sampling plan of type ChSP (0, 1, 2) developed by Soundararajan and Raju (1984) is a generalization of Dodge (1955) chain sampling plan ChSP – 1 and Dodge and Stephens (1966) chain sampling plan ChSP – (0, 1). Soundararajan and Raju (1984) gives the structure and operating procedure of generalized three – stage chain sampling plan and expressions for OC curve of certain three – stage plans are also given. Raju (1991) has extended the two-stage chain sampling to three stages. The three stage cumulation procedure becomes complex, and will pay a limited role for costly or destructive inspection. The three-stage plan will however be useful for general type B lot by lot inspection. Soundararajan (1984) and Raju (1984) gave the structure and operating procedure for generalized three-stage chain sampling plan and expression for OC curve of certain three-stage plans are also given.Suresh and Sripriya (2007) has developed a method for designing plans from the desired operating ratio where procedures and tables for the construction of three stage chain sampling plan of type ChSP (0, 1, 2) and for selection of plans by specified parameters are given. Suresh and Anamiya (2012) has developed three –stage Chain Sampling Plans indexed through Minimum angel method with producer quality level and consumer quality level.

The concept of Repetitive Group Sampling plan (RGS) plan was introduced by Sherman (1965) in which acceptance and rejection of the lot is based on the repeated sample results of the same lot. The detailed procedure and tables for the construction and selection of RGS plans have been given by Soundarajan and Ramaswamy (1984) and Singh et al.(1989).The purpose of present investigation is twofold. Firstly, following Stephens and Dodge (1976) proposed plan which uses different sample size in the normal and tightened phases of inspection.

2. Selection of Sampling Plan

Repetitive Group Sampling plan (RGS). Conditions for the Repetitive Group Sampling Plan

1. The size of the lot is taken to be sufficiently large
2. Under normal conditions, the lots are expected to be of eventually same quality
3. The producer comes from a source in which the consumer has confidence Operating Procedure
4. Draw a random sample of size n1 from the lot of normal inspection and determine the number of defectives (d) found therein.
5. Accept the lot if d $\leq c_1$
6. Reject the lot if d $\leq c_2$

If $c_1 \leq d \leq c_2$ repeat the steps (1),(2) and (3).
It is also noted that $c_1 < c_2$. Thus, the RGS plan is determined by the parameters n, c_1, c_2 the probability of acceptance in a particular group sample is
The probability of acceptance in a particular group sample is

$$P_1(p) = \sum_{k_1=0}^{c_1} \frac{e^{-np}(np)^{k_1}}{k_1!} \tag{1}$$

The probability of rejection in a particular group sample is

$$P_1^1(p) = \sum_{k_2=c_2}^{\infty} \frac{e^{-np}(np)^{k_1}}{k_2!} \tag{2}$$

The probability of eventually accepting the lot is given as

$$P_1(n, c_1c_2/p) = \frac{P_1}{p_1 + p_1^1}$$

Then from (1) and (2)

$$P_a = \frac{\sum_{k_1=0}^{c_1} \frac{e^{-np}(np)^{k_1}}{k_1!}}{\sum_{k_2=c_2}^{\infty} \frac{e^{-np}(np)^{k_1}}{k_2!}}$$

The three-stage chain sampling plan has 7 parameters which are defined below:
n = sample size
k_1 = The maximum number of samples over which the cumulation of the defectives take place in the first stage of procedure.
k_2= The maximum number of samples over which the cumulation of the defectives take place in the second stage of procedure.
k_3 = The maximum number of samples over which the cumulation of the defectives take place in the first of procedure.
c_1= The allowable number of defectives in the cumulative results from k_1 or fewer sample of n. Thus, c_1 is an acceptance number for cumulative results. It is the cumulative results criterion (CRC) that must be met by cumulative sampling results during the first stage of of the restart period in order to permit acceptance of a lot.
c_2= The allowable number of defectives in the cumulative results from k_1+ 1 to k_2 sample of n. Thus, c_2 is also an acceptance number for cumulative results and the CRC that must be met by cumulative sampling results during the second stage of the restart period in order to permit acceptance of a lot.
c_3= The allowable number of defectives in the cumulative results from $k_2 + 1$ to k_3 sample of n. Thus, c_3 is also an acceptance number for cumulative results and the CRC that must be met by cumulative sampling results during the third stage of the restart period in order to permit acceptance of a lot.

3. Selection of ChSPRGS (0,1,2)

For construction and evaluation of the Three Stage Chain Repetitive Group Sampling plan, the np values presented in tables were derived under the procedure stated by Duncan [1965]. Tables are used to derive individual plan to meet specified values of fraction defectives and probability of acceptance. It requires the specifications of $AQL(p1), LTPD(p2)$, Producers risk (α), Consumers risk (β) and acceptance criteria i. The steps to be followed are,

1. Specify p_1 - Acceptable Quality Level (AQL), p_2 - Lot Tolerance Proportion Defective (LTPD), producer risk (α) and consumer risk (β).
2. The operating ratio is OR $=p_2/p_1$and m = np.
3. Choose the plan parameters having $k_1, k_2, k_3, c_1, c_2, c_3$ and i associated with an operating ratio which is nearest in the corresponding table.
4. Determine the sample size n = np_2/p_1.
5. The OC Curve may be drawn by dividing the values of np shown for the plan by sample size n to obtain p associated with 0.95 for Pa(p).
6. Thus, the plan consists of six parameters namely: n, $k_1, k_2, k_3, c_1, c_2, c_3$ and i may choose from the given tables

4. Construction and Evaluation of the Plan

Based on the principle of two points on the OC curve, the designing methodology of the ChSPRGS plan is explained below.According to Raju, the OC function of an ChSP (0,1,2) plan is given by,

where,
P_0= Probability of getting exactly zero non- conforming in a sample of size n
P_1= Probability of getting exactly one non- conforming in a sample of size n
P_2= Probability of getting exactly two non- conforming in a sample of size n

5. A Review on Minimum Angle Method

The practical performance of any sampling plan is generally revealed through its operating characteristic curve. When producer and consumer are negotiating for quality limits and designing sampling plans, it is important especially for the minimize the consumer risk. In order to minimize the consumer's risk, the ideal OC curve could be made to pass as closely through (AQL, 1-α) was proposed by Norman Bush (1953) considering the tangent of the angle between the lines joining the points (AQL, 1-α), (AQL, β). Norman Bush et al. (1953) have considered two points on the OC curve as (AQL, 1-α) and (IOL, 0.50) for minimize the consumer's risk. But Peach and Littauer (1946) have taken two points on the OC curves as (p_1,1-α) and (p_2,β) for ideal condition to minimize the consumers risks here another approach with minimization of angle between the lines joining the points (AQL,1-α), (AQL, β) and (AQL, 1- α), (LQL, β) was proposed by Singaravelu (1993). Applying this method one can get a better plan which has an OC curve approaching to the ideal OC curve. Govindaraju K (1990), Soundararajan V (1981) and many others have studied AQL.
The formula for tan θ is given as $tan\theta = \frac{oppositesite}{adjacentside}$

Tangent of angle made by AB and AC is
$tan\theta = \frac{(p_2-p_1)}{(P_a(p_1)-P_a(p_2))}$,
where $p1 = AQL$and $p2 = LQL$. This may be expressed as,
$ntan\theta = (np_2 - np_1)/(1 - \alpha - \beta), ntan\theta = (np_2 - np_1)/(1 - \alpha - \beta)$
The smaller value of this $tan\theta$closer is the angle θapproaching zero, and the chord AB approaching AC, the ideal condition through$(AQL, 1-\alpha), \theta = tan-1(ntan\theta/n)$ Using this formula, the minimum angle θ is obtained, for the given np_1 and np_2 values.

6. Construction of Tables

The binomial model for the OC curve will be exact in the case of fraction non-conforming. It can be satisfactorily approximated with the Poisson model where p is small, n is large, and np ¡ 5 when the quality is measured in terms of non-conformities, the Poisson model is the appropriate one. Under the Poisson assumption, the expression for P_0= e-np, P_1= np e-np, P_2= ((np) 2/2) e-np. The equation cannot easily solve. The solutions for np for a given Pa have been found by Newton's method of successive approximation and are tabulated in Table 1 for different values of k_1, k_2, k_3.

Example. 1. Given p_1= 0.50 and p_2= 2.24, then the Associated sets of values corresponding to the computed OR values from Table 2 is, k_1=13, k_2=14, k_3=16, np_1 = 0.1522, np_2 =2.302 and $ntan\theta$= 43.7467 from the above results, one can find, n = np_1/p_1 = 0.0695/0.50 = 0.3044 $\theta = tan-1(ntan\theta/n)\theta$ = tan-1(43.7467)/ 0.3044 = 0.9999 Now the minimum angle is θ= 0.9999. Hence the selected parameters for the three-stage chain sampling plan of type ChSP (0, 1, 2) for given p_1= 0.07 and p_2 = 2.24 with minimum angle θ= 0.9999.

7. Conclusion

Acceptance sampling is the technique which deals with the procedures in which decision either to accept or reject lots or process which are based on the examination of samples. The work presented in this paper relates to the new procedure for the construction and selection of tables for designing sampling inspection plan through Minimum Angle Method. This procedure reduces the cost of inspection for the producer and the consumer, gets good items. In practice it is desirable to design any sampling plan with the associated quality levels which concern to producer and consumer. Tables provided in this paper are tailor – made which are handy and ready made, which are also well considered for comparison purposes. Tables are also useful for developing and under developing countries, which have limited resources to the Industrial shop floor- situations.

Table 1: np values for given probability of acceptance by three stage chain sampling plan ChSP (0, 1, 2) with Repetitive Group Sampling plan i=1

k_1	k_2	k_3	0.99	0.95	0.90	0.75	0.50	0.1	0.05	0.01
1	2	3	0.5591	0.5941	0.6414	0.8063	1.1813	2.565	3.152	4.629
1	2	5	0.4442	0.4854	0.5406	0.7324	0.1453	2.561	3.151	4.575
2	2	5	0.0442	0.474	0.5164	0.6611	0.9939	2.394	3.043	4.562
2	2	5	0.0442	0.474	0.5164	0.6611	0.9939	2.394	3.043	4.562
2	3	4	0.4127	0.4398	0.4766	0.6079	0.928	2.334	3.005	4.561
2	4	5	0.3366	0.3643	0.4023	0.5419	0.878	2.321	3.002	4.561
2	5	4	0.3364	0.3549	0.3974	0.5101	0.8354	2.319	3.001	4.559
3	4	5	0.3357	0.3594	0.3917	0.5101	0.8167	2.314	2.995	4.559
4	5	6	0.2869	0.3085	0.3384	0.4515	0.759	2.311	2.9338	4.558
5	6	7	0.2521	0.2729	0.3012	0.4117	0.729	2.310	2.9048	4.558
6	7	8	0.2265	0.2462	0.2735	0.3831	0.712	2.304	2.8758	4.557
7	8	9	0.2067	0.2254	0.252	0.3619	0.7029	2.306	2.8463	4.557
8	9	10	0.1902	0.2086	0.2347	0.3456	0.698	2.302	2.8178	4.557
9	10	11	0.1771	0.1947	0.2204	0.3329	0.6954	2.299	2.788	4.559
9	10	11	0.1771	0.1947	0.2204	0.3329	0.6954	2.299	2.759	4.559
10	11	12	0.1653	0.183	0.2084	0.3227	0.394	2.298	2.7308	4.559
11	12	13	0.1558	0.1729	0.1981	0.3157	0.6932	2.297	2.7018	4.559
11	12	13	0.1558	0.1729	0.1981	0.3157	0.6932	2.297	2.6728	4.559
11	12	14	0.1501	0.1676	0.1935	0.3141	0.6931	2.296	2.6438	4.559
12	13	14	0.1472	0.1642	0.1892	0.3096	0.6928	2.295	2.994	4.559
13	14	15	0.1397	0.1565	0.1821	0.3047	0.6925	2.293	2.994	4.558
13	14	16	0.1353	0.1522	0.1784	0.3036	0.6925	2.302	2.994	4.567
13	14	16	0.1353	0.1522	0.1784	0.3036	0.6925	2.302	2.994	4.568
13	15	16	0.1334	0.1502	0.1763	0.3022	0.6924	2.302	2.994	4.569
14	15	16	0.1289	0.1499	0.1753	0.3007	0.6924	2.302	2.994	4.564
14	15	17	0.1223	0.1458	0.1721	0.2999	0.6923	2.302	2.994	4.563
14	17	18	0.1223	0.1442	0.1656	0.2974	0.6919	2.302	2.994	4.560
15	17	18	0.1236	0.1441	0.1656	0.2973	0.6919	2.302	2.994	4.564
15	16	18	0.1238	0.1480	0.1653	0.2971	0.6919	2.302	2.994	4.565
15	14	19	0.1215	0.1473	0.1651	0.2964	0.6919	2.302	2.994	4.566
15	16	19	0.1174	0.1367	0.1636	0.2950	0.6913	2.302	2.994	4.564

Table 2. Certain characteristic values for three stage chain sampling plan ChSP (0, 1, 2) with Repetitive Group Sampling plan through minimum angle method.

k_1	k_2	k_3	np_1	np_2	$P_a(p_1)$	$P_a(p_2)$	N$\tan\theta$
1	2	3	0.5941	2.565	0.9505	0.1002	38.89678
1	2	5	0.4854	2.561	0.9514	0.1005	39.99152
2	2	5	0.474	2.394	0.9539	0.1009	35.05883
2	3	4	0.4398	2.334	0.9526	0.1001	35.92167
2	4	5	0.3643	2.321	0.9530	0.1005	36.57769
2	5	4	0.3549	2.3194	0.9502	0.1004	38.80135
3	4	5	0.3594	2.3142	0.9546	0.1006	35.41116
4	5	6	0.3085	2.3111	0.9503	0.1033	37.33386
5	6	7	0.2729	2.3104	0.9486	0.1036	39.05108
6	7	8	0.2462	2.3086	0.9494	0.1038	38.77749
7	8	9	0.2254	2.3061	0.9511	0.1041	37.67877
8	9	10	0.2086	2.3029	0.9488	0.1044	39.40172
9	10	11	0.1947	2.299	0.9520	0.1046	37.13062
9	10	11	0.1947	2.299	0.9495	0.1002	42.29015
10	11	12	0.183	2.298	0.9501	0.1049	38.43621
11	12	13	0.1729	2.297	0.9485	0.1052	39.61233
11	12	13	0.1729	2.297	0.9457	0.1003	46.23851
11	12	14	0.1676	2.296	0.9534	0.1003	39.6274
12	13	14	0.1642	2.295	0.9471	0.1054	40.5533
13	14	15	0.1565	2.293	0.9517	0.1057	37.24622
13	14	16	0.1522	2.302	0.9489	0.1002	43.74675
13	14	16	0.1522	2.302	0.9496	0.1002	43.09218
13	15	16	0.1502	2.302	0.9517	0.1003	41.43933
14	15	16	0.1499	2.302	0.9486	0.1059	39.48172
14	15	17	0.1458	2.302	0.9496	0.1059	38.78392
14	17	18	0.1442	2.302	0.9527	0.1002	40.82499
15	17	18	0.1441	2.302	0.9483	0.1002	44.50932
15	16	18	0.1480	2.302	0.9515	0.1002	41.80192
15	14	19	0.1473	2.302	0.9492	0.1062	38.9942
15	16	19	0.1367	2.302	0.9453	0.1002	47.51054

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