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# CHARACTERIZATION ON INTUITIONISTIC FUZZY SUB $L$-RING

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**Abstract:** The relation between a non-empty subset of an $L$-ring and the Intuitionistic fuzzy sub $L$-ring is discussed.

**Keywords and Phrases:** Fuzzy,Intitionistic fuzzy set, Fuzzy sub $L$-ring, Intuitionistic fuzzy sub $L$-ring.

**2010 Mathematics Subject Classification:** 05C25, 05C50.

## 1. Introduction and Preliminaries

Intuitionistic fuzzy set is an extension of fuzzy sets done by Prof. K. T. Atanassov [2] introduced a new component which determines the degree of non-membership. R. Natarajan and S. Mohanavali [3] applied the concept of fuzzy sets to Lattice ordered ring. N. Ajmal and K. V. Thomas [1] discussed the lattice of fuzzy ideals of a ring. In this paper, characterization on intuitionistic fuzzy sub $L$-ring is established. Also proved the converse relation is true.

**Definition 1.1.** *A fuzzy set A is defined as* $A=\{ (x, \mu_A(x))/ x\in A, \mu_A(x)\in[0,1]\}$ *where* $\mu_A(x)$ *is a membership function belongs to the interval [0,1].*

**Definition 1.2.** *Let X be a non-empty. An intuitionistic fuzzy set A of X is an object of the following from* $A = \{\langle X, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, *where* $\mu_A : X \to [0, 1]$ *and* $\nu_A: X \to [0, 1]$, *define the degree of membership and the degree of non-membership of the element* $x \in X$, *respectively and*
$\forall\ x \in X,\ 0 \leq \mu_A(x) + \nu_A(x) \leq 1.$

**Definition 1.3.** *A fuzzy subset $\mu$ of a lattice ordered ring (or $\ell$-ring)R is called a fuzzy sub L-ring of R, if the following conditions are satisfied, for all x,y $\in$ R.*

1. *(i)* $\mu(x-y) \geq min\{\mu(x), \mu(y)\}$
2. *(ii)* $\mu(xy) \geq min\{\mu(x), \mu(y)\}$
3. *(iii)* $\mu(x \vee y) \geq min\{\mu(x), \mu(y)\}$
4. *(iv)* $\mu(x \wedge y) \geq min\{\mu(x), \mu(y)\}$

**Definition 1.4.** *Let R be a lattice ordered ring(or L-ring in short) and A=$\{x,\mu_A(x), \nu_A(x)/x \in R\}$ be an intuitionistic fuzzy subset of R. Then A is called an intuitionistic fuzzy sub L-ring of R, if the following conditions are satisfied:*

*(i)* $\mu_A(x \vee y); \geq min\{\mu_A(x), ; \mu_A(y)\}$

*(ii)* $\mu_A(x \wedge y); \geq min\{\mu_A(x), ; \mu_A(y)\}$

*(iii)* $\mu_A(x-y) \geq min\{\mu_A(x), \mu_A(y)\}$

*(iv)* $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$

*(v)* $\nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}$

*(vi)* $\nu_A(x \wedge y) \leq max\{\nu_A(x), \nu_A(y)\}$

*(vii)* $\nu_A(x-y) \leq max\{\nu_A(x), \nu_A(y)\}$

*(viii)* $\nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}$ *for all* $x, y \in R$.

## 2. Characterization on Intuitionistic fuzzy sub $L$-ring

**Theorem 2.1.** *Let A be any non-empty subset of an L-ring R, A$\neq$R. If $\mu_A$, $\nu_A$ is an intuitionistic fuzzy sub L-ring of R, defined by*

$$\mu_A(x) = \begin{cases} s & \text{if } x \in A \\ t & \text{if } x \in R \sim A \end{cases} \qquad \nu_A(x) = \begin{cases} m & \text{if } x \in A \\ n & \text{if } x \in R \sim A \end{cases}$$

*where s,t,m,n $\in$ [0,1], s>t and m<n, then A is an sub L-ring of R.*

**Proof.** Let A be any non-empty subset of an $L$-ring R, A$\neq$R.

Let x,y∈ R be arbitrary.
Consider the function

$$\mu_A(x) = \begin{cases} s & \text{if } x \in A \\ t & \text{if } x \in R \sim A \end{cases} \qquad \nu_A(x) = \begin{cases} m & \text{if } x \in A \\ n & \text{if } x \in R \sim A \end{cases}$$

where s,t,m,n ∈ [0,1], s>t and m<n, then A is an sub $L$-ring of R.
Assume that $\mu_A$, $\nu_A$ is an intuitionistic fuzzy sub $L$-ring of R, then

(i) $\mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}$

(ii) $\mu_A(x \wedge y) \geq min\{\mu_A(x), \mu_A(y)\}$

(iii) $\mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}$

(iv) $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$

(v) $\nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}$

(vi) $\nu_A(x \wedge y) \leq max\{\nu_A(x), \nu_A(y)\}$

(vii) $\nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}$

(viii) $\nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}$ $for\ all\ x, y \in R.$

To prove:
A is an sub $L$-ring of R.
Let a,b∈A be arbitrary. Then $\mu_A(a)$=s and $\mu_A$(b)=s. Here min { $\mu_A(a)$, $\mu_A(b)$} = min {s, s} = s. Hence all the values of $\mu_A(a \vee b)$, $\mu_A(a \wedge b)$, $\mu_A(a - b)$ and $\mu_A(ab)$ are greater than or equal to s. But $\mu_A$ has only two values s and t with s > t. Therefore, all the values of $\mu_A(a \vee b)$, $\mu_A(a \wedge b)$, $\mu_A(a - b)$ and $\mu_A(ab)$ are equal to s. This implies (a ∨ b), (a ∧ b), (a - b) and (ab) ∈ A. Also $\nu_A(a)$=m, and $\nu_A(b)$=m. Here max{ $\nu_A(a)$, $\nu_A(b)$ }= max { m, m } =m.
Hence all the values of $\mu_A(a \vee b)$, $\mu_A(a \wedge b)$, $\mu_A(a - b)$ and $\mu_A(ab)$ are less than or equal to m. But $\nu_A$ has only two values m and n with m<n. This implies $(a \vee b)$, $(a \wedge b)$, $(a - b)$ and $(ab) \in$ A. $\mu_A(a \vee b)$, $\mu_A(a \wedge b)$, $\mu_A(a - b)$ and $\mu_A(ab)$ are all equal to m.
This proves that A is an sub $l$ ring of R.

**Theorem 2.2.** *If A is any sub L-ring of R, A≠R, then the intuitionistic fuzzy subset $\mu_A$,$\nu_A$ of R, defined by*

$$\mu_A(x) = \begin{cases} s & \text{if } x \in A \\ t & \text{if } x \in R \sim A \end{cases} \qquad \nu_A(x) = \begin{cases} m & \text{if } x \in A \\ n & \text{if } x \in R \sim A \end{cases}$$

*where s,t,m,n ∈ [0,1], s>t and m<n, is an intuitionistic fuzzy sub L-ring of R.*
**Proof.** Let A be any sub $L$-ring of R, A≠R. Consider the intuitionistic fuzzy subset $\mu_A$,$\nu_A$ of R, defined by

$$\mu_A(x) = \begin{cases} s & \text{if } x \in A \\ t & \text{if } x \in R \sim A \end{cases} \qquad \nu_A(x) = \begin{cases} m & \text{if } x \in A \\ n & \text{if } x \in R \sim A \end{cases}$$

where s,t,m,n ∈ [0,1], s>t and m<n, then A is an sub $L$-ring of R.
Let x,y ∈ R be arbitrary. To prove: $\mu_A$, $\nu_A$ is an intuitionistic fuzzy sub $L$-ring of R. It is enough to prove that,

(i) $\mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}$

(ii) $\mu_A(x \wedge y) \geq min\{\mu_A(x), \mu_A(y)\}$

(iii) $\mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}$

(iv) $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$

(v) $\nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}$

(vi) $\nu_A(x \wedge y) \leq max\{\nu_A(x), \nu_A(y)\}$

(vii) $\nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}$

(viii) $\nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}$ $for\ all\ x, y \in R.$

We prove this cases:
*Case:(i)* x,y∈ A.
Then $\mu_A$(x)= s, $\mu_A$(y)= s. ⟹ min{$\mu_A$(x),$\mu_A$(y)} =min{s,s}=s. Here x,y∈ A ⟹ x ∨ y, x ∧ y, x - y, xy ∈ A. Since A is an sub $L$-ring of R. ⟹ $\mu_A$(x ∨ y)=s, $\mu_A$(x ∧ y)=s, $\mu_A$(x-y)=s, $\mu_A$(xy)=s.

$$\mu_A(x \vee y) = s = min\{\mu_A(x), \mu_A(y)\} \Longrightarrow \mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$\mu_A(x \wedge y) = s = min\{\mu_A(x), \mu_A(y)\} \Longrightarrow \mu_A(x \wedge y) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$\mu_A(x - y) = s = min\{\mu_A(x), \mu_A(y)\} \Longrightarrow \mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$\mu_A(xy) = s = min\{\mu_A(x), \mu_A(y)\} \Longrightarrow \mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}.$$

Then all the inequalities are satisfied in this case.
x,y∈ A.
Then $\nu_A$(x)= m, $\nu_A$(y)= m. ⟹ max{$\nu_A$(x),$\nu_A$(y)} =max{m,m}=m. Here x,y∈ A

$\Longrightarrow$ x $\vee$ y, x $\wedge$ y, x - y, xy $\in$ A. Since A is an sub $L$-ring of R. $\Longrightarrow$ $\nu_A$(x $\vee$ y)=m, $\nu_A$(x $\wedge$ y)=m, $\nu_A$(x-y)=m, $\nu_A$(xy)=m.

$$\begin{aligned}
\nu_A(x \vee y) = s = max\{\nu_A(x), \nu_A(y)\} &\Longrightarrow \nu_A(x \vee y) \leq min\{\nu_A(x), \nu_A(y)\}.\\
\nu_A(x \wedge y) = m = max\{\nu_A(x), \nu_A(y)\} &\Longrightarrow \nu_A(x \wedge y) \leq max\{\nu_A(x), \nu_A(y)\}.\\
\nu_A(x - y) = m = max\{\nu_A(x), \nu_A(y)\} &\Longrightarrow \nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}.\\
\nu_A(xy) = m = max\{\nu_A(x), \nu_A(y)\} &\Longrightarrow \nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}.
\end{aligned}$$

Then all the inequalities are satisfied in this case.
*Case:(ii)* X,YR $\sim$ A
Then $\mu_A(x)$=t,$\mu_A(y)$=t, and min{ $\mu_A$(x),$\mu_A$(y)}=min{t,t} = t. Here x$\vee$y,x$\wedge$y,x-y,xy $\in$ R. Then all the values of $\mu_A$(x $\vee$ y), $\mu_A$(x $\wedge$ y),$\mu_A$(x - y), $\mu_A$(xy) are either t or s.

$$\begin{aligned}
If \mu_A(x \vee y) = s, then \mu_A(x \vee y) = s > t = min\{\mu_A(x), \mu_A(y)\}\\
\Longrightarrow \mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}.\\
If \mu_A(x \vee y) = t, then \mu_A(x \vee y) = t = min\{\mu_A(x), \mu_A(y)\}\\
\Longrightarrow \mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}.\\
If \mu_A(x \wedge y) = s, then \mu_A(x \wedge y) = s > t = min\{\mu_A(x), \mu_A(y)\}\\
\Longrightarrow \mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}.\\
If \mu_A(x \wedge y) = t, then \mu_A(x \wedge y) = t = min\{\mu_A(x), \mu_A(y)\}\\
\Longrightarrow \mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}.\\
If \mu_A(x - y) = s, then \mu_A(x - y) = s > t = min\{\mu_A(x), \mu_A(y)\}\\
\Longrightarrow \mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}.\\
If \mu_A(x - y) = t, then \mu_A(x - y) = t = min\{\mu_A(x), \mu_A(y)\}\\
\Longrightarrow \mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}.\\
If \mu_A(xy) = s, then \mu_A(xy) = s > t = min\{\mu_A(x), \mu_A(y)\}\\
\Longrightarrow \mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}.\\
If \mu_A(xy) = t, then \mu_A(xy) = t = min\{\mu_A(x), \mu_A(y)\}\\
\Longrightarrow \mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}.
\end{aligned}$$

Then all the inequalities are satisfied in this case also.
X,YR $\sim$ A
Then $\nu_A(x)$=n,$\nu_A(y)$=n, and max{ $\nu_A$(x),$\nu_A$(y)}=max{n,n} = n. Here x$\vee$y,x$\wedge$y,x-y,xy $\in$ R. Then all the values of $\nu_A$(x $\vee$ y), $\nu_A$(x $\wedge$ y),$\nu_A$(x - y), $\nu_A$(xy) are either

m or n.

$$If \nu_A(x \vee y) = m, then \nu_A(x \vee y) = m < n = min\{\nu_A(x), \nu_A(y)\} \Longrightarrow \nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x \vee y) = n, then \nu_A(x \vee y) = n = max\{\nu_A(x), \nu_A(y)\} \Longrightarrow \nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x \wedge y) = m, then \nu_A(x \wedge y) = m < n = max\{\nu_A(x), \nu_A(y)\} \Longrightarrow \nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x \wedge y) = n, then \nu_A(x \wedge y) = n = max\{\nu_A(x), \nu_A(y)\} \Longrightarrow \nu_A(x - y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x - y) = m, then \nu_A(x - y) = m < n = max\{\nu_A(x), \nu_A(y)\} \Longrightarrow \nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x - y) = n, then \nu_A(x - y) = n = max\{\nu_A(x), \nu_A(y)\} \Longrightarrow \nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(xy) = m, then \nu_A(xy) = m < n = max\{\nu_A(x), \nu_A(y)\} \Longrightarrow \nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(xy) = n, then \nu_A(xy) = n = max\{\nu_A(x), \nu_A(y)\} \Longrightarrow \nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}.$$

Then all the inequalities are satisfied in this case also.
*Case:(iii)* X,YR $\sim$ A
Here $\mu_A(x)$=s, $\mu_A$(y)=t and min $\{\mu_A$(x),$\mu_A$(y)$\}$ = min{s,t} = t. Clearly x $\vee$ y, x $\wedge$ y, x - y, xy $\in$ R. Then all the values of $\mu_A$(x $\vee$ y), $\mu_A$(x $\wedge$ y),$\mu_A$(x - y), $\mu_A$(xy) are either t or s.

$$If \mu_A(x \vee y) = s, then \mu_A(x \vee y) = s > t = min\{\mu_A(x), \mu_A(y)\} \Longrightarrow \mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$If \mu_A(x \vee y) = t, then \mu_A(x \vee y) = t = min\{\mu_A(x), \mu_A(y)\} \Longrightarrow \mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$If \mu_A(x \wedge y) = s, then \mu_A(x \wedge y) = s > t = min\{\mu_A(x), \mu_A(y)\} \Longrightarrow \mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$If \mu_A(x \wedge y) = t, then \mu_A(x \wedge y) = t = min\{\mu_A(x), \mu_A(y)\} \Longrightarrow \mu_A(x - y) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$If \mu_A(x - y) = s, then \mu_A(x - y) = s > t = min\{\mu_A(x), \mu_A(y)\}$$

$$\Longrightarrow \mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$If \mu_A(x-y) = t, then \mu_A(x-y) = t = min\{\mu_A(x), \mu_A(y)\}$$
$$\Longrightarrow \mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$If \mu_A(xy) = s, then \mu_A(xy) = s > t = min\{\mu_A(x), \mu_A(y)\}$$
$$\Longrightarrow \mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}.$$
$$If \mu_A(xy) = t, then \mu_A(xy) = t = min\{\mu_A(x), \mu_A(y)\}$$
$$\Longrightarrow \mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}.$$

Then all the inequalities are satisfied in this case also.
X,YR $\sim$ A
Here $\nu_A$(x)=m, $\nu_A$(y)=n and max $\{\nu_A$(x),$\nu_A$(y)$\}$ = max{m,n} = n. Clearly x $\vee$ y, x $\wedge$ y, x - y, xy $\in$ R. Then all the values of $\nu_A$(x $\vee$ y), $\nu_A$(x $\wedge$ y),$\nu_A$(x - y), $\nu_A$(xy) are either m or n.

$$If \nu_A(x \vee y) = m, then \nu_A(x \vee y) = m < n = min\{\nu_A(x), \nu_A(y)\}$$
$$\Longrightarrow \nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x \vee y) = n, then \nu_A(x \vee y) = n = max\{\nu_A(x), \nu_A(y)\}$$
$$\Longrightarrow \nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x \wedge y) = m, then \nu_A(x \wedge y) = m < n = max\{\nu_A(x), \nu_A(y)\}$$
$$\Longrightarrow \nu_A(x-y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x \wedge y) = n, then \nu_A(x \wedge y) = n = max\{\nu_A(x), \nu_A(y)\}$$
$$\Longrightarrow \nu_A(x-y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x-y) = m, then \nu_A(x-y) = m < n = max\{\nu_A(x), \nu_A(y)\}$$
$$\Longrightarrow \nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(x-y) = n, then \nu_A(x-y) = n = max\{\nu_A(x), \nu_A(y)\}$$
$$\Longrightarrow \nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}.$$

$$If \nu_A(xy) = m, then \nu_A(xy) = m < n = max\{\nu_A(x), \nu_A(y)\}$$
$$\Longrightarrow \nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}.$$
$$If \nu_A(xy) = n, then \nu_A(xy) = n = max\{\nu_A(x), \nu_A(y)\}$$
$$\Longrightarrow \nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}.$$

Then all the inequalities are satisfied in this case also.
Thus $\mu_A$, $\nu_A$ is an intutionistic fuzzy sub $L$-ring of R.

**3. Conclusion:**

This paper can be extended using the characterization for Intuitionistic fuzzy sub L-ring with respect to the level subsets.

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