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Applications of Successive half turns in paper folding and paper cutting

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Abstract: The term 'half turn' is a geometric transformation as a function under which to each point on a plane a corresponding point. In geometrical transformation and construction many times the half turn is applicable. In the construction point of view it may be expressed as an equation of translation or half turn according to even or odd number of half turns. The main focus of this paper is to explore the importance of half turn which is highly applicable in our daily life for any geometric construction and Generalization of result of successive half turns.
Keywords: Half turn, Geometric transformation, product, folding, successive, origami, polygon, hexagon, circle, rectangle, strip.

General Background:

Mathematics is a practical art-science. Its practicality is to be developed in the form of arts. Paper folding and paper cutting is an effective subject matter on the field of artistry among various teaching arts.

The behavioral theories like read and understand, write and remember and do and know i.e. learning by doing and learning by observation are very effective in mathematics teaching. Therefore, the techniques like paper folding and paper cutting are adopted to make mathematics teaching more practical and to drive mathematics towards practicality.

By using waste paper, different types of mathematical models can be constructed and the same paper can be reused for multipurpose. It helps students to grasp mathematical concepts that the students themselves learn or the teacher, by using a paper, can easily demonstrate and teach complex mathematical concepts directly.

It is important to show concrete object to teach mathematical skill and concepts, for this, paper folding and paper cutting can be used as a special technique.

The learners love to play games, especially in the lower level, we can introduce paper folding and paper cutting as a special technique to teach mathematics in the

form of different games which help to motivate learners and also facilitate teaching and learning for example, different types of Tan gram, Facto game, etc.

This technique is more practical to develop concepts of different types of models and on engineering, architecture and construction. Slow learners and mentally retarded learners are specially benefited by playing with such materials like dolls and different play things (visual aids) such as boat, camera, polygons etc. made up of simple paper that help to enhance mental and physical development. Such visual teaching aids help to refresh mind, develop (enhance) excitement, courage, activeness and relaxation. Therefore the new thinking has been put forward to enhance effective teaching and enrich the interest of the learners through this study. It is hoped that this study will equally benefit the students and teachers of school as well as of higher level.

Small children cannot understand and explain complex abstract concepts therefore, teaching must be directed from concrete to abstract. Paper folding and paper cutting can be used as an alternative tool for learning mathematical concepts. The students participate and learn well with the help of these aids and these aids also help to motivate the learners. This is a very important aspect of learning. The students can demonstrate the learned facts by paper folding and paper cutting and even expected to develop new facts and theories.

In Japan, origami has reached its greatest development, with hundreds of traditional folds and extensive literature dealing with the art. Japanese folds divided roughly into categories: Figures used in ceremonial etiquette (such as nosh, folded, decoration attached to gifts); and birds, animals, fish insects, flowers, human figures, furniture, and other objects. Some of the animals have amusing action features; best known art the bird that flaps its wings when it's tail is pulled and the frog that hops when its back is tapped. Yoshizawa Akira of Tokyo is considered the greatest of modern paper folders. He wrote several books of origami and created a large number of new, often realism and delicate beauty.

Paper folding also has flourished in Spain and South America. Miguelde Unamuno Spanish writer and Philosopher, made a hobby of paper folding. He invented many new animal constructions and wrote Amory among pedagogia (1902), a humorous easy on the art.

In South America, Vicente Solorzano Sagredo of Argentina was the leading expert on paper folding and another of the most comprehensive manuals on the art in Spanish. George Rhoads of Evanston, III; and Giuseppe Baggi of New York also achieved distinction in this art.

Apart from the oriental tradition the folding of coloured papers into ornamental designed was introduced by Friedrich Froebel into the kindergarten movement that

he initiated in Germany in the 19^{th} century. Latter the Bauhaus, a famous German school of design, stressed the folding of paper as a method of training students of commercial design. The use of folded paper in mathematical recreation is similarly independent of origami. Particularly intriguing are A.H. Stone's Flexagons (1939), a variety of paper structure that alters their faces in curious ways when properly flexed. Arts of paper folding were used in china at first then in Japan and other Countries.

The Mathematics related to paper folding and paper cutting is generally known as Origami mathematics. It gives the collective information on the mathematics of paper folding and cutting. The various models are prepared by the method of paper folding and paper cutting. The crease of paper forms the blue print of the fold and cut, since there are some particular rules for crease of papers. We also note that there are some Origami mathematics such as origami geometry, Origami algebra, Origami arithmetic and Origami statistics.

Since in origami Mathematics the crease or fold of paper create various vertices and edges and such non-empty set which is known as a graph. The graph is as a network with a mathematical model. It is related to fields of agriculture, engineering air aviation. Transportations network business and e-commerce at the 20^{th} century. The Origami Mathematic and they are inter-related to each other. They functions as bridges among the various field of Mathematics.

The origami Mathematics and tessellation are parallel. Being the parallelism of Origami Mathematics and tessellation implies the parallel concept to the graph, which is related to tessellation. On the other hand Origami Mathematics is also conceptual to chemistry for formation of molecular formula of molecules and bond between the atoms, physics and architecture are highly attached with Origami Mathematics.

In effective and practical teaching of Mathematics, we should use various teaching materials prepared with papers by the method of paper folding and paper cutting. Through this method various models are prepared in various shapes and sizes as our daily practical uses such methods are related to our practical life day to day.

Since the objectives of mathematics teaching is to teach mathematics at the practical approach. Thus in teaching of mathematics we do not focus only the formulate way and its rote-learning through which we determine the answer. We do not emphasize only rote-learning through formulae. For this we should follow up new teaching technologies and methods. Thus here we should follow up the methods with our soul or heart. In this way new technology and methods are guided and discovered as well and the teaching is becoming practical and achievable. Through which effective teaching technology is to be researched.

There are many teaching strategies and tools in Mathematics. But, in our practice only a few and less effective strategies and tools are being used. Besides this the tools and techniques used in mathematics are limited. The paper folding and paper cutting as a teaching tool is the least used one in the current teaching strategies. The limitation of the Mathematics teaching and the progress of Mathematics is not well developed. Representing, consisting of formed by, or characterized by abstract mathematical points, lines, surfaces, etc. rather than actual physical shapes. A geometric figure is usually taken to be a diagram, but mathematics is concerned only with its abstract and not with its physical properties, so that diagrams are not themselves geometric figures properly so called, but merely representation of them.

Half turn is a geometric transformation as a Geometric representation. It follows as a construction. The numbers of half turn is applicable in construction and it may be expressed as translation according to even or odd number of half turns. The various expression of half turn is given but the generalization of the above assertion is not formulated as a mathematical theory merely used as a product of half turns. We also fold or cut papers throughout half turn in order to make beautiful designs, flowers and garlands. In various countries, mostly people prepared various decorated materials by the process of folding and cutting of papers, clothes. The process of folding and cutting is based in origami. Japan is one of the highly popular for origami.

Objectives

The main part of the study is the application of successive half turn can be generalized to transform into either half turn or translation. Here the following are the main objectives of this paper.

(i) Generalization of result of successive half turns,

(ii) To explore the applications of half turn in origami mathematics.

(iii) Application of paper folding in half turns for construction.

Hypothesis

1. *The product of successive half turn gives the equation of translation or equation of half turn if the product is taken even number of half turns or odd number of half turns.*

2. *Half turn is useful in the construction of paper made and wooden blocks for preparation of dolls, bags, decorated materials from low-cost materials.*

Review of related literature

The review of literature shows the importance of research approach of investigations and practices. Here various resource books and papers to be reviewed to justify the reliability of this paper with strengthening the generalization and use

of the geometrical tool in theoretical form.

An Introduction to the History of Mathematics, (1911), Eves Howard Whitley has given the transformation of areas and the existence of five regular polyhedrons; tetrahedron, cube, octahedron, dodecahedron, icosahedrons.

Dictionary of Mathematics, (reprint, 2006), Sunny Sareen, has given Half angle formula for establishing trigonometric identities, half Cauchy distribution; refers to a special case of the folded Cauchy distribution which, in standard form is having the density function $\frac{2}{\pi(1+x^2)}$, $(0 < x)$. Half line, or ray, is the portion of a line on one side of a fixed point P of the line. P is the initial point of the half line. For example, an angle consist of two half- lines with a common initial point; also in a rectangular co-ordinate system, the origin divides the x- axis into two half lines namely, the positive x- axis and the negative x-axis. A half plane is the portion of a plane on one side of a fixed line ℓ of the plane, Where ℓ is the edge of the half plane. For example, a dihedral angle in space consists of two half planes with a common edge; also the y- axis in a plane co-ordinate system divides the plane into two-half planes, one containing the points (x, y) with x positive and the other with negative. A half space is the portion of the space on one side of a fixed plane in space for example, the xy - co-ordinate plane divides space into two half space, one with z positive, the other with z negative. It refers to convex polyhedron and convex polygon. Half normal distribution refers to a special case of the folded normal distribution. Half normal plots, half width, half side formula (spherical trigonometry).

"Golbodh", (1977), Prof. Nayaraj Pant had explained that the Earth has no base; the Earth is round, the position of the stars, altitude and latitude of the Earth. In my view the half turn concept plead a role for convincing the concept of these comprehensive phenomenon.

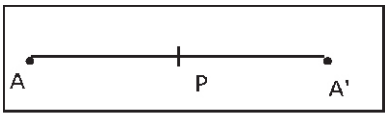
To Fold, Cut of paper and teaching training mathematics, (1995), Hari Narayan Upadhya, Takayukee Kitadai, CDC, JOCV has introducing the paper folding and paper cutting as a main tool of teaching mathematics with applying reflection, rotation, here the half turn is used to formulating /constructing and examining the rectangle, square, vertically opposite angles, adjacent angle, interior angle of a triangle, angle bisector, altitudes of triangle, parallelogram, rhombus, circles, cylinder, cone, sphere and preparation of various dolls such as propeller, basket, jewellery box, flapping pigeon. The mathematical formulas such as the area of triangle, surface area of rectangle, etc. are formed as models through half turn with very use of reflection, rotation etc.

In 'Introduction to Modern Mathematics, volume-1, third edition, (1997)', Prof.

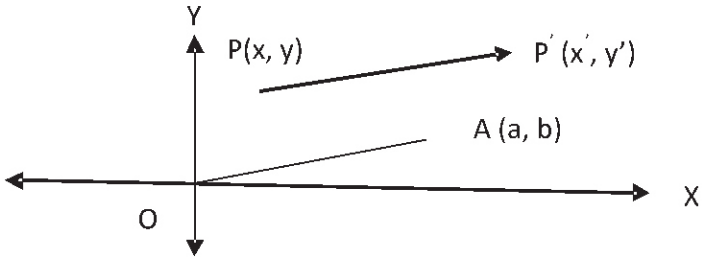
S.M. Maskey has explain geometric transformations with introducing one-to-one onto mapping is the transformation. He discussed the following types of isometries; reflection, translation, half-turn, rotation and glide reflection. An isometric transformation or isometry is a mapping which does not distort the figure so I sure that the theorems; relating to preserving a distance, collinearity, and betweenness, segment rays, lines, triangles, angle, perpendicularity and so on obviates the proofs.

The half turn is introduced as,
"A half-turn σ_p of a point A about the point P is defined as the point A' such that P is the mid-point of the segment AA'.

$$I.e. \quad PA = PA'$$



For comparing half turn and translation, we need the concept of translation. So let's defining translation; translation is geometric transformation which correspondence between points and their image points such that each image point at the same distance in the same direction from the respective original points.



If P(x, y) be an arbitrary point is translating about the point A (a, b) through the direction of OA gives the image point P' (x', y') then the equations of translation are, x'= x+a, y'= y+b.

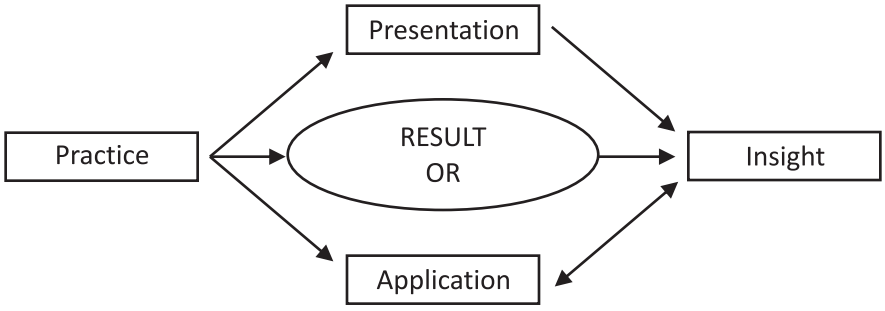
If the point is half turned two or more than two times (finitely) about any point then this is known as successive half turn. It is also note that for a half turn we should turn an object through 180o.Here the point A (consider as an object) turned through 180o then we get A as the image point of A and it is termed as half turn of a point A.

"Curriculum Evaluation and Teaching in Higher Education", (2004), E.R. Acharya has explained simple methods of construction of circle, square, regular hexagon, tetrahedron, and order of vertex of a polygon with applying the half turn as a geometric transformation.

Folding and cutting of papers in the Teaching of Mathematics, (2002), H. N. Upadhyaya has expressed the geometric transformation through the process of paper folding and paper cutting with giving the introduction of paper folding or cutting, preliminary activities with paper folding and cutting experimental verifications of some plane geometric shapes and surface area of solids. Here I notify that the half turn is highly used even not mentioned as use of half turn. This process is guided by 'Origami', which a phenomenon used to crease, fold and deform the shape.

Methodological Consideration

The research is based on geometrical model in practice; theoretically the explanation and descriptions based in library and historical background. The tools and instruments used here are most of theoretical concept and are based on books and papers, no concrete solid instruments are used for the preparation of this research paper but the approach of the paper is quite practical and highly applicable in practice. The sample and size of the research is based on practical approach so it is historical and theoretical but the concept is properly based on practical which is applicable. The primary and secondary data are used to give the concept of the research. In this research the design is given as theoretical concept. The model of the study based on the following mathematical flow chart.



Equations of half turn

Here we have to discuss about the equations of half turn. If A(x, y) be a point and A' (x', y') be the half turn of A about P (a, b) then,
x' = 2a x, y' = 2b y be the equations of the image point, which termed as equation of half turn .The verification and example of the above statement as abstract is carried out as below.
The product of successive half turn is illustrated by considering a point A(x,y).If the point P(x, y) is half turned about the point P (a, b) then we obtained the equation of the image point,

$\sigma_p(x, y) = A'(x', y') = (2ax, 2b - y)$, which gives the equation of half turn. Again if half turned the point (x', y') about the point Q(c, d) then image point is,
$\sigma_Q(\sigma_{P(x,y)}) = \sigma_Q(2ax, 2by)$
= (2c-2a+x, 2d-2b+y)
= {x+2(c-a), y+2(d-b)}
= A"(x", y")
which gives the equation of translation, since the point is half turned two times. Similarly if we half turned the point A"(x",y") about the point R(e ,f) we will get the equation of half turn about another new point .since the point is half turned three times i.e. $\sigma_R(\sigma_Q(\sigma_P(x, y))) = \{2(e+a-c)-x, 2(f+b-d)-y)\} = A"(x", y")$. The diagrammatic representation is given as
Now continuously half turned the point A" about the point S (g, h) then we get the equation of image point of S.
I.e. $\sigma_S(\sigma_R(\sigma_Q(\sigma_P(x, y)))) = A'''$. This gives the equation of translation about the new point. Continue this process then we get the equation of half turn if the point is half turned an odd number of times and we get the equation of translation if the point is half turn an even number of times. Thus we conclude that,

The product of half turns

$$= \begin{cases} \textit{the equation of halfturn, if the point is half turned odd number of times} \\ \textit{the equation of translation, if the point is half turned even number of times.} \end{cases}$$

Hence we claim that if we want to displace an object from an initial position then we should half turn the object by even number of times and if we want to half turn the object only then we should half turn the object by an odd numbers of times. From the view of above we can determine the distance of the final position of the object from its initial position and can be locate the position with expressing co-ordinate as described above figure.

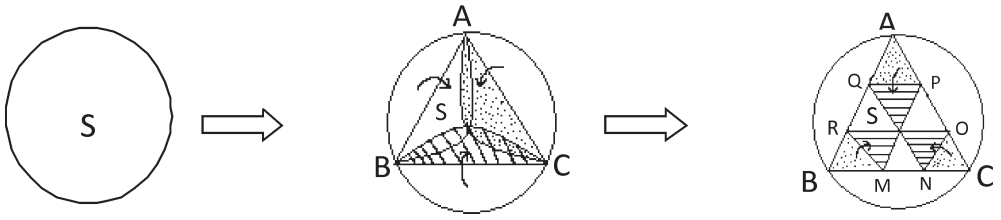
Evidences of applications of half turns

The practical demonstration of half turn is carried out through paper folding and paper cutting which are illustrated in the following points although the equations of half turn are not used here.

(i) Formation of a Regular hexagon from circular piece of a paper

For formation of a regular hexagon by this method we should take a circular piece of paper of any diameter as our choice. For this purpose we should take help with ruler, compass or bangle or other circular rings etc.

In this circular paper we should prepare an equilateral triangle by the method of paper folding whose centre of gravity lies on the centre of the circle as show in the figure.

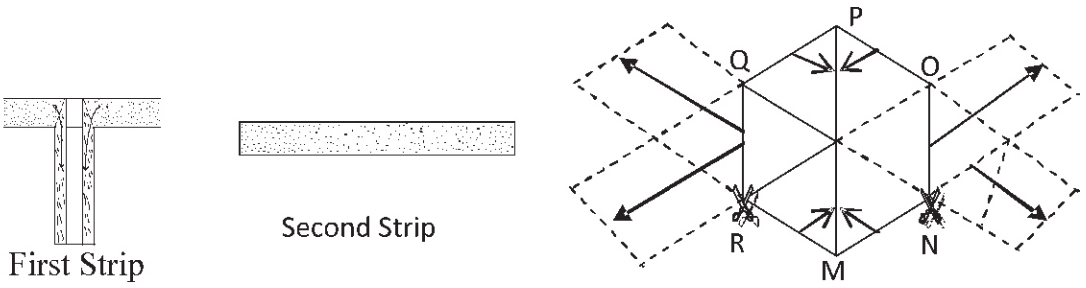


Now, fold the vertices A, B and C by drawing the lines PQ, RM and NO on the point S which is the centre of the circle and centroide or centre of gravity of the equilateral triangle formed by us. Then we should get a regular hexagon named by MNOPQR.

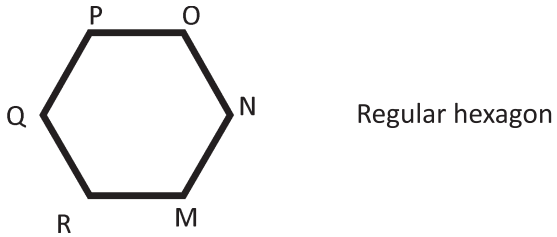
(ii) Formation of a regular hexagon by using two strips of paper

We should take two paper strips with equal width and equal length. Then we fold those two strips as shown in the figure.

The folds of the first strip is shown, similarly we should fold strip second and prepare a node as shown in the figure.

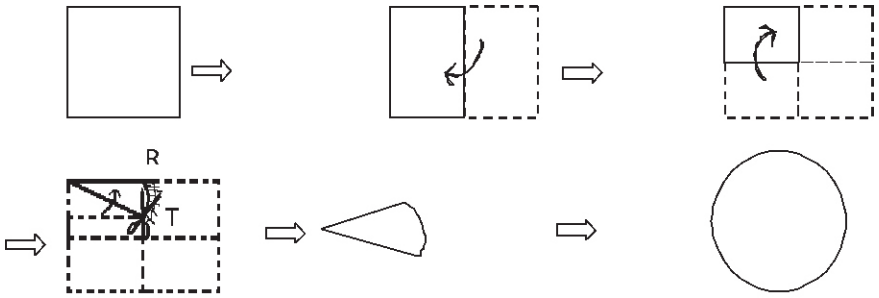


Now, cut the tip of the strip after folding along NO and QR then we should get a regular hexagon MNOPQR.



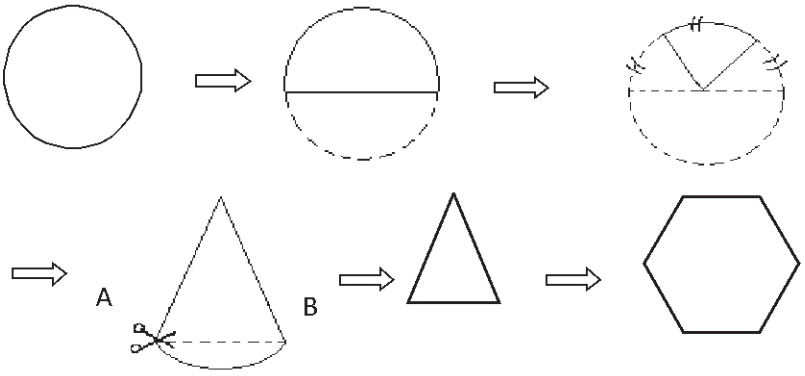
(iii) Formation of a circle by cutting to a rectangular piece of paper

To fold twice a rectangular piece of paper by making half on each time and cut by a scissor along RT then opens the paper which is a circle as shown in the figure.



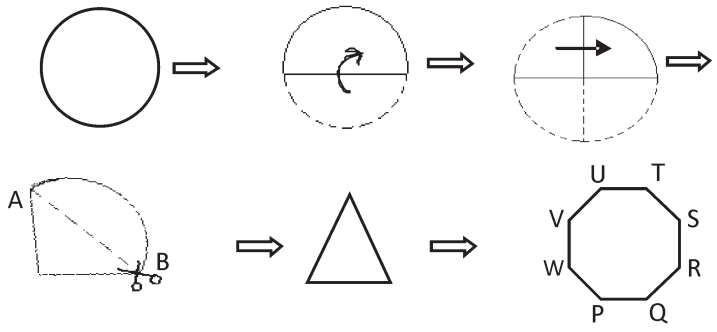
(iv) Formation of a regular hexagon from the circle obtaining above

At first fold the circle into a half circular position then again folds it into one-third circular position that is as a cone. Then join the two circular edges denoted by A and B by a line AB and cut it then by unfold it to get a regular hexagon. The process is shown in the following figures.



(v) Formation of a regular octagon from the circle

Fold the circle thrice by making half in each fold and we get a cone then join the two circular edges by a line AB and cut it along AB then open, it will changes into a regular octagon as shown in the following figure.



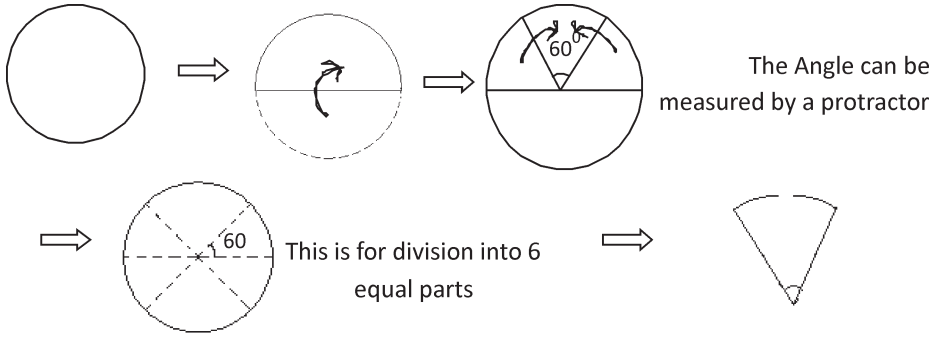
(vi) Division of a circle on equal parts

For six equal parts, now fold the circle into half, then fold the half circle into one-third form, that divides the half circle into three equal parts and form a cone, open the circle with marking by any sigh on the folding circular edge, count the marking signs, they are six. Hence a circle is divided into six equal parts. From which we also conclude that the required angle=60 degree for division of a circle into six equal parts theoretically.

Next for division of a circle into eight equal parts we should fold the circle thrice by making half in each fold. From this we also conclude that the angle at centre $= 45^0$ for division of a circle into eight equal parts theoretically.

Similarly, for division of a circle into sixteen equal parts, we should fold the circular paper four times by making half in each fold.

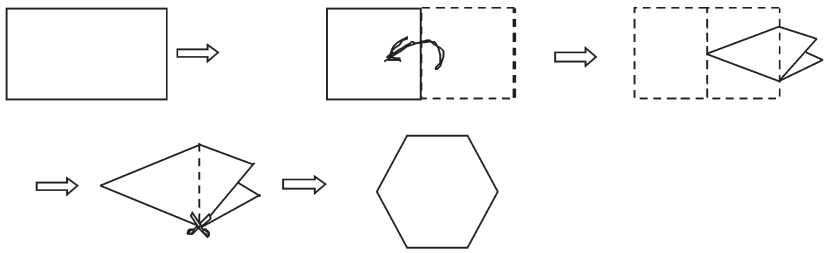
For division of a circle into 12 equal parts, we should fold the half after folding for division into six equal parts. The processes of folding are shown as:



If we fold the configuration obtained in the third step into two equal halves then the circle is divided into 12 equal parts. The figures of folding for 8 equal and 16 equal parts can be done easily so they are not shown here.

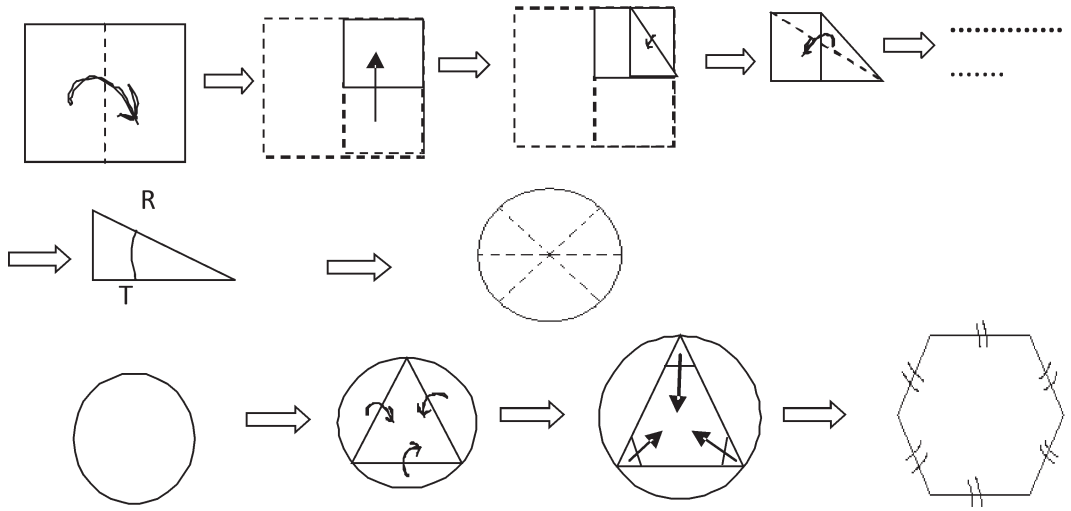
(vii) Formation of a regular hexagon from a rectangular piece of paper

Fold a rectangular paper into half, then fold it into three lobs as shown in figure then cut it by a scissor by forming an equilateral triangle. Now unfold it, it would change into a regular hexagon.

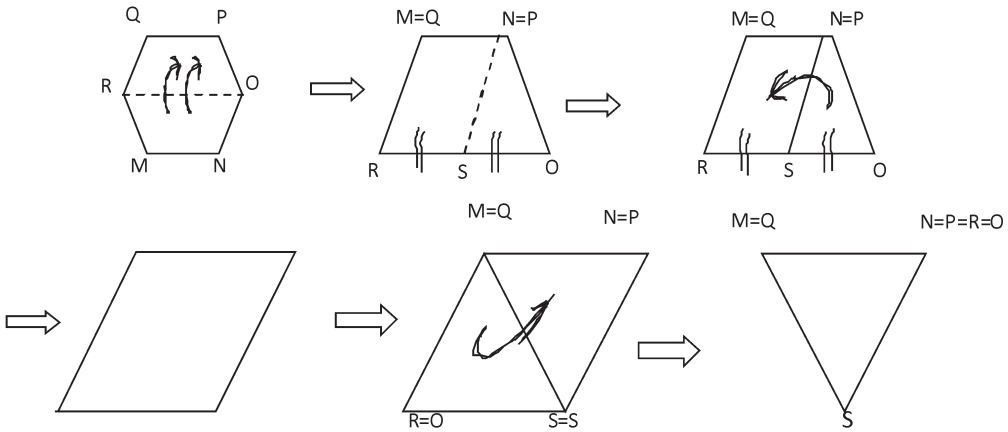


(viii) Construction of various geometrical figures

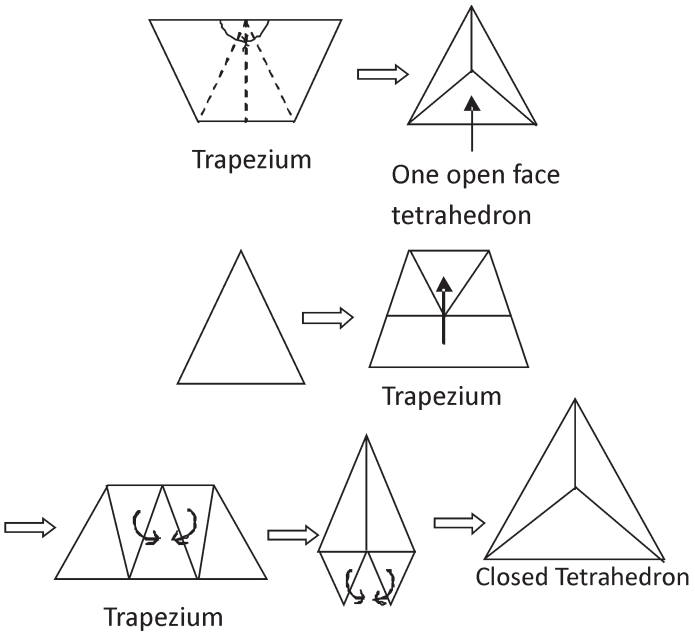
The following processes can be repeated to construct a hexagon from a circular piece of paper.



(ix)The other evidences of half turn for the techniques of formation of geometrical figures



The following constructive models can be prepared which to be used in the field of teaching geometrical shapes.



Other examples of half turn are shown in the diagrams below which are uses in our daily life, they are ladder, and Nordstrom's surface and manipulative squared as kids doll.

Results and Discussion

The result of this research is half turn is a geometrical tool which can transformed into geometrical translation and half turn as the product of the half turn is even or odd .On the other hand the application of half turn is used as the geometrical transformation to construct the various geometrical models regarding with geometrical mathematics. In theme the half turn is highly applicable for constructing and formulating the various solid materials by using the waste materials; paper, wood, clothes, leaf of trees etc. The application of geometric transformation especially successive half turns and their product is explorer to the discussion with application.

Content Analysis

The content of the research highly based in geometrical transformations. So many mathematical models can be formed. The analytical study properly based in half turns with paper folding. Its general concept and the mathematical model and equation are analyzed as above.

Conclusions

In conclusion the successive half turns together with geometric transformation is applicable for constructing mathematical models, solid materials in our daily used. Hence the conclusion of this paper is we get the equation of half turn if the point is half turned an odd number of times and we get the equation of translation if the point is half turn an even number of times. On the other hand the application of half turn is highly applicable in the field of origami. Thus it can be generate job opportunity, so it is as the market oriented geometrical phenomena. The Mathematics related to paper folding and paper cutting is generally known as Origami mathematics. It gives the collective information on the mathematics of paper folding and cutting. The various models are prepared by the method of paper folding and paper cutting. The crease of paper forms the blue print of the fold and cut, since there are some particular rules for crease of papers.

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