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CORDIAL LABELING FOR FIVE STAR GRAPH

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Abstract: In this paper, we proved that the five star graph $K_{1,\eta_1} \wedge K_{1,\eta_2} \wedge K_{1,\eta_3} \wedge K_{1,\eta_4} \wedge K_{1,\eta_5}$ is a cordial graph for all $\eta_1 \geq 1, \eta_2 \geq 1, \eta_3 \geq 1, \eta_4 \geq 1$ and $\eta_5 \geq 1$.

Keywords and Phrases: Star graph, Cordial graph and Wedge.
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1. Introduction and Preliminaries

In [4], we considered undirected, finite and simple graph $R = (N(R), L(R))$, where $N(R)$ denotes node set of R and $L(R)$ denotes link set of R. In [5], cordial graphs for smaller graphs are given. In [2], Cahit proved that the following graphs are cordial: Every tree is cordial; K_η is cordial if and only if $\eta \leq 3$; K_{η_1,η_2} is cordial for all η_1 and η_2; all fans are cordial; the wheel W_η is cordial if and only if $\eta \neq 3(mod 4)$; maximal outerplanar graphs are cordial; and an Eulerian graph is not cordial if its size is congruent to $2(mod 4)$. In [3], [6] and [7] they proved that the two star graph $K_{1,\eta_1} \wedge K_{1,\eta_2}$, three star graph $K_{1,\eta_1} \wedge K_{1,\eta_2} \wedge K_{1,\eta_3}$ and four star graph $K_{1,\eta_1} \wedge K_{1,\eta_2} \wedge K_{1,\eta_3} \wedge K_{1,\eta_4}$ is a cordial labeling. We provided some definitions which are used for our present study. After referring all these results we got inspired and found that every five star graph is cordial.

Wedge. A wedge is a link which is used for connecting two components of a graph.

It is denoted as $\wedge$. $\lambda(R\wedge) < \lambda(R)$, where λ denotes the number of components of graph R.

Cordial Graph. In [1], let t be a function from the nodes of R to $\{0, 1\}$ and for each link $\lambda\mu$ assigns the label $|t(\lambda) - t(\mu)|$, call t a cordial labeling of R if the number of nodes labeled 0 and the number of nodes labeled 1 differs by atmost 1 and the number of links labeled 0 and number links labeled 1 differs by atmost 1.

2. Main Results

Theorem 2.1. *Every five star graph* $K_{1,\eta_1} \wedge K_{1,\eta_2} \wedge K_{1,\eta_3} \wedge K_{1,\eta_4} \wedge K_{1,\eta_5}$ *is a cordial graph for all* $\eta_1 \geq 1, \eta_2 \geq 1, \eta_3 \geq 1, \eta_4 \geq 1$ *and* $\eta_5 \geq 1$.

Proof. Let the graph $R = K_{1,\eta_1} \wedge K_{1,\eta_2} \wedge K_{1,\eta_3} \wedge K_{1,\eta_4} \wedge K_{1,\eta_5}$.
Let $N(R)$ be the node set of R and $L(R)$ be the link set of R. Then we have that,
$N(R) = \{\alpha, \beta, \gamma, \lambda, \mu\} \cup \{\alpha_\xi : 1 \leq \xi \leq \eta_1\} \cup \{\beta_\xi : 1 \leq \xi \leq \eta_2\} \cup \{\gamma_\xi : 1 \leq \xi \leq \eta_3\} \cup \{\lambda_\xi : 1 \leq \xi \leq \eta_4\} \cup \{\mu_\xi : 1 \leq \xi \leq \eta_5\}$.
$L(R) = \{\alpha\alpha_\xi : 1 \leq \xi \leq \eta_1\} \cup \{\beta\beta_\xi : 1 \leq \xi \leq \eta_2\} \cup \{\gamma\gamma_\xi : 1 \leq \xi \leq \eta_3\} \cup \{\lambda\lambda_\xi : 1 \leq \xi \leq \eta_4\} \cup \{\mu\mu_\xi : 1 \leq \xi \leq \eta_5\} \cup \{\alpha_\xi\beta_\xi\} \cup \{\beta_\xi\gamma_\xi\} \cup \{\gamma_\xi\lambda_\xi\} \cup \{\lambda_\xi\mu_\xi\}$ then R has $\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + 5$ nodes and $\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + 4$ links.
Now we have to prove that R is a cordial graph for all
$\eta_1 \geq 1, \eta_2 \geq 1, \eta_3 \geq 1, \eta_4 \geq 1$ and $\eta_5 \geq 1$. Let $t : N(R) \to \{0, 1\}$ and $t^* : L(R) \to \{0, 1\}$. Assume $N_t(i) = |n_t(0) - n_t(1)|$ and $L_t(i) = |l_t(0) - l_t(0)|$.
We will discuss about node and link labeling of the following 32 cases of odd and even combinations of $\eta_1, \eta_2, \eta_3, \eta_4$ and η_5.
The cases which given in below tabular column will obey the following node labeling of R : $t(\alpha) = 0$; $t(\beta) = 1$; $t(\gamma) = 0$; $t(\lambda) = 1$; $t(\mu) = 0$; $t(\alpha_{2\xi-1}) = 1$ for $1 \leq \xi \leq \lceil \frac{\eta_1}{2} \rceil$; $t(\alpha_{2\xi}) = 0$ for $1 \leq \xi \leq \lfloor \frac{\eta_1}{2} \rfloor$; $t(\beta_{2\xi-1}) = 1$ for $1 \leq \xi \leq \lceil \frac{\eta_2}{2} \rceil$; $t(\beta_{2\xi}) = 0$ for $1 \leq \xi \leq \lfloor \frac{\eta_2}{2} \rfloor$; $t(\gamma_{2\xi-1}) = 1$ for $1 \leq \xi \leq \lceil \frac{\eta_3}{2} \rceil$; $t(\gamma_{2\xi}) = 0$ for $1 \leq \xi \leq \lfloor \frac{\eta_3}{2} \rfloor$; $t(\lambda_{2\xi-1}) = 0$ for $1 \leq \xi \leq \lceil \frac{\eta_4}{2} \rceil$; $t(\lambda_{2\xi}) = 1$ for $1 \leq \xi \leq \lfloor \frac{\eta_4}{2} \rfloor$; $t(\mu_{2\xi-1}) = 1$ for $1 \leq \xi \leq \lceil \frac{\eta_5}{2} \rceil$ and $t(\mu_{2\xi}) = 0$ for $1 \leq \xi \leq \lfloor \frac{\eta_5}{2} \rfloor$. Then the link labeling is given by $\alpha\alpha_{2\xi-1} = 1$ for $1 \leq \xi \leq \left\lceil \frac{\eta_1}{2} \right\rceil$; $\alpha\alpha_{2\xi} = 0$ for $1 \leq \xi \leq \left\lfloor \frac{\eta_1}{2} \right\rfloor$; $\beta\beta_{2\xi-1} = 0$ for $1 \leq \xi \leq \left\lceil \frac{\eta_2}{2} \right\rceil$; $\beta\beta_{2\xi} = 1$ for $1 \leq \xi \leq \left\lfloor \frac{\eta_2}{2} \right\rfloor$; $\gamma\gamma_{2\xi-1} = 1$ for $1 \leq \xi \leq \left\lceil \frac{\eta_3}{2} \right\rceil$; $\gamma\gamma_{2\xi} = 0$ for $1 \leq \xi \leq \left\lfloor \frac{\eta_3}{2} \right\rfloor$; $\lambda\lambda_{2\xi-1} = 1$ for $1 \leq \xi \leq \left\lceil \frac{\eta_4}{2} \right\rceil$; $\lambda\lambda_{2\xi} = 0$ for $1 \leq \xi \leq \left\lfloor \frac{\eta_4}{2} \right\rfloor$; $\mu\mu_{2\xi-1} = 1$ for $1 \leq \xi \leq \left\lceil \frac{\eta_5}{2} \right\rceil$ and $\mu\mu_{2\xi} = 0$ for $1 \leq \xi \leq \left\lfloor \frac{\eta_5}{2} \right\rfloor$. Also wedge is labeled by $\alpha_1\beta_1 = 0$; $\beta_1\gamma_1 = 0$; $\gamma_1\lambda_1 = 1$ and $\lambda_1\mu_1 = 1$.

Cases	$n_t(0)$	$n_t(1)$	$N_t(i)$	$l_t(0)$	$l_t(1)$	$L_t(i)$
$\eta_1,\eta_2,\eta_3,\eta_4$ and η_5 are even	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+1+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	1	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	0
η_1 is odd and $\eta_2,\eta_3,\eta_4,\eta_5$ are even	$\lceil\frac{\eta_1}{2}\rceil+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\lceil\frac{\eta_1}{2}\rceil+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	0	$\lceil\frac{\eta_1}{2}\rceil+1+\frac{\eta_2}{2}+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\lceil\frac{\eta_1}{2}\rceil+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	1
η_2 is odd and $\eta_1,\eta_3,\eta_4,\eta_5$ are even	$\frac{\eta_1}{2}+1+\lfloor\frac{\eta_2}{2}\rfloor+1+\frac{\eta_3}{2}+1+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\lfloor\frac{\eta_2}{2}\rfloor+1+\frac{\eta_3}{2}+1+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	0	$\frac{\eta_1}{2}+1+\lfloor\frac{\eta_2}{2}\rfloor+1+\frac{\eta_3}{2}+1+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\lfloor\frac{\eta_2}{2}\rfloor+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	1
η_3 is odd and $\eta_1,\eta_2,\eta_4,\eta_5$ are even	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\lfloor\frac{\eta_3}{2}\rfloor+1+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\lfloor\frac{\eta_3}{2}\rfloor+1+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	0	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\lfloor\frac{\eta_3}{2}\rfloor+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\lfloor\frac{\eta_3}{2}\rfloor+1+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	1
η_5 is odd and $\eta_1,\eta_2,\eta_3,\eta_4$ are even	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+1+\frac{\eta_4}{2}+\lfloor\frac{\eta_5}{2}\rfloor$	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+1+\frac{\eta_4}{2}+\lfloor\frac{\eta_5}{2}\rfloor$	0	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\lfloor\frac{\eta_5}{2}\rfloor$	$\frac{\eta_1}{2}+1+\frac{\eta_2}{2}+1+\frac{\eta_3}{2}+1+\frac{\eta_4}{2}+\lfloor\frac{\eta_5}{2}\rfloor$	1
η_1,η_2 are odd and η_3,η_4,η_5 are even	$\lceil\frac{\eta_1}{2}\rceil+1+\lfloor\frac{\eta_2}{2}\rfloor+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\lceil\frac{\eta_1}{2}\rceil+1+\lfloor\frac{\eta_2}{2}\rfloor+1+\frac{\eta_3}{2}+1+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	1	$\lceil\frac{\eta_1}{2}\rceil+1+\lfloor\frac{\eta_2}{2}\rfloor+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\lceil\frac{\eta_1}{2}\rceil+1+\lfloor\frac{\eta_2}{2}\rfloor+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	0
η_2,η_3 are odd and η_1,η_4,η_5 are even	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\lfloor\frac{\eta_3}{2}\rfloor+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\lfloor\frac{\eta_3}{2}\rfloor+1+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	1	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\lfloor\frac{\eta_3}{2}\rfloor+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\lfloor\frac{\eta_3}{2}\rfloor+\frac{\eta_4}{2}+\frac{\eta_5}{2}$	0
η_2,η_4 are odd and η_1,η_3,η_5 are even	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\frac{\eta_3}{2}+1+\lfloor\frac{\eta_4}{2}\rfloor+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\frac{\eta_3}{2}+\lfloor\frac{\eta_4}{2}\rfloor+\frac{\eta_5}{2}$	1	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\frac{\eta_3}{2}+\lfloor\frac{\eta_4}{2}\rfloor+\frac{\eta_5}{2}$	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\frac{\eta_3}{2}+\lfloor\frac{\eta_4}{2}\rfloor+\frac{\eta_5}{2}$	0
η_2,η_5 are odd and η_1,η_3,η_4 are even	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\lfloor\frac{\eta_5}{2}\rfloor$	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\frac{\eta_3}{2}+1+\frac{\eta_4}{2}+\lfloor\frac{\eta_5}{2}\rfloor$	1	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\lfloor\frac{\eta_5}{2}\rfloor$	$\frac{\eta_1}{2}+1+\lceil\frac{\eta_2}{2}\rceil+1+\frac{\eta_3}{2}+\frac{\eta_4}{2}+\lfloor\frac{\eta_5}{2}\rfloor$	0

The cases which given in below tabular column will obey the following node labeling of R : $t(\alpha)=0; t(\beta)=1; t(\gamma)=0; t(\lambda)=0; t(\mu)=1$; $t(\alpha_{2\xi-1})=1$ for $1\le\xi\le\lceil\frac{\eta_1}{2}\rceil$; $t(\alpha_{2\xi})=0$ for $1\le\xi\le\lfloor\frac{\eta_1}{2}\rfloor$; $t(\beta_{2\xi-1})=0$ for $1\le\xi\le\lceil\frac{\eta_2}{2}\rceil$; $t(\beta_{2\xi})=1$ for $1\le\xi\le\lfloor\frac{\eta_2}{2}\rfloor$; $t(\gamma_{2\xi-1})=1$ for $1\le\xi\le\lceil\frac{\eta_3}{2}\rceil$; $t(\gamma_{2\xi})=0$ for $1\le\xi\le\lfloor\frac{\eta_3}{2}\rfloor$; $t(\lambda_{2\xi-1})=0$ for $1\le\xi\le\lceil\frac{\eta_4}{2}\rceil$; $t(\lambda_{2\xi})=1$ for $1\le\xi\le\lfloor\frac{\eta_4}{2}\rfloor$; $t(\mu_{2\xi-1})=1$ for $1\le\xi\le\lceil\frac{\eta_5}{2}\rceil$ and $t(\mu_{2\xi})=0$ for $1\le\xi\le\lfloor\frac{\eta_5}{2}\rfloor$. Then the link labeling is given by $\alpha\alpha_{2\xi-1}=1$ for $1\le\xi\le\left\lceil\frac{\eta_1}{2}\right\rceil$; $\alpha\alpha_{2\xi}=0$ for $1\le\xi\le\left\lfloor\frac{\eta_1}{2}\right\rfloor$; $\beta\beta_{2\xi-1}=1$ for $1\le\xi\le\left\lceil\frac{\eta_2}{2}\right\rceil$; $\beta\beta_{2\xi}=0$ for $1\le\xi\le\left\lfloor\frac{\eta_2}{2}\right\rfloor$; $\gamma\gamma_{2\xi-1}=1$ for $1\le\xi\le\left\lceil\frac{\eta_3}{2}\right\rceil$; $\gamma\gamma_{2\xi}=0$ for $1\le\xi\le\left\lfloor\frac{\eta_3}{2}\right\rfloor$; $\lambda\lambda_{2\xi-1}=0$ for $1\le\xi\le\left\lceil\frac{\eta_4}{2}\right\rceil$; $\lambda\lambda_{2\xi}=1$ for

$1 \le \xi \le \left\lfloor \frac{\eta_4}{2} \right\rfloor$; $\mu\mu_{2\xi-1} = 0$ for $1 \le \xi \le \left\lceil \frac{\eta_5}{2} \right\rceil$ and $\mu\mu_{2\xi} = 1$ for $1 \le \xi \le \left\lfloor \frac{\eta_5}{2} \right\rfloor$. Also wedge is labelled by $\alpha_1\beta_1 = 1$; $\beta_1\gamma_1 = 1$; $\gamma_2\lambda_1 = 0$ and $\lambda_1\mu_2 = 0$.

Cases	$n_t(0)$	$n_t(1)$	$N_t(i)$	$l_t(0)$	$l_t(1)$	$L_t(i)$
η_1 is even and $\eta_2, \eta_3, \eta_4, \eta_5$ are odd	$\frac{\eta_1}{2} + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	$\frac{\eta_1}{2} + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	1	$\frac{\eta_1}{2} + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	$\frac{\eta_1}{2} + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	0
η_2 is even and $\eta_1, \eta_3, \eta_4, \eta_5$ are odd	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	1	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	0
η_3 is even and $\eta_1, \eta_2, \eta_4, \eta_5$ are odd	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + 1 + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	1	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + \lceil \frac{\eta_4}{2} \rceil + \lceil \frac{\eta_5}{2} \rceil$	0
η_1, η_2 are even and η_3, η_4, η_5 are odd	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	0	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	1
η_2, η_3 are even and η_1, η_4, η_5 are odd	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \lfloor \frac{\eta_5}{2} \rfloor$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \lfloor \frac{\eta_5}{2} \rfloor$	0	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \lfloor \frac{\eta_5}{2} \rfloor$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \lfloor \frac{\eta_4}{2} \rfloor + \lfloor \frac{\eta_5}{2} \rfloor$	1
η_3, η_4 are even and η_1, η_2, η_5 are odd	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	0	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	1
η_3, η_4 are odd and η_1, η_2, η_5 are even	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lceil \frac{\eta_3}{2} \rceil + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lceil \frac{\eta_3}{2} \rceil + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	1	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lceil \frac{\eta_3}{2} \rceil + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lceil \frac{\eta_3}{2} \rceil + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	0
$\eta_1, \eta_2, \eta_3, \eta_4$ and η_5 are odd	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	0	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	1

The cases which given in below tabular column will obey the following node labeling of R : $t(\alpha) = 0; t(\beta) = 1; t(\gamma) = 0; t(\lambda) = 0; t(\mu) = 0$; $t(\alpha_{2\xi-1}) = 1$ for $1 \le \xi \le \lceil \frac{\eta_1}{2} \rceil$; $t(\alpha_{2\xi}) = 0$ for $1 \le \xi \le \lfloor \frac{\eta_1}{2} \rfloor$; $t(\beta_{2\xi-1}) = 1$ for $1 \le \xi \le \lceil \frac{\eta_2}{2} \rceil$; $t(\beta_{2\xi}) = 0$ for $1 \le \xi \le \lfloor \frac{\eta_2}{2} \rfloor$; $t(\gamma_{2\xi-1}) = 1$ for $1 \le \xi \le \lceil \frac{\eta_3}{2} \rceil$; $t(\gamma_{2\xi}) = 0$ for $1 \le \xi \le \lfloor \frac{\eta_3}{2} \rfloor$;

$t(\lambda_{2\xi-1}) = 1$ for $1 \le \xi \le \lceil \frac{\eta_4}{2} \rceil$; $t(\lambda_{2\xi}) = 0$ for $1 \le \xi \le \lfloor \frac{\eta_4}{2} \rfloor$; $t(\mu_{2\xi-1}) = 1$ for $1 \le \xi \le \lceil \frac{\eta_5}{2} \rceil$ and $t(\mu_{2\xi}) = 0$ for $1 \le \xi \le \lfloor \frac{\eta_5}{2} \rfloor$. Then the link labeling is given by $\alpha\alpha_{2\xi-1} = 1$ for $1 \le \xi \le \left\lceil \frac{\eta_1}{2} \right\rceil$; $\alpha\alpha_{2\xi} = 0$ for $1 \le \xi \le \left\lfloor \frac{\eta_1}{2} \right\rfloor$; $\beta\beta_{2\xi-1} = 0$ for $1 \le \xi \le \left\lceil \frac{\eta_2}{2} \right\rceil$; $\beta\beta_{2\xi} = 1$ for $1 \le \xi \le \left\lfloor \frac{\eta_2}{2} \right\rfloor$; $\gamma\gamma_{2\xi-1} = 1$ for $1 \le \xi \le \left\lceil \frac{\eta_3}{2} \right\rceil$; $\gamma\gamma_{2\xi} = 0$ for $1 \le \xi \le \left\lfloor \frac{\eta_3}{2} \right\rfloor$; $\lambda\lambda_{2\xi-1} = 1$ for $1 \le \xi \le \left\lceil \frac{\eta_4}{2} \right\rceil$; $\lambda\lambda_{2\xi} = 0$ for $1 \le \xi \le \left\lfloor \frac{\eta_4}{2} \right\rfloor$; $\mu\mu_{2\xi-1} = 1$ for $1 \le \xi \le \left\lceil \frac{\eta_5}{2} \right\rceil$ and $\mu\mu_{2\xi} = 0$ for $1 \le \xi \le \left\lfloor \frac{\eta_5}{2} \right\rfloor$. Also wedge is labelled by $\alpha_1\beta_1 = 0$; $\beta_1\gamma_1 = 0$; $\gamma_1\lambda_1 = 0$ and $\lambda_1\mu_1 = 1$.

Cases	$n_t(0)$	$n_t(1)$	$N_t(i)$	$l_t(0)$	$l_t(1)$	$L_t(i)$
η_4 is even and $\eta_1, \eta_2, \eta_3, \eta_5$ are odd	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lceil \frac{\eta_3}{2} \rceil + 1 + \frac{\eta_4}{2} + \lceil \frac{\eta_5}{2} \rceil$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lceil \frac{\eta_3}{2} \rceil + \frac{\eta_4}{2} + \lceil \frac{\eta_5}{2} \rceil$	1	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lceil \frac{\eta_3}{2} \rceil + \frac{\eta_4}{2} + \lceil \frac{\eta_5}{2} \rceil$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lceil \frac{\eta_3}{2} \rceil + \frac{\eta_4}{2} + \lceil \frac{\eta_5}{2} \rceil$	0
η_5 is even and $\eta_1, \eta_2, \eta_3, \eta_4$ are odd	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lceil \frac{\eta_3}{2} \rceil + \lceil \frac{\eta_4}{2} \rceil + \frac{\eta_5}{2}$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lceil \frac{\eta_3}{2} \rceil + 1 + \lceil \frac{\eta_4}{2} \rceil + \frac{\eta_5}{2}$	1	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lceil \frac{\eta_3}{2} \rceil + \lceil \frac{\eta_4}{2} \rceil + \frac{\eta_5}{2}$	$\lfloor \frac{\eta_1}{2} \rfloor + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lceil \frac{\eta_3}{2} \rceil + \lceil \frac{\eta_4}{2} \rceil + \frac{\eta_5}{2}$	0
η_4, η_5 are even and η_1, η_2, η_3 are odd	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \frac{\eta_5}{2}$	0	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \frac{\eta_4}{2} + \frac{\eta_5}{2}$	1
η_4, η_5 are odd and η_1, η_2, η_3 are even	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + 1 + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	1	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \lceil \frac{\eta_4}{2} \rceil + \lfloor \frac{\eta_5}{2} \rfloor$	0
η_1, η_3 are odd and η_2, η_4, η_5 are even	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \frac{\eta_4}{2} + \frac{\eta_5}{2}$	1	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \frac{\eta_4}{2} + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \frac{\eta_4}{2} + \frac{\eta_5}{2}$	0
η_1, η_3 are even and η_2, η_4, η_5 are odd	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \lfloor \frac{\eta_5}{2} \rfloor$	0	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \frac{\eta_3}{2} + \lfloor \frac{\eta_4}{2} \rfloor + \lfloor \frac{\eta_5}{2} \rfloor$	1
η_1, η_4 are odd and η_2, η_3, η_5 are even	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	1	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	0
η_1, η_5 are odd and η_2, η_3, η_4 are even	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	1	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	0

η_1, η_4 are even and η_2, η_3, η_5 are odd	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	0	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	1
η_1, η_5 are even and η_2, η_3, η_4 are odd	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	0	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\frac{\eta_1}{2} + 1 + \lceil \frac{\eta_2}{2} \rceil + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	1
η_2, η_4 are even and η_1, η_3, η_5 are odd	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	0	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	1
η_2, η_5 are even and η_1, η_3, η_4 are odd	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	0	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \frac{\eta_2}{2} + 1 + \lfloor \frac{\eta_3}{2} \rfloor + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	1
η_3, η_5 are odd and η_1, η_2, η_4 are even	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lceil \frac{\eta_3}{2} \rceil + 1 + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lceil \frac{\eta_3}{2} \rceil + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	1	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lceil \frac{\eta_3}{2} \rceil + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \lceil \frac{\eta_3}{2} \rceil + \frac{\eta_4}{2} + \lfloor \frac{\eta_5}{2} \rfloor$	1
η_3, η_5 are even and η_1, η_2, η_4 are odd	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	0	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\lceil \frac{\eta_1}{2} \rceil + 1 + \lfloor \frac{\eta_2}{2} \rfloor + 1 + \frac{\eta_3}{2} + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	1

The cases which given in below tabular column will obey the following node labeling of R are $t(\alpha) = 0; t(\beta) = 1; t(\gamma) = 0; t(\lambda) = 1; t(\mu) = 1$; $t(\alpha_{2\xi-1}) = 1$ for $1 \le \xi \le \lceil \frac{\eta_1}{2} \rceil$; $t(\alpha_{2\xi}) = 0$ for $1 \le \xi \le \lfloor \frac{\eta_1}{2} \rfloor$; $t(\beta_{2\xi-1}) = 1$ for $1 \le \xi \le \lceil \frac{\eta_2}{2} \rceil$; $t(\beta_{2\xi}) = 0$ for $1 \le \xi \le \lfloor \frac{\eta_2}{2} \rfloor$; $t(\gamma_{2\xi-1}) = 1$ for $1 \le \xi \le \lceil \frac{\eta_3}{2} \rceil$; $t(\gamma_{2\xi}) = 0$ for $1 \le \xi \le \lfloor \frac{\eta_3}{2} \rfloor$; $t(\lambda_{2\xi-1}) = 0$ for $1 \le \xi \le \lceil \frac{\eta_4}{2} \rceil$; $t(\lambda_{2\xi}) = 1$ for $1 \le \xi \le \lfloor \frac{\eta_4}{2} \rfloor$; $t(\mu_{2\xi-1}) = 0$ for $1 \le \xi \le \lceil \frac{\eta_5}{2} \rceil$ and $t(\mu_{2\xi}) = 1$ for $1 \le \xi \le \lfloor \frac{\eta_5}{2} \rfloor$. Then the link labeling is given by $\alpha\alpha_{2\xi-1} = 1$ for $1 \le \xi \le \left\lceil \frac{\eta_1}{2} \right\rceil$; $\alpha\alpha_{2\xi} = 0$ for $1 \le \xi \le \left\lfloor \frac{\eta_1}{2} \right\rfloor$; $\beta\beta_{2\xi-1} = 0$ for $1 \le \xi \le \left\lceil \frac{\eta_2}{2} \right\rceil$; $\beta\beta_{2\xi} = 1$ for $1 \le \xi \le \left\lfloor \frac{\eta_2}{2} \right\rfloor$; $\gamma\gamma_{2\xi-1} = 1$ for $1 \le \xi \le \left\lceil \frac{\eta_3}{2} \right\rceil$; $\gamma\gamma_{2\xi} = 0$ for $1 \le \xi \le \left\lfloor \frac{\eta_3}{2} \right\rfloor$; $\lambda\lambda_{2\xi-1} = 1$ for $1 \le \xi \le \left\lceil \frac{\eta_4}{2} \right\rceil$; $\lambda\lambda_{2\xi} = 0$ for $1 \le \xi \le \left\lfloor \frac{\eta_4}{2} \right\rfloor$; $\mu\mu_{2\xi-1} = 1$ for $1 \le \xi \le \left\lceil \frac{\eta_5}{2} \right\rceil$ and $\mu\mu_{2\xi} = 0$ for $1 \le \xi \le \left\lfloor \frac{\eta_5}{2} \right\rfloor$. Also wedge is labelled by $\alpha_1\beta_1 = 0$; $\beta_1\gamma_1 = 0$; $\gamma_1\lambda_1 = 1$ and $\lambda_1\mu_2 = 1$.

Cases	$n_t(0)$	$n_t(1)$	$N_t(i)$	$l_t(0)$	$l_t(1)$	$L_t(i)$
η_4 is odd and $\eta_1, \eta_2, \eta_3, \eta_5$ are even	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	0	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	$\frac{\eta_1}{2} + 1 + \frac{\eta_2}{2} + 1 + \frac{\eta_3}{2} + 1 + \lfloor \frac{\eta_4}{2} \rfloor + \frac{\eta_5}{2}$	1

Therefore every five star graph $K_{1,\eta_1} \wedge K_{1,\eta_2} \wedge K_{1,\eta_3} \wedge K_{1,\eta_4} \wedge K_{1,\eta_5}$ is a cordial graph for all $\eta_1 \geq 1, \eta_2 \geq 1, \eta_3 \geq 1, \eta_4 \geq 1$ and $\eta_5 \geq 1$.

3. Conclusion

In this paper, we proved that every five star graph is cordial. We intend to focus on cordial labeling for six star and seven star graph in the future.

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