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THE BLAST DOMINATION IN MYCIELSKI'S GRAPH OF GRAPHS AND ZERO DIVISOR GRAPHS

P. Rajakumari and K. Ameenal Bibi

P.G. and Research Department of Mathematics,
D.K.M.College for Women (Autonomous),
Vellore-632001, Tamilnadu, INDIA

E-mail : rajakumari0990@gmail.com

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Abstract: In this paper, interesting results regarding the Mycielskian number of the Mycielski's graph of path $\mu(P_n)$, the Mycielski's graph of cycle $\mu(C_n)$, the Mycielski's graph of complete graph $\mu(K_n)$, the Mycielski's graph of complete bi-partite graph $\mu(K_{m,n})$, the Mycielski's graph of wheel graph $\mu(W_n)$, the Mycielski's graph off an graph $\mu(F_{1,n})$, the Mycielski's graph of tadpole graph $\mu(T_{m,n})$, the Mycielski's graph of snake graph $\mu(T_n)$ and the Mycielski's graph of zero divisor graphs$\mu(\Gamma(Z_n))$under the domination parameters such as blast domination,distance-2 domination, blast distance-2 domination and independent distance-2 domination are investigated. The exact values of these new parameters for some special graphs are attained. The relation with other domination parameters have been discussed.

Keywords and Phrases: Blast domination number, Blast distance-2 domination number, Mycielski's graph.

2010 Mathematics Subject Classification: 05C76, 05C69.

1. Introduction and Preliminaries [6]

The concept of triple connected graphs was introduced by Paulraj Joseph et.al [9]. A graph is said to be triple connected if any three vertices lie on a path in G. Mahadevan et.al introduced the concept of complementary triple connected domination number of a graph and A.Ahila et.al., introduced Blast Domination Number of a graph with a real life application.

The Mycielski's Construction [6]

The open neighborhood of a vertex v in a graph G, denoted by $N_G(v)$, is the set of all vertices of G, which are adjacent to v. Also, $N_G[v] = N_G(v) \cup \{v\}$is called the closed neighborhood of v in the graph G. Here, G is a simple, finite, undirected and connected graph.
From a graph G, by Mycielski's construction obtained a graph μ (G) with $V(\mu(G)) = V \cup U \cup \{w\}$, where $V = V(G) = \{v_1, \ldots v_n\}$, $U = \{u_1, \ldots, u_n\}$,and $E(\mu(== G)) = E(G) \cup \{u_i v : v \in N_G(v_i) \cup \{w\}, i = 1, \ldots, n\}$ .For each $0 \leq i \leq n$, $v_i$and u_i are called the corresponding vertices of μ (G) and denote $C(v_i) = u_i$, $C(u_i) = v_i$. Moreover, for the subsets A ⊆U, B ⊆V ,$C(A) = \{C(u_i) : u_i \in A\}$, $C(B) = \{C(v_i) : v_i \in B\}$. Also, $x \leftrightarrow y$ is implies that {x, y} is an edge.

Definition 1.1. *A distance -2 dominating set $D \subseteq V$ of a graph G is an independent distance -2 dominating set if the induced sub graph $< D >$ has no edges. The independent distance -2 domination number$i_{\leq 2}(G)$is the minimum cardinality of a minimal independent distance -2 dominating set of G.*
The minimal independent distance -2 dominating set in a graph G is an independent distance -2 dominating set that contains no independent distance - 2 dominating set as a proper subset of G.
The distance -2 open neighborhood of a vertex $v \in V$ is the set, $N_{\leq 2}(v)$ of vertices within distance of two from v.

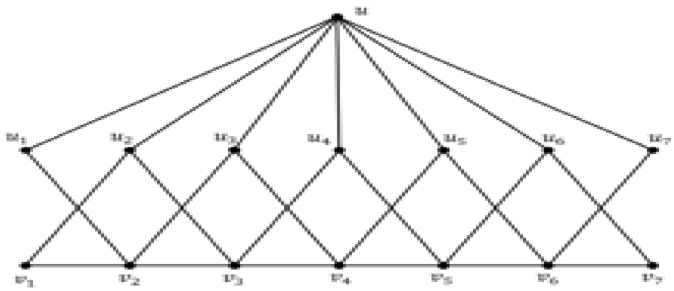
Definition 1.2. *The Harary graph $H_{k,n}$is a particular example of a k connected graph with n, graph vertices having the smallest possible number of edges.*

2. Blast domination in the Mycielski's graph of $C_n, K_n, W_n, K_{m,n}, F_{1,n}, T_{m,n}$ and T_n graphs

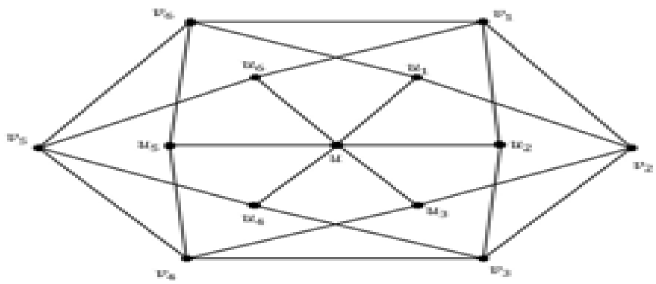
In this section, blast domination number of some graphs are investigated. Many bounds for this parameter is obtained.

Example 2.1. See Figure 1. Mycielskian graph

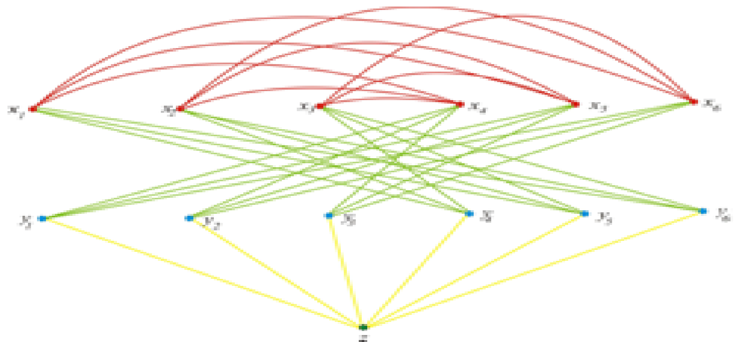
Theorem 2.2. *In a Cycle graph, for $n \geq 4$, $\gamma_c^{tc}[\mu(C_n)] = n - 1$.*
Proof. A cycle in a graph is a closed walk consists of a sequence ofvertices starting and ending at the same vertex, with two consecutive vertices in the sequence adjacent to each other in the graph.
Let $V(C_n) = \{x_i : 1 \leq i \leq n\}$ be the set of vertices of C_n taken in cyclic order. By the construction of Mycielski's graph,



(a) Mycielski graph of P_7



(b) Mycielski graph of C_6



(c)The Mycielskian graph of complete bipartite graph $K_{3,3}$

Figure 1: Mycielskian graph

$V\ (\mu\ (C_n)) = V\ (C_n) \cup \{y_i : 1 \leq i \leq n\} \cup \{z\}$ and$E\ (\mu\ (C_n)) = E\ (C_n) \cup \{y_i(x) : x \in N_{C_n}\ (x_i) \cup \{z\}\ ,\ i = 1,\ 2,\ \ldots,\ n\}$.
Assume that $D = \{u, u_i : i = 1, 2, \ldots n-2\}$ is a blast dominating set in $\mu\,(C_n)$. If $n = 5$,fix the vertex set $\{u, u_1, u_2, u_3\}$ which is connected and its complement $V - D$, $\{v_i \cup u_4, u_5\}$ which is lie on a path. Therefore $\gamma_c^{tc}\,[\mu\,(C_n)] = n - 1$.

Successively, hence $\gamma_c^{tc}\left[\mu\left(C_n\right)\right]=n-1$.

Result 2.3. *Let G be the Mycielski's graph of Cycle graph such that* $\mu\left(C_n\right)$ *and* $\overline{\mu\left(C_n\right)}$ *have no isolated vertices of order* $2n+1$.
(i) $\gamma_c^{tc}\left[\mu\left(C_n\right)\right]+\gamma_c^{tc}\left[\overline{\mu\left(C_n\right)}\right]\ \leq n+2$ *and*
(ii) $\gamma_c^{tc}\left[\mu\left(C_n\right)\right].\gamma_c^{tc}\left[\overline{\mu\left(C_n\right)}\right]\ \leq 3(n-1)$.

Proposition 2.4. *For the graphs*$\mu\left(C_n\right)$ *and* $\overline{\mu\left(C_n\right)}$*with maximum independence number* $\beta_0\left(\mu\left(C_n\right)\right)$ *and* $\beta_0\left(\overline{\mu\left(C_n\right)}\right)$

$$(i)\beta_0\left(\mu\left(C_n\right)\right)+\beta_0\left(\overline{\mu\left(C_n\right)}\right)=2n$$

$$(ii)\beta_0\left(\mu\left(C_n\right)\right).\beta_0\left(\overline{\mu\left(C_n\right)}\right)=n^2.$$

Theorem 2.5. *In a Complete graph, where* $n\geq 3$, $\gamma_c^{tc}\left[\mu\left(K_n\right)\right]=3$.
Proof. A complete graph is a simple, connected,undirected graph in which every pair of distinct vertices is connected by a unique edge.
Let $V\ (K_n)\ =\ \{x_i\ :\ 1\ \leq\ i\ \leq\ n\}$. By the construction of Mycielski's graph, $V\ (\mu\ (K_n))\ =\ V\ (K_n)\cup\{y_i\ :\ 1\ \leq\ i\ \leq\ n\}\cup\{z\}andE\ (\mu\ (K_n))\ =\ E\ (K_n)\cup\{y_i x\ :\ x\ \in N_{K_n}\ (x_i)\ \cup\ \{z\}\ ,\ i\ =\ 1,\ 2,\ .\ .\ .\ n\}$. Since z is adjacent with each vertex of $\{y_i\ :\ 1\ \leq\ i\ \leq\ n\}$, also $\mu\ (K_n)$ contains a n-clique. Assume that $D=\{z,y_i,x_i:\ i\ =\ 1,\ 2,\ .\ .\ .\ n\}$ is a blast dominating set in $\mu\left(K_n\right)$. If $n=3$, choose and fix any of the corresponding vertices $\{z,y_1,x_2\}$,$\{z,y_2,x_1\}$,$\{z,y_3,x_3\}$, which are connected and its complement $\{y_2,y_3,x_1,x_3\}$, $\{y_1,y_3,x_2,x_3\}$,$\{y_1,y_2,x_1,x_2\}$ forms a blast dominating set. Therefore,$\gamma_c^{tc}\left[\mu\left(K_3\right)\right]=3$.Successively, the assumption is true. Hence $\gamma_c^{tc}\left[\mu\left(K_n\right)\right]=3$.

Result 2.6. *Let G be a Mycielski's graph of Complete graph such that* $\mu\left(K_n\right)$*and* $\overline{\mu\left(K_n\right)}$ *have no isolated vertices of order* $2n+1$.
(i) $\gamma_c^{tc}\left[\mu\left(K_n\right)\right]+\gamma_c^{tc}\left[\overline{\mu\left(K_n\right)}\right]=6$ *and*
(ii) $\gamma_c^{tc}\left[\mu\left(K_n\right)\right].\gamma_c^{tc}\left[\overline{\mu\left(K_n\right)}\right]=9$.

Remark 2.7. *For the graphs* $\mu\left(K_n\right)$ *and* $\overline{\mu\left(K_n\right)}$ *with the maximum independence number* $\beta_0\left(\mu\left(K_n\right)\right)$*and* $\beta_0\left(\overline{\mu\left(K_n\right)}\right)$,

$$(i)\beta_0\left(\mu\left(K_n\right)\right)+\beta_0\left(\overline{\mu\left(K_n\right)}\right)=2n$$

$$(ii)\beta_0\left(\mu\left(K_n\right)\right).\beta_0\left(\overline{\mu\left(K_n\right)}\right)=n^2.$$

Theorem 2.8. *In a Complete-bipartite graph $K_{m,n}$ where $m, n \geq 2, \gamma_c^{tc}\left[\mu\left(K_{m,n}\right)\right] = 3$.*

Proof. Let $V\ (K_{m,n}) = \{x_i\ : 1 \leq i \leq m\} \cup \{y_i\ : 1 \leq j \leq n\}$ and

$$E\left(K_{m,n}\right) = \cup_{i=1}^{m}[\{e_{ij} = x_i y_j\ : 1 \leq i \leq n\}\ .$$

By Mycielski's construction,

$$V\ (\mu\ (K_{m,n}))\ =\ V\ (K_{m,n}) \cup\ \{x'_i\ :\ 1\ \leq\ i\ \leq m\}\ [\ y'_j\ :\ 1\ \leq\ j\ \leq n\ \ [\{z\}]]\ .$$

Choose the minimal blast dominating sets$D_1 = \{z, y_1, y_{m+n}\}$,

$$D_2 = \{z, y_2, y_{m+n-1}\}\,,$$

$D_n = \{z, y_i, y_m\}$and$|D_1|, |D_2|... |D_n| = 3$ which are connected and its complement $< V - D >$ is triple connected in $\mu(K_{m,n})$. Hence $\gamma_c^{tc}[\mu\left(K_{m,n}\right)] = 3$.

Result 2.9. *Let G be a Mycielski's graph of Complete-bipartite graph $K_{m,n}$ such that $\mu\left(K_{m,n}\right)$and $\overline{\mu\left(K_{m,n}\right)}$ have no isolated vertices of order $2n+1$,*
(i) $\gamma_c^{tc}\left[\mu\left(K_{m,n}\right)\right] + \gamma_c^{tc}[\overline{\mu\left(K_{m,n}\right)}] = 6$ and
(ii) $\gamma_c^{tc}\left[\mu\left(K_{m,n}\right)\right] .\gamma_c^{tc}[\overline{\mu\left(K_{m,n}\right)}] = 9$.

Proposition 2.10. *For the graphs $\mu\left(K_{m,n}\right)$ and $\overline{\mu\left(K_{m,n}\right)}$ with the maximum independence number $\beta_0\left(\mu\left(K_{m,n}\right)\right)$and $\beta_0\left(\overline{\mu\left(K_{m,n}\right)}\right)$, $\beta_0\left(\overline{\mu\left(K_{m,n}\right)}\right) \leq \beta_0\left(\mu\left(K_{m,n}\right)\right)$.*

Theorem 2.11. *In a Wheel graph, with $n \geq 3$, $\gamma_c^{tc}\left[\mu\left(W_n\right)\right] = 3$.*

Proof. A wheel graph is a graph formed by connecting a single vertex to all the vertices of a cycle. The wheel graph has n vertices and $2(n-1)$ edges. Let $V(W_n) = \{v \cup v_i\ :\ 1\ \leq i\ \leq\ n\}$ and $E(W_n) = \{vv_i\ : 1\ \leq\ i\ \leq\ n\}$.By the Mycielski's construction, $V(\mu(W_n)) = V(W_n) \cup \{uu_i\ :\ 1\ \leq\ i\ \leq\ n\} \cup \{w\}$.In $\mu(W_n)$, each $u$is adjacent with each vertex of $N_{W_n}(v)$, and w is adjacent with each vertex of $\{uu_i\ :\ 1\ \leq\ i\ \leq\ n\}$.

By the definition of Mycielskian, v is adjacent with each of $\{v_i\ :\ 1\ \leq\ i\ \leq n\} \cup \{u_i\ :\ 1\ \leq\ i\ \leq\ n\}$.Assume that $D = \{w, u, v_i\}$ is a blast dominating set in $\mu\left(W_n\right)$.Suppose if $n = 4$ then $V(\mu(W_4)) = V(W_4) \cup \{uu_i\ :\ 1\ \leq\ i\ \leq\ 4\} \cup \{w\}$. Choose the minimal blast dominating sets $D_1 = \{w, u, v_1\}, D_2 = \{w, u, v_2\}, D_3 = \{w, u, v_3\}$, $D_4 = \{w, u, v_4\}$ which are connected and their complements $V - D_1 = \{v \cup v_2, v_3, v_4\} \cup \{u_i : 1 \leq i \leq 4\}$, $V - D_2 = \{v \cup v_1, v_3, v_4\} \cup \{u_i : 1 \leq i \leq 4\}, V - D_3 = \{v \cup v_1, v_2, v_4\} \cup \{u_i : 1 \leq i \leq 4\}, V - D_4 = \{v \cup v_1, v_2, v_3\} \cup \{u_i : 1 \leq i \leq 4\}$ are triple connected. Also $|D_1|, |D_2|, |D_3|, |D_4| = 3$. Therefore, the assumption result holds. Hence $\gamma_c^{tc}\left[\mu\left(W_n\right)\right] = 3$.

Proposition 2.12. *Let G be the Mycielski's graph of Wheel graph such that $\mu(W_n)$ and $\overline{\mu(W_n)}$ have no isolated vertices and of order $2n+1$,*
(i) $\gamma_c^{tc}[\mu(W_n)] + \gamma_c^{tc}[\overline{\mu(W_n)}] = 6$ *and*
(ii) $\gamma_c^{tc}[\mu(W_n)] . \gamma_c^{tc}[\overline{\mu(W_n)}] = 9$.
Proof. By the above theorem the result is true.

Result 2.13. *For the graphs $\mu(W_n)$ and $\overline{\mu(W_n)}$ with the maximum independence number $\beta_0(\mu(W_n))$ and $\beta_0\left(\overline{\mu(W_n)}\right)$, $\beta_0\left(\overline{\mu(W_n)}\right) \leq \beta_0(\mu(W_n))$*

Theorem 2.14. *For the Tadpole graph $T_{m,n}$ where $n = 1$ and $m \geq 4$, $\gamma_c^{tc}[\mu(T_{m,n})] = n-1$.*
Proof. Let $T_{m,1}$ be the Tadpole graph with joining the cycle C_n and path $P_n$with $m = 4$, $n = 1$. Let $V\ (T_{m,1}) = \{V_1, V_2, ..V_n\}$. By the construction of Mycielski's graph,
$V(\mu(T_{m,n})) = V\ (T_{m,1}) \cup \{U_i : 1 \leq i \leq n\} \cup \{w\}$and $E(\mu(T_{m,n})) = E(T_{m,1}) \cup \{U_iV : V \in N_{T_{m,n}}(V_i), i = 1, 2 \ldots n$
Consider$D = \{w \cup U_i : i = 1, 2 \ldots n-2\}$ be the dominating set in $\mu(T_{m,n})$and $V - D = \{V_i : 1 \leq i \leq n-1\}$. Since D is a connected dominating set and its complement set is triple connected. Hence,$\gamma_c^{tc}[\mu(T_{m,n})] = n-1$.

Proposition 2.15. *For the graphs $\mu(T_{m,n})$ and $\overline{\mu(T_{m,n})}$ where $n = 1$and $m \geq 4$,*

$$(i)\ \gamma_c^{tc}[\mu(T_{m,n})] + \gamma_c^{tc}[\overline{\mu(T_{m,n})}] = n+1$$

$$(i)\ \gamma_c^{tc}[\mu(T_{m,n})] . \gamma_c^{tc}[\overline{\mu(T_{m,n})}] = 2(n-1).$$

Observation 2.16. *For the graphs $\mu(T_{m,n})$ and $\overline{\mu(T_{m,n})}$, $\gamma_{\leq 2}(\mu(T_{m,n})) \leq \gamma(\overline{\mu(T_{m,n})})$ $\leq \gamma_c^{tc}\overline{[\mu(T_{m,n})} \leq \gamma(\mu(T_{m,n})) \leq \gamma_c^{tc}(\mu(T_{m,n}))$.*

Theorem 2.17. *For the Fan graph $F_{1,n}$ where $n \geq 4$, then$\gamma_c^{tc}[\mu(F_{1,n})] = 3$.*
Proof. The graph F_{n+1} has 2n + 3 vertices. By the construction of Mycielski's graph,
$V(\mu(F_{1,n})) = V(F_{1,n}) \cup \{u \cup u_i : 1 \leq i \leq n)\} \cup \{w\}$ and
$E(\mu(F_{1,n})) = E(F_{1,n}) \cup \{uu_iv : \text{where } v \in N_{F_{n+1}}(v_i), 1 \leq i \leq n)\}$
Choose $D = \{w \cup u_i \cup v : 1 \leq i \leq n\}$ as the dominating set of $\mu(F_{1,n})$and $V - D = \{v_i : 1 \leq i \leq n\}$.Here$D$ is a connected dominating set and its complement$V - D$ lies on the path. Therefore,D forms a blast dominating set. Hence, $\gamma_c^{tc}[\mu(F_{1,n})] = 3$.

Result 2.18. *For the graphs $\mu(F_{1,n})$ and $\overline{\mu(F_{1,n})}$ where $n \geq 3$,*

$$(i)\ \gamma[\mu(F_{1,n})] + \gamma[\overline{\mu(F_{1,n})}] = 4$$

$$(ii)\, \gamma\left[\mu\left(F_{1,n}\right)\right].\gamma\overline{\left[\mu\left(F_{1,n}\right)\right]} = 4.$$

$$(iii)\, \gamma_c^{tc}\left[\mu\left(F_{1,n}\right)\right] + \gamma_c^{tc}\overline{\left[\mu\left(F_{1,n}\right)\right]} = 6$$

$$(iv)\, \gamma_c^{tc}\left[\mu\left(F_{1,n}\right)\right].\gamma_c^{tc}\overline{\left[\mu\left(F_{1,n}\right)\right]} = 9.$$

Theorem 2.19. *For the Snake graph T_n where $n \geq 3$, $\gamma_c^{tc}\left[\mu\left(T_n\right)\right] = n$.*
Proof. By the definition, triangular snake graph is a connected graph all of whose blocks are triangles. A triangular snake graph is a triangular cactus whose block-cut vertex graph is a path. and it is obtained from the path $P = \{v_1,\ v_2, ..., v_{n+1}\}$ by joining $v_i$and $v_{i+1}$to a new vertex $u_1,\ u_2, ..., u_n$. By the construction of Mycielski's graph,
$V\left(\mu\left(T_n\right)\right) = V\left(T_n\right) \cup \{V_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n+1\} \cup \{w\}$ and
$E\left(\mu\left(T_n\right)\right) = E\left(T_n\right) \cup \{V_i \cup u_i UV : N_{T_n}\left(U_i V_i\right), 1 \leq i \leq n\}$
Assume that $D = w \cup \{V_i : 1, 2, 3 \ldots n-1\}$ is a connected dominating set in $\mu\left(T_n\right)$ and $V - D = \{U_i : 1, 2, ..., n\} \cup \{v_i : 1, 2 \ldots n+1\}$ is triple connected in $\mu\left(T_n\right)$. Therefore $\gamma_c^{tc}\left[\mu\left(T_n\right)\right] = n$.

Result 2.20. *For the graphs $\mu\left(T_n\right)$ and $\overline{\mu\left(T_n\right)}$ where $n \geq 3$,*

$$(i)\, \gamma\left[\mu\left(T_n\right)\right] + \gamma\overline{\left[\mu\left(T_n\right)\right]} \leq n+2$$

$$(ii)\, \gamma\left[\mu\left(T_n\right)\right].\gamma\overline{\left[\mu\left(T_n\right)\right]} \leq (n+2)^2.$$

$$(iii)\, \gamma_c^{tc}\left[\mu\left(T_n\right)\right] + \gamma_c^{tc}\overline{\left[\mu\left(T_n\right)\right]} = n+2$$

$$(iv)\, \gamma_c^{tc}\left[\mu\left(T_n\right)\right].\gamma_c^{tc}\overline{\left[\mu\left(T_n\right)\right]} = 2n.$$

3. Relationship of Blast domination with other domination parameters

In this section, some results and bounds related to distance-2 domination and independent distance-2 domination of Mycielski's graphs are discussed. Also, exact values of some special graphs are obtained.

Proposition 3.1. *Let $\mu\left(C_n\right)$ be a graph for $n \geq 3$,$\gamma_{\leq 2}\left(\mu\left(C_n\right)\right) \leq i_{\leq 2}\left(\mu\left(C_n\right)\right)$.*
Proof. Every independence distance-2 dominating set of $\mu\left(C_n\right)$is a distance-2 dominating set of $\mu\left(C_n\right)$. Thus, $\gamma_{\leq 2}\left(\mu\left(C_n\right)\right) \leq i_{\leq 2}\left(\mu\left(C_n\right)\right)$.

Proposition 3.2. *For the graphs G and $\overline{G}$, $n \geq 3$, $\gamma_{\leq 2}(G) \leq \gamma(\overline{G}) \leq \gamma(G) \leq \gamma_c^{tc}(\overline{G}) \leq \gamma_c^{tc}(G)$ if G is any one of the graphs $\mu\left(C_n\right)$or $\mu\left(K_n\right)$.*

Proposition 3.3. *For any graph G,$\gamma_{c\leq 2}^{tc}\left(G\right) = \gamma_{\leq 2}\left(G\right) = 1$ if and only if G is Mycielski's graph ofP_n or C_n or K_n or $K_{m,n}$ or W_n or $F_{1,n}$ or $T_{m,n}$ or T_n.*

Proposition 3.4. *For the Mycielski's graph of Complete-bipartite graphs* $\mu(K_{m,n})$ *and* $\overline{\mu(K_{m,n})}$, $(m,n \geq 2)$

$$(i)\,\gamma(\mu(K_{m,n})) = \gamma_c^{tc}(\mu(K_{m,n})).$$

$$(ii)\gamma(\overline{\mu(K_{m,n})}) \leq \gamma_c^{tc}(\overline{\mu(K_{m,n})}).$$

Proposition 3.5. *For the Mycielski's graph of Wheel graphs,* $\mu(W_n)$ *and* $\overline{\mu(W_n)}$ $(n \geq 3)$,

$$(i)\gamma_{\leq 2}(\mu(W_n)) \leq \gamma(\mu(W_n)) \leq \gamma_c^{tc}(\mu(W_n)).$$

$$(ii)\gamma_{\leq 2}(\overline{\mu(W_n)}) \leq \gamma(\overline{\mu(W_n)}) \leq \gamma_c^{tc}(\overline{\mu(W_n)}).$$

Proposition 3.6. *For any graph G,* $\gamma_{\leq 2}(G) \leq i_{\leq 2}(G) \leq i(G) \leq \gamma(G) \leq \beta_0(G)$ *if and only if the graph G is any one of the following graphs* $\mu(C_n)$, $\mu(P_n)$, $\mu(K_n)$, $\mu(K_{m,n})$,$\mu(W_n)$, $F_{1,n}$, $T_{m,n}$ and T_n.

Proposition 3.7. *For any graph G,*$\gamma_{c\leq 2}^{tc}(\overline{G}) = \gamma_{\leq 2}\left(\overline{G}\right) = 1$ *if and only if G is the Mycielski's graph of* C_n *or* K_n *or* $K_{m,n}$ *or* W_n *or* $F_{1,n}$ *or* $T_{m,n}$*or* T_n.

Observation 3.8. *(i) Every dominating set is a distance-2 dominating set if and only if Gis the Mycielski's graph of*P_n*or*C_n *or* K_n *or* $K_{m,n}$ *or* W_n *or* $F_{1,n}$ *or* $T_{m,n}$*or* T_n*.But the converse is not true.*
(ii) Every dominating set is a distance-2 dominating set if and only if $\overline{G}$ *is the Mycielski's graph of*P_n*or*C_n *or* K_n *or* $K_{m,n}$ *or* W_n *or* $F_{1,n}$ *or* $T_{m,n}$ *or* T_n.
(iii) Every distance-2 dominating set is a blast dominating set if and only if Gis a Mycielski's graph ofP_n *or*C_n *or* K_n *or* $K_{m,n}$ *or* W_n *or or*$F_{1,n}$ *or* $T_{m,n}$*or* T_n.
(iv) Every distance-2 dominating set is a blast dominating set if and only if $\overline{G}$ *is a Mycielski's graph of*P_n *or*C_n *or* K_n *or* $K_{m,n}$ *or* W_n *or* $F_{1,n}$ *or* $T_{m,n}$*or* T_n.

Exact values and bounds of some standard graphs

(i) If any graph $G = CP_k$*is the cocktail party graph of k vertices where* $k = 2n$ *for all* $n \geq 2$, *then* $\gamma_c^{tc}(G) = \gamma_{c\leq 2}^{tc}(G) = 1$.
(ii) For the n- Andrasfai graph of k vertices where k = 3n-1 for all $n \geq 1$,

$$\gamma_c^{tc}(A_k) = \gamma_{c\leq 2}^{tc}(A_k) = 1.$$

(iii) For any Harary graph $H_{k,n}$,$\gamma_c^{tc}(H_{k,n}) = \gamma_{c\leq 2}^{tc}(H_{k,n}) = 1$ *for* $k \geq 3$.

4. Blast domination number for the Mycielski's graph of Zero divisor graphs

This section deals with blast domination of Mycielski's graph of zero divisor graph and its complement graph and the bounds for these graphs are attained.The concept zero divisor graphs are introduced by I. Beck in 1988 and further studied by D. D. Anderson and M. Naseer.

Example 4.1.

$\Gamma(Z_6) = \{2,3,4\}$

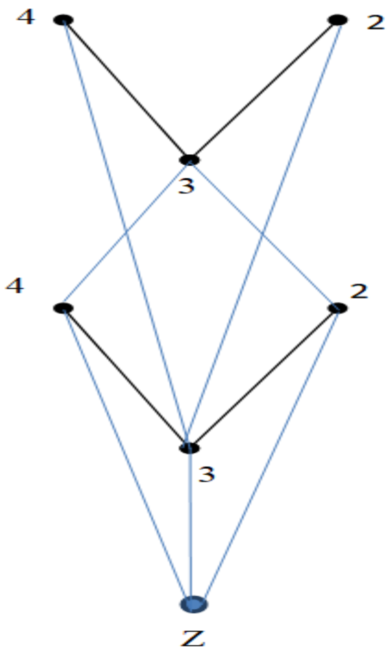


Figure 2: The Mycielskian graph of $\Gamma(\mathrm{Z}_6)$

Proposition 4.2. *If $G = \mu(\Gamma(\mathrm{Z}_{2p}))$ is the Mycielski's graph of star zero divisor graph, then the blast domination number of G is given by, $\gamma_c^{tc}[\mu(\Gamma(Z_{2p}))] = 3$.*

Result 4.3. *If the graphs $G = \mu(\Gamma(Z_{2p}))$ and $\overline{G} = \overline{\mu(\Gamma(Z_{2p}))}$ are the Mycielski's graphs of star zero divisor graph where p is a prime number then*

$$(i)\, \gamma[\mu\,(\Gamma(Z_{2p}))] + \gamma\left[\overline{\mu(\Gamma(Z_{2p}))}\right] = 4$$

$$(ii)\, \gamma[\mu\,(\Gamma(Z_{2p}))]\,.\gamma\left[\overline{\mu(\Gamma(Z_{2p}))}\right] = 4$$

$$(iii)\, \gamma_c^{tc}[\mu\,(\Gamma(Z_{2p}))] + \gamma_c^{tc}\left[\overline{\mu(\Gamma(Z_{2p}))}\right] = 5$$

$$(iv)\, \gamma_c^{tc}[\mu\left(\Gamma(Z_{2p})\right)] .\gamma_c^{tc}\left[\overline{\mu(\Gamma(Z_{2p}))}\right] = 6.$$

Proposition 4.4. *For $G = \mu(\Gamma(Z_n))$, if$n = 3p$ where p is a prime number, the blast domination number of G is given by, $\gamma_c^{tc}[\mu(\Gamma(Z_{3p}))] = 3$.*

Result 4.5. *If the graphs$G = \mu(\Gamma(Z_{3p}))$and $\overline{G} = \overline{\mu(\Gamma(Z_{3p}))}$ are the Mycielski's graphs of zero divisor graph where p is a prime number ($p > 3$) then*

$$(i)\, \gamma[\mu(\Gamma(Z_{3p}))] + \gamma\left[\overline{\mu(\Gamma(Z_{3p}))}\right] = 5$$

$$(ii)\, \gamma[\mu(\Gamma(Z_{3p}))].\gamma\left[\overline{\mu(\Gamma(Z_{3p}))}\right] = 6$$

$$(iii)\, \gamma_c^{tc}[\mu(\Gamma(Z_{3p}))] + \gamma_c^{tc}\left[\overline{\mu(\Gamma(Z_{3p}))}\right] = 6$$

$$(iv)\, \gamma_c^{tc}[\mu(\Gamma(Z_{3p}))].\gamma_c^{tc}\left[\overline{\mu(\Gamma(Z_{3p}))}\right] = 9.$$

Proposition 4.6. *If $G = \mu(\Gamma(Z_n))$ and $n = p^2$ where p is an odd prime number,then the blast domination number of G is given by, $\gamma_c^{tc}[\mu(\Gamma(Z_n))] = 3$.*
Proposition 4.7. *For $G = \mu(\Gamma(Z_n))$ and $n = pq$ where $p = 5,\ q = 7$,then the blast domination number of G is given by, $\gamma_c^{tc}[\mu(\Gamma(Z_n))] = 3$.*
Proposition 4.8. *For the graphs$\mu(\Gamma(Z_n))$and $\overline{\mu(\Gamma(Z_n))}$,*

$$\gamma_{\leq 2}[\overline{\mu(\Gamma(Z_n))}] \leq \gamma[\overline{\mu(\Gamma(Z_n))}] \leq \gamma[\mu(\Gamma(Z_n))] \leq \gamma_c^{tc}[\mu(\Gamma(Z_n))] \leq \gamma_c^{tc}[\overline{\mu(\Gamma(Z_n))}]$$

Observation 4.9. *For the graphs $G = \mu(\Gamma(Z_n))$and $\overline{G} = \overline{\mu(\Gamma(Z_n))}$,*

$$\gamma_{\leq 2}[\mu\left(\Gamma(Z_n)\right)] = i_{\leq 2}[\mu\left(\Gamma(Z_n)\right)] = \gamma_{\leq 2}\left[\overline{\mu(\Gamma(Z_n))}\right] = i_{\leq 2}\left[\overline{\mu(\Gamma(Z_n))}\right]$$

Proposition 4.10. *If the graphs$G = \mu(\Gamma(Z_n))$and $\overline{G} = \overline{\mu(\Gamma(Z_n))}$ are the Mycielski's graphs of the star zero divisor graph then $(i)\, \gamma_{\leq 2}[\mu\left(\Gamma(Z_n)\right)] + \gamma_{\leq 2}\left[\overline{\mu(\Gamma(Z_n))}\right] = 2$*

$$(ii)\, \gamma_{\leq 2}[\mu\left(\Gamma(Z_n)\right)] .\gamma_{\leq 2}\left[\overline{\mu(\Gamma(Z_n))}\right] = 1$$

$$(iii)\, i_{\leq 2}[\mu\left(\Gamma(Z_n)\right)] + i_{\leq 2}\left[\overline{\mu(\Gamma(Z_n))}\right] = 2$$

$$(iv)\, i_{\leq 2}[\mu\left(\Gamma(Z_n)\right)] .i_{\leq 2}\left[\overline{\mu(\Gamma(Z_n))}\right] = 1$$

Proof. By the above observation the result is true.
Proposition 4.11. *(i) Every distance-2 dominating set is a blast distance-2 dominating set in $\mu(\Gamma(Z_n))$ and $[\overline{\mu(\Gamma(Z_n))}]$. Converse is also true. (ii) Every independent distance-2 dominating set is a blast distance-2 dominating set in $\mu(\Gamma(Z_n))$and*

$[\overline{\mu(\Gamma(Z_n))}]$. *Converse is also true.*
Proof. The results follow from
$(i)\,\gamma_{\leq 2}[\mu\,(\Gamma(Z_n))] \leq \gamma^{tc}_{c\leq 2}[\mu(\Gamma(Z_n))]$ and $(ii)\,i_{\leq 2}[\mu\,(\Gamma(Z_n))] \leq \gamma^{tc}_{c\leq 2}[\mu(\Gamma(Z_n))]$.

5. Application of Blast distance - 2 dominating sets

This section extended the concept of dominating sets to Blast and Blast distance - 2 dominating sets. There are more useful models of these sets to many real-world problems. Indeed, much of the motivation for the study of Blast and Blast distance 2 domination arises from problems involving an optimal location of a hospital, police station, fire station, or any other emergency service facility.

5.1. Radio stations

Suppose that there is a collection of small villages in a remote part of the world. The need is to locate radio stations in some of these villages so that messages can be broad casted to all the villages in the region. But since the installations of radio stations are costly, locate as few stations as possible which can cover all the other villages. Let each village be represented by a vertex. An edge between two villages is labeled with the distance, say in kilometers. The distance between the two villages is shown in fig.5.1. Assume that a radio station has a broadcast range

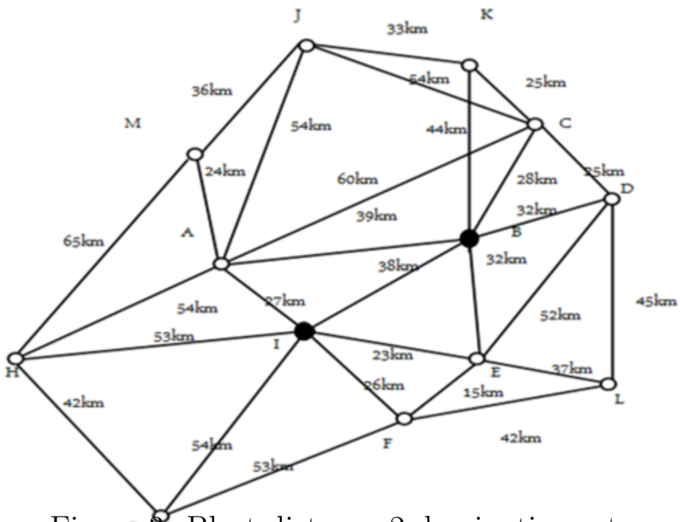


Figure 3: Blast distance-2 dominating set

of hundred kilometers. In this case, seek a Blast and Blast distance - 2 dominating sets among all the vertices within the distance of 100 kilometers. It is seen that the set I,B is connected (blast) and its complement $\{A, C, D, E, F, G, H, J, K, L, M\}$ is triple connected and it is of distance almost 2. Hence, it forms a blast and blast distance-2 dominating set.

6. Conclusion

The hub of this article, we defined the notion of blast and blast distance-2 domination for Mycielskis graph of C_n, K_n,$K_{m,n}$ and W_n graphs. We attained many bounds on these two new parameters. Also Applications of Blast and Blast distance - 2 dominating sets have been discussed. It would be interesting to determine the results for general graphs.

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