South East Asian J. of Mathematics and Mathematical Sciences
Vol. 16, No. 1 (A) (2020), pp. 75-88

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

MILDLY α GENERALIZED CLOSED SETS AND ITS CLOSED MAPPINGS

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(**Received:** *Mar. 08, 2020* **Accepted:** *April. 24, 2020* **Published:** *Apr. 30, 2020)*

Abstract: In this paper, we define new types of closed sets called mildly α generalized closed sets and mildly α generalized closed mappings and study some of their properties. The relations with other notions directly or indirectly connected with mildly αgeneralized closed sets are investigated.

Keywords and Phrases: Mildly α generalized closed sets, closed mappings.

2010 Mathematics Subject Classification: 54A05, 54C10.

1. Introduction and Preliminaries

In 1970, N. Levine [8] introduced the concept of generalized closed sets in the topological space by comparing the closure of subset with its open supersets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axioms, covering properties and generalization of continuity. Kong et.al [6] shown some of the properties of generalized closed set have been found to be useful in computer science and digital topology. A. S. Mashhour et.al [13], M. Sheik John [22], J. K. Park et.al [17], Benchalli and Walli [3] introduced preclosed sets, weakly closed sets, mildly g–closed sets and rw-closed sets in topological spaces respectively. Also Walli et.al [26] introduced regular mildly generalized closed sets in 2016 and investigated some of its properties.

Also different types of closed and open mappings were studied by various researchers in general topology. In 1982, Malghan [12] introduced and investigated some properties of generalized closed maps. El-Deeb et.al [4], M. Sheik John [22], N. Nagaveni [14], I. Arockiarani [1] and Benchalli et.al [3] have introduced and studied pre-closed and preopen maps, w-closed and w-open maps, wg-closed, rwg-closed and wg-open, rwg-open maps, rg-closed and rg-open maps and rw-closed, rw-open maps respectively. In this paper new generalization of closed sets called mildly α-generalized closed sets are introduced and some of their properties and its closed mappings are discussed. Preliminaries

Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) always denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ) Cl(A), Int(A) and A^c denote the Closure of A, Interior of A and Compliment of A respectively.

Definition 1.1. *1. A subset A of X is called semi open set [7] if A $\subseteq$ cl(int(A)) and semi closed set [7] if int(cl(A)) $\subseteq$ A,*
2. A subset A of X is called preopen set [13] if A $\subseteq$ int(cl(A)) and preclosed set [13] if cl(int(A)) $\subseteq$ A,
3. A subset A of X is called regular open set [24] if A = int(cl(A)) and regular closed set [24] if A = cl(int(A)),
4. A subset A of X is called α-open set [15] if A $\subseteq$ int(cl(int(A))) and α-closed set [15] if cl(int(cl(A))) $\subseteq$A,
5. A subset A of X is called β-open set [2] if A $\subseteq$ cl(int(cl(A))) and β-closed set [2] if int(cl(int(A))) $\subseteq$ A,

Definition 1.2. *A subset A of a topological space (X,τ) is called*
1. Generalized closed (briefly g-closed) [7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
2. Generalized α-closed (briefly gα-closed) [10] if αcl(A)) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in X.
3. Weakly generalized closed (briefly wg-closed) [14] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
4. Strongly generalized closed (briefly g^*-closed) [18] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.
5. Weakly closed (briefly w-closed) [18] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi open in X.
6. Mildly generalized closed (briefly mildly g-closed) [17] if cl(int(A)) $\subseteq$ U when-

ever $A \subseteq U$ and U is g-open in X.
7. Regular weakly generalized closed (briefly rwg-closed) [14] if cl(int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is regular open in X.
8. Weakly π-generalized closed (briefly wπg-closed) [21] if cl(int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is π-open in X.
9. Regular weakly closed (briefly rw-closed) [3] if cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is regular semi open in X.
10. generalized preclosed (briefly gp-closed) [14] if pcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is open in X.
11. α-generalized closed (briefly αg-closed) [16] if αcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is open in X.
12. Regular generalized closed (briefly rg-closed) [20] if cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is regular open in X.
13. π-generalized closed (briefly πg-closed) [5] if cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is π-open in X.
14. Regular mildly generalized closed (briefly RMG-closed) [26] if cl(int(A))$\subseteq$ U whenever $A \subseteq U$ and U is regular generalized open in X.

Definition 1.3. *A function $f : X \longrightarrow Y$ is called*
1. Regular closed [1] if f(F) is closed in Y for every regular closed set F of X.
2. g-closed [12] if f(F) is g-closed in Y for every closed set F of X.
3. w-closed [23] if f(F) is w-closed in Y for every closed set F of X.
4. Pre-closed [4] if f(F) is pre-closed in Y for every closed set F of X.
5. w g^*(mildly-g)-closed [20] if f(F) w g^*-closed in Y for every closed set F of X.
6. wg-closed [14] if f(F) is wg-closed in Y for every closed set F of X.
7. rwg-closed [14] if f(F) is rwg-closed in Y for every closed set F of X.
8. rw-closed [3] if f(F) is rw-closed in Y for every closed set F of X.
9. rg-closed [14] if f(F) is rg-closed in Y for every closed set F of X.
10.g^*-closed [19] if f(F) is g^*-closed in Y for every closed set F

2. Mildly α generalized Closed Sets

In this section we introduce mildly α generalized closed sets and study some of their properties.

Definition 2.1. *A subset A of a topological (X, τ) is called mildly α generalized closed (mildly αg closed) if cl(int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is αg-open in X.*

Theorem 2.2. *Every closed set is mildly α generalized closed set in X.*
Proof. Let A be any closed set in X. Let U be a αg-open set in X such that A $\subseteq$

U. Since A is closed, cl(A)= A and hence cl(A) $\subseteq$ U. But cl(int(A) $\subseteq$ cl(A) $\subseteq$ U . Therefore cl(int(A)) $\subseteq$ U. Hence A is a mildly αgeneralized closed set in X. The converse of the above theorem need not true as seen from the following example.

Example 2.3. Let $X = \{a, b, c, d\}$ with $\tau = \{X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \emptyset\}$ mildly α generalized closed sets of (X, τ): $\{X, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, c, d\}, \{a, c, d\}, \emptyset\}$. Closed sets of (X, τ):$\{X, \{c\}, \{d\}, \{c, d\}\{b, c, d\}, \{a, c, d\}, \emptyset\}$. Here A = $\{a, d\}$ is mildly α generalized closed set but not a closed set.

Theorem 2.4. *Every pre-closed set is mildly α generalized closed set in X.*
Proof. Let A be any pre-closed set in X. Suppose U is αg-open in X such that A $\subseteq$ U. Since A is a pre-closed set in X, cl(int(A) $\subseteq$ A. We have cl(int(A) $\subseteq$ A $\subseteq$ U. (i.e) cl(int(A) $\subseteq$ U. Hence A is mildly α generalized closed set in X. The converse of the above theorem need not true as seen from the following example.

Example 2.5. Let $X = \{a, b, c\}$ with topology $\tau = \{X, \{a, c\}, \emptyset\}$. Pre-closed sets of (X, τ):$\{X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \emptyset\}$. mildly α generalized closed sets of (X, τ) :$\{X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \emptyset\}$. Here A = $\{a, c\}$is mildly α generalized closed set but not a pre-closed set.

Theorem 2.6. *Every α closed set is mildly α generalized closed set in X.*
Proof. Let A be a α-closed set in X. Suppose U is αg-open in X such that A $\subseteq$ U. By hypothesis, cl(int(cl(A))) $\subseteq$ A. Therefore cl(int(A)) $\subseteq$ cl(int(cl(A))) $\subseteq$ A $\subseteq$ U. Hence cl(int(A)) $\subseteq$ U . Therefore A is a mildly α generalized closed set in X. The converse of the above theorem need not true as seen from the following example.

Example 2.7. Let$X = \{a, b, c\}$ with topology $\tau = \{X, \{a, c\}, \emptyset\}$. α-closed sets of (X, τ):$\{\emptyset\}, \{b\}, X\}$ mildly α generalized closed sets of (X, τ) :$\{X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \emptyset\}$. Here A = $\{a\}$ is mildly α generalized closed set but not α-closed.

Theorem 2.8. *Every regular closed set is mildly α generalized closed set in X.*
Proof. Let A be any regular closed set in X. Suppose U is αg-open in X such that A $\subseteq$ U. Since A is a regular closed set, cl(int(A)) = A $\subseteq$ U. Every regular closed set is closed,Hence by Theorem 3.2, every closed set is mildly α generalized closed set. (i.e) cl(int(A)) $\subseteq$ U. Hence A is a mildly α generalized closed set in X. The converse of the above theorem need not true as seen from the following example.

Example 2.9. Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \emptyset\}$. mildly$\alpha$ generalized closed sets of (X, τ): $\{X, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, c, d\}, \{a, c, d\}, \emptyset\}$. Regular closed sets of (X, τ) :$\{X, \{c\}, \{d\}, \{c, d\}, \{b, c, d\},$

$\{a, c, d\}, \emptyset\}$. Here A $=\{a, d\}$ is mildly α generalized closed set but not a regular closed set.

Theorem 2.10. *Every mildly α generalized closed set is weakly generalized closed set in X.*

Proof. Let A be a mildly α generalized closed set in X. Suppose A $\subseteq$ U, where U is open in X.. Since every open set is αg -open in X, U is αg-open in X. As A is mildly αgeneralized cl(int(A)) $\subseteq$ U. Therefore A is a weakly generalized closed in X. The converse of the above theorem need not true as seen from the following example.

Example 2.11. Let$X = \{a, b, c, d\}$ with topology $\tau = \{X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. mildly α generalized closed sets of (X, τ):$\{X, \{c\}, \{d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, \emptyset\}$. Weakly generalized closed sets of (X, τ) : $\{X, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \emptyset\}$ Here A $= \{a, d\}$ weakly generalized closed set but not mildly α generalized closed set.

Theorem 2.12. *Every mildly α generalized closed set is mildly generalized closed set in X.*

Proof. Let A be any mildly α generalized closed set in X. Suppose U is g-open in X such that A $\subseteq$ U. Since A is mildly α generalized closed,(i.e) cl(int(A))$\subseteq$ U. U is αg-open in X. But every αg-open set is g-open in X. where cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U, Therefore A is a mildly generalized closed set in X. The converse of the above theorem need not true as seen from the following example.

Example 2.13. Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \emptyset\}$. Mildly g closed sets (X, τ) :$\{X, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}\emptyset\}$ Mildly α generalized closed sets (X, τ):$\{X, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \emptyset\}$ Here A $=\{b, d\}$ is a mildly α generalized closed set but not a mildly generalized closed set.

Remark 2.14. *The mildly α generalized closed set is a independent of generalized closed sets, Regular mildly generalized closed sets, regular weakly closed sets, strongly generalized closed sets α-generalized closed sets and regular generalized closed sets.*

Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Then, mildly α generalized closed sets of (X, τ) :$\{X, \{c\}, \{d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, \emptyset\}$. generalized closed sets are of (X, τ): $\{X, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \emptyset\}$. Regular mildly generalized closed sets of (X, τ): $\{X, \{d\}, \{a, b\}, \{a, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \emptyset\}$. Regular weakly closed sets of (X, τ): $\{X, \{d\}, \{a, b\}, \{a, d\}, \{a, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\},$

$\emptyset\}$. Strongly generalized closed of (X,τ) :$\{X,\{d\},\{a,d\},\{b,d\}$, $\{c,d\}$, $\{b,c,d\}$, $\{a,c,d\}$, $\{a,b,d\}$, $\emptyset\}$ Regular generalized closed sets of (X,τ): $\{X,\{d\},\{a,b\}$, $\{a,d\}$, $\{a,c\}$, $\{b,d\}$, $\{c,d\}$, $\{b,c,d\}$, $\{a,c,d\}$, $\{a,b,d\}$, $\{a,b,c\}$, $\emptyset\}$.

Remark 2.15. *The intersection of two mildly α generalized closed sets in X is mildly α generalized closed set in X.*

Proof. Let A and B be any two mildly α generalized closed sets of X. Then αg cl (A) $\subseteq$ U, αg cl (B $\subseteq$ U whenever A $\subseteq$ U and B $\subseteq$ U, U is αg-open. Let U be an αg -open set in X such that A$\bigcap$ B$\subseteq$ U, A$\neq$ U, B$\neq$ U, Now , αg cl (A$\bigcap$ B) $\subseteq$ αg cl (A)$\bigcap$ αg cl (B) $\subseteq$ U, U is αg -open in X. Hence A $\bigcap$ B is a mildly α generalized closed set.

Remark 2.16. *The union of two mildly α generalized closed sets in X need not be mildly α generalized closed set in X.*

Example 2.17. Let X = a, b, c with topology $\tau = \{X,\{b,c\},\emptyset\}$. mildly α generalized closed sets in X are$\{X,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\emptyset\}$. Then, A={b} B={c} are mildlyα generalized closed set in X. But A∪B=$\{b,c\}$ is not mildly α generalized closed set in X.

Remark 2.18. *The complement of a mildly α generalized closed set need not be mildly α generalized closed set in X.*

Example 2.19. Let $X = \{a,b,c,d\}$ with $\tau = \{X,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},\emptyset\}$ Then $A = \{d\}$ is mildly α generalized closed set in X. But $X-\{d\}$=$\{a,b,c\}$ is not mildly α generalized closed set in X.

Remark 2.20. *A subset A of X is mildly α generalized closed if and only if cl(int(A))-A does not contain any non empty α generalized closed in X.*

Proof. Let A be a mildly α generalized closed set and F be a non empty α generalized closed subset of cl(int(A))-A. Now F$\subseteq$ cl(int(A))-A= cl(int(A)) $\bigcap$ A^c which implies F$\subseteq$ cl(int(A)) and F$\subseteq$ A^c. Therefore, A$\subseteq$ F^c. Since F^c is αg open and A is mildly α generalized closed set in X, we have cl(int(A)) $\subseteq$ F ^{c} which implies that F$\subseteq$ $(\text{cl(int(A))})^c$. Hence cl(int(A))-A does not contain any non empty α generalized closed set.

Theorem 2.21. *For an element $x \in X$, X-x is mildly α generalized closed set or α generalized open set in X.*

Proof. Assume that xas α generalized open set of X. Therefore X-xis α generalized open set of X.Then X is the only α g open set containing X-x. Hence cl(int(X-x))$\subseteq$ X which implies that X-x is mildly α generalized closed set in X.

Theorem 2.22. *If A is an mildly α generalized closed set in X such that A$\subset$ B$\subset$*

cl(int (A)),then B is an mildly αgeneralized closed set in X.
Proof. Let A be a mildly α generalized closed set in X,such that A⊂ B⊂ cl(int(A)). Let U be αg open set such that B⊂ U ,then A⊂ U. Since A is mildly α generalized closed set, we have cl(int(A))⊂ U. Now as B ⊂ cl(int(A)), so cl(int(B))⊂ cl(int(cl(int(A))) ⊂ cl(int(A))⊂ U.Thus cl(int(B))⊂ U. whenever B ⊂ U, and U is α g open in X. Hence B is mildly α generalized closed set in X. The converse of the theorem need not be true in general.

Example 2.23. Let $X = \{a, b, c, d\}$ with $\tau = \{X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \emptyset\}$ Closed sets are:(X, τ): $\{X, \{d\}, \{c, d\}, \{a, d\}, \{b, c, d\}, \{a, c, d\}, \emptyset\}$ mildlyα generalized closed sets:(X, τ): $\{X, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, c, d\}, \{a, c, d\}, \emptyset\}$ Let $A = \{a, d\}$, $B = \{a, c, d\}$,Now A and B both are mildly αgeneralized closed set. But A⊂ B⊄ cl(int(A)).

Theorem 2.24. *If A is g-open and mildly g closed in X. Then A is mildly α generalized closed set in X.*
Proof. Let A be g-open and mildly g closed in X. suppose U is an α g- open set in X,such that A ⊆ U.As A is mildly g closed and A is g-open. cl(int(A))⊆ A⊆ U. Hence A is mildly α generalized closed set in X.

Theorem 2.25. *If A is both open and mildly α generalized closed set in X, then A is closed set in X.*
Proof. Let A is both open and mildly α generalized closed set in X. To prove A is closed in X. Now cl(int(A)) ⊆ cl(A) ⊆ U,A⊆U, U is open in X. Thus A is open in X .(i.e) cl(A⊆) A. Also A ⊆ cl(A). Hence A= cl(A). Therefore A is closed set in X.

3. Mildly α generalized Closed Maps in Topological Spaces

In this section we introduce mildly α generalized closed sets and study some of their properties.

Definition 3.1. *A map f: $X \longrightarrow Y$ is said to be mildly α generalized closed map, if the image of every closed set in X is mildly α generalized closed in Y.*

Theorem 3.2. *Every closed map is mildlyα generalized closed map but not conversely.*
Proof. Let f: X $\longrightarrow Y$ is closed map. Let F be any closed set in X. Then f(F) is closed but every closed set is mildly α generalized closed set Hence f is mildly α generalized closed map.

Example 3.3. Let $X = \{p, q, r\}$ with topology $\tau = \{X, \{p, q\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a\}, \{b, c\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(p)=c, f(q)=b and f(r)=c. Then f is mildly α generalized closed map but not closed, since the image of closed set $\{r\}$in X is $\{c\}$, which is not closed in Y

Theorem 3.4. *Every pre-closed map is mildly α generalized closed map but not conversely.*

Proof. Let f: X $\longrightarrow Y$ is pre-closed map. Let F be any closed set in X. Then f(F) is pre-closed but every pre-closed set is mildly α generalized closed set. Hence f is mildly α generalized closed map.

Example 3.5. Let$X = \{p, q, r\}$ with topology $\tau = \{X, \{q\}, \{q, r\}\emptyset\}$ and $Y = \{a, b, c\}$ with topology $\sigma = \{Y, \{a, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(p)=c, f(p,r)=$\{a, c\}$Then f is mildly α generalized closed map but not pre-closed, since the image of closed set $\{p, r\}$ in X is $\{a, c\}$, which is not pre-closed in Y.

Theorem 3.6. *Every mildly αclosed map is mildly generalized closed map but not conversely.*

Proof. Let f: X $\longrightarrow Y$ is mildly αg closed map. Let F be any closed set in X. Then f(F) is mildly generalized but every mildly α g closed set is mildly generalized closed set.Hence f is mildly generalized closed map.

Example 3.7. Let $X = \{r, s, t, u\}$ with topology $\tau = \{X, \{r, s\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a, c\}, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(r)=c, f(s)=a and f(t)=d f(u)=b. Then f is mildly generalized closed map but not mildly α g closed, since the image of closed set $\{t, u\}$ in X is $\{b, d\}$, which is not mildly αg closed in Y.

Theorem 3.8. *Every α -closed map is mildly α generalized closed map but not conversely.*

Proof. Let f: X $\longrightarrow Y$ is α g-closed map. Let F be any closed set in X. Then f(F) is α closed but every α closed set is mildly α generalized closed set.Hence f is mildlyα generalized closed map.

Example 3.9. Let $X = \{p, q, r\}$ with topology $\tau = \{X, \{p, q\}, \{p, r\}, \{r\}, \emptyset\}$ and $Y = \{a, b, c, \}$ with topology $\sigma = \{Y, \{a, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(r)=c, f(p)= b and f(q)=a. Then f is mildly α generalized closed map but not α closed, since the image of closed set $\{q\}$ in X is $\{a\}$, which is not α closed in Y.

Theorem 3.10. *Every regular closed map is mildly αgeneralized closed map but not conversely.*

Proof. Let Let f: X $\longrightarrow Y$ is regular closed map. Let F be any closed set in X. Then f(F) is regular closed but every regular closed set is mildly α generalized closed set. Hence f is mildly α generalized closed map.

Example 3.11. Let $X = \{m, n, o\}$ with topology $\tau = \{X, \{m, n\}, \{n\}\emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(m)=a, f(n)=b,f(o)=d. Then f is mildly α generalized closed map but not regular closed, since the image of closed set $\{m, o\}$ in X is $\{a, d\}$, which is not regular closed in Y.

Theorem 3.12. *Every mildly α generalized closed map is wg-closed map but not conversely.*

Proof. Let f: X $\longrightarrow Y$ is mildly α generalized closed map. Let F be any closed set in X. Then f(F) is mildly α generalized closed but every mildly α generalized closed set is wg-closed set. Hence f is wg-closed map.

Example 3.13. Let $X = \{p, q, r\}$ with topology $\tau = \{X, \{p, q\}, \{p\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y\}, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(p)=b, f(q)=a and f(r)=d. Then f is wg-closed map but not mildly α generalized closed, since the image of closed set $\{q, r\}$ in X is $\{a, d\}$, which is not mildly α generalized closed in Y.

Theorem 3.14. *Every mildly α generalized closed map is rwg-closed map but not conversely.*

Proof. Let f: X $\longrightarrow Y$ is mildly α generalized closed map. Let F be any closed set in X. Then f(F) is mildly α generalized closed but every mildly α generalized closed set is rwg-closed set. Hence f is rwg-closed map.

Example 3.15. Let $X = \{r, s, t\}$with topology $\tau = \{X, \{r, s\}, \{r\}$ $\emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a, c\}, \{a\}, \{b\}, \{c\}, \{d\},$ $\{b, c\}, \{a, b\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}\emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(r)=c, f(s)=b and f(t)=a. Then f is rwg-closed map but not mildly α generalized closed, since the image of closed set $\{s, t\}$ in X is $\{a, b\}$, which is not mildly α generalized closed.

Remark 3.16. *The following example shows generalized closed maps and mildly α generalized closed maps are independent.*

Example 3.17. Let $X = \{p, q, r\}$ with topology $\tau = \{X, \{r, s\}, \{p, q\}, \{r, \}\{p, q, r\}$ $\emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a\}, \{b\}, \{b, c\}, \{a, b\},$ $\{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(p)=d, f(q)=a and f(r)=b, f(s)=c. Then f is mildly α generalized closed map but not g-closed, since the image of closed set $\{p, q, r\}$ in X is $\{a, d\}$, which is not g-closed in Y.

Example 3.18. Let $X = \{p, q, r, s\}$ with topology $\tau = \{X, \{p, q, s\}, \{p, q, r\}, \emptyset\}$

and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$.

Let a map f: X $\longrightarrow Y$ be defined by f(r)=c, f(s)=d and f(p)=b,f(q)=a. Then f is g-closed map but not mildly α generalized closed, since the image of closed set $\{s$ in X is $\{d\}$, which is not mildly α generalized closed in Y.

Remark 3.19. *The following example shows RMG closed maps and mildly α generalized closed maps are independent.*

Example 3.20. Let $X = \{p, q, r\}$ with topology $\tau = \{X, \{p, q\}, \{r\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a, c\}, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(p)=c, f(q)=d and f(r)=b. Then f is mildly α generalized closed map but not RMG-closed, since the image of closed set $\{p, q\}$ in X is $\{c, d\}$, which is not RMG-closed in Y.

Example 3.21. Let $X = \{p, q, r\}$ with topology $\tau = \{X, \{p, q\}, \{r\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology
$sigma = \{Y, \{a, c\}, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(p)=c, f(q)=d and f(r)=b. Then f is RMG closed map but not mildly α generalized closed map, since the image of closed set $\{p, q\}$ in X is $\{a, b\}$, which is not mildly α generalized closed in Y.

Remark 3.22. *The following example shows RW closed maps and mildly α generalized closed maps are independent.*

Example 3.23. Let $X = \{r, s, t, u\}$ with topology $\tau = \{X, \{r, s\}, \{s, t\}, \{t\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(r)=c, f(s)=a, and f(t)=b. Then f is mildly α generalized closed map but not RW-closed, since the image of closed set $\{r\}$ in X is $\{c\}$, which is not RW-closed in Y.

Example 3.24. Let $X = \{r, s, t\}$ with topology $\tau = \{X, \{s, t\}, \{t\}\emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined as f(r)=b, f(s)=a, and f(t)=c. Then f is mildly α generalized closed map but not mildly α generalized closed, since the image of closed set $\{r, s\}$ in X is $\{a, b\}$, which is not mildly α generalized closed in Y.

Remark 3.25. *The following example shows strongly generalized closed maps and mildly α generalized closed maps are independent.*

Example 3.26. Let $X = \{p, q, r, s\}$ with topology $\tau = \{X, \{r, s\}, \{p, q\}, \{s, p\}, \{q, r\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined by f(p)=b, f(q)=a and f(r)=d, f(s)=c. Then f is mildly α generalized closed map but not strongly generalized closed, since the image of closed set $\{s\}$ in X is $\{c\}$, which is not strongly generalized -closed in Y.

Example 3.27. Let $X = \{p, q, r, s\}$ with topology $\tau = \{X, \{r, s\}, \{p\}, \{r\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\tau = \{Y, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X$\longrightarrow Y$ be defined by f(p)=a, f(q)=d and f(r)=b, f(s)=c. Then f is strongly generalized closed map but not mildly α generalized closed map, since the image of closed set $\{p, q\}$ in X is $\{a, d\}$, which is not mildly α generalized closed map in Y.

Remark 3.28. *The following example shows regular generalized closed maps and mildly α generalized closed maps are independent.*

Example 3.29. Let $X = \{p, q, r, s\}$ with topology $\tau = \{X, \{r, s\}, \{p, q\}, \{s, p\}, \{q, r\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a\}, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined by f(p)=b, f(q)=a and f(r)=d, f(s)=c. Then f is mildly α generalized closed map but not regular generalized closed, since the image of closed set $\{s\}$ in X is $\{c\}$, which is not regular generalized closed in Y.

Example 3.30. Let $X = \{p, q, r\}$ with topology $\tau = \{X, \{r\}, \{p\}, \{s\}, \emptyset\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{Y, \{a\}, \{b, d\}, \{b, c, d\}, \{a, b, d\}, \emptyset\}$. Let a map f: X $\longrightarrow Y$ be defined by f(p)=b, f(q)=a and f(r)=d, f(s)=c. Then f is regular generalized closed map but not mildly α generalized closed , since the image of closed set $\{p, q\}$ in X is $\{a, b\}$, which is not regular generalized closed in Y.

3. Conclusion:

In this paper mildly α generalized closed set is introduced and its closed mappings were studied.In future we extend concept of mildly α generalized closed set to binary ideal topological spaces.

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