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PERFORMANCE OF CONTROL CHARTS BASED ON THE TRANSMUTED MUKHERJEE-ISLAM DISTRIBUTION

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Abstract: Control charts are designed by assuming that the quantitative characteristics of interest follow a normal distribution, which is not always the case in practice. The variable of interest may follow some non-normal distribution such as an exponential distribution or a gamma distribution or any other. The use of control charts designed for a normal distribution may not be workable in this situation and may cause an increase in the proportion of non-conforming products.

In this paper, the techniques of Transmuted distributions are used to the Mukherjee-Islam distribution to the applications of statistical process control to check the performances of the production process.

The main objective of this paper is to introduce a control chart using Transmuted Mukherjee-Islam distribution in order to study the production system and monitor the same.

Keywords and Phrases: Control Charts, Transmuted Distributions, Mukherjee-Islam distribution.

2010 Mathematics Subject Classification: 62P30.

1. Introduction

The quality of the product depends on the variation of the production process. This variation in the process may be due to some controllable and uncontrollable factors. Control charts are important tools to ensure high quality of the products.

The control charts were introduced by Prof. Shewhart Walter in 1920s and have now become an important tool in quality improvement. Control charts are applied for monitoring a process during the manufacturing of a product. Timely action about the process should be taken based on control charts, that is, no action is taken when the analysis of a control chart shows that the process is in control. However, if the control chart declares that the process has shifted, then appropriate action should be taken to bring the process back under control. Therefore, a control chart indicates the right time or to which observation corrective action should be taken. The two control limits, called the upper control limits (UCL) and the lower control limit (LCL), are used to monitor the process mean or variance. These limits are very helpful for reducing defective products and alternately for increasing the profits of industries. The maintenance of quality through control charts brings a good reputation for an industry in the market.

Shewhart control charts are often used in the area to monitor processes when the quality of interest follows a normal distribution. In practice, it is not always true that the variable of interest follows the normal distribution, but may also follow non-normal distributions. For decades, several researchers have developed different types of control charts. Nelson (1984) proposed the Shewhart control chart-tests for special causes. Santiago and Smith (2013) have proposed the control charts, called the t-chart, when the time between events follows the exponential distribution. They used the variable transformation proposed by Nelson (1994) to transform exponentially distributed data to an approximate normal data. Aslam et al. (2014) proposed a new control charts for the exponential distribution using the transformed variable and the repetitive sampling. Amin and Venkatesan (2017) proposed the comparison of Bayesian method and classical charts in detection of small shifts in the control charts. Amin and Venkatesan (2019) have discussed on the recent developments in control charts techniques. Amin and Venkatesan (2019) proposed the SPC using transmuted generalized uniform distribution. Control charts are designed on the assumption that the quantitative trait of interest follows the normal distribution, which is not always the case in practice. The variable of interest may follow some non-normal distribution such as an exponential distribution or a gamma distribution or any other distribution. The use of control charts designed for a normal distribution may not be workable in this situation and may result in an increasing of the number of non-conforming products. In addition, the normal distribution is applied in situations where data are collected in subgroups, so that the central limit theorem can be applied when designing the control charts. Again, in practice, it is not always possible to collect data in groups. In many practical situations, classical distributions therefore offer insufficient adjustment for real data. For example, if the data are asymmetric, the normal distribution is not a good option. That's why different generators have been proposed based on one or more parameters to generate new distributions.

The concept of Transmuted distributions has been discussed dynamically in frequently occurring large-scale experimental statistical data for model selection and related issues. The techniques of Transmuted distributions have been used to the Mukherjee-Islam distribution, which was introduced by Mukherjee and Islam (1983). Recently, Aafaq and Subramanian (2018) discussed on Transmuted Mukherjee Islam Failure Model. It is preferred to be used over more complex distributions such as Normal, Beta, Weibull, and others distributions, due to its simple form and can be handled easily. Several distributions can be used to monitor the production process, and in this paper, the control limits are derived and control chart is monitored using the Transmuted Mukherjee-Islam distribution. The organization of this paper is as follows.

Description of the distribution is provided in section 2. Section 3 provides the Performance measures of the Transmuted Mukherjee Islam distribution. Control limits using the Transmuted Mukherjee Islam distribution is provided in section 4. A numerical example is provided in section 5, finally section 6 provides the Conclusion.

2. Description of the Distribution

Definition 2.1. *Transmuted distributions are extended models. A random variable X is said to have a transmuted distribution if its cumulative distribution function (cdf) is given by*

$$F(x) = (1+\delta)H(x) - \delta H^2(x); \qquad |\delta| \leq 1 \tag{2.1}$$

where; $F(x)$ is the cdf of the transmuted distribution and $H(x)$ is the cdf of the base distribution.
We can see that when $\delta = 0$, we have the base distribution of the random variable X.
By differentiating equation (2.1) with respect to x, one can get the pdf of the transmuted distribution as

$$f(x) = h(x)\left[1 + \delta - 2\delta H(x)\right] \tag{2.2}$$

where; $f(x)$ and $h(x)$ are the corresponding pdf of $F(x)$ and $H(x)$ respectively.

Definition 2.2. *The density function (pdf) of Mukherjee Islam distribution is*

$$h(x) = \frac{k}{\theta^k} x^{k-1}; \qquad 0 < x < \theta, \quad k > 0, \quad \theta > 0 \tag{2.3}$$

and the respective cumulative distribution function (cdf) is

$$H(x) = \left(\frac{x}{\theta}\right)^k; \qquad 0 < x < \theta, \quad k > 0, \quad \theta > 0 \tag{2.4}$$

where θ is a scale parameter and k is a shape parameter.
By using equation (2.4) in (2.1), one can get the cdf of Transmuted Mukherjee Islam distribution, that is,

$$F(x) = \left(\frac{x}{\theta}\right)^k \left[1 + \delta - \delta\left(\frac{x}{\theta}\right)^k\right] \tag{2.5}$$

With transmuted pdf

$$f(x) = \frac{k}{\theta^k} x^{k-1} \left[1 + \delta - 2\delta\left(\frac{x}{\theta}\right)^k\right] \tag{2.6}$$

3. Performance measures of the Transmuted Mukherjee Islam distribution

From the above transmuted density function (pdf), one can derive the r^{th} moment $E(X^r)$ of a Transmuted Mukherjee Islam distribution, thus

$$\begin{aligned} E(X^r) &= \int_0^\theta x^r f(x) dx \\ &= \int_0^\theta x^r \frac{k}{\theta^k} x^{k-1} \left[1 + \delta - 2\delta\left(\frac{x}{\theta}\right)^k\right] dx \\ &= \int_0^\theta x^{r+k-1} \frac{k(1+\delta)}{\theta^k} dx - 2\delta k \int_0^\theta \frac{x^{r+2k-1}}{\theta^{2k}} dx \\ &= \frac{(1+\delta)k\theta^r}{r+k} - \frac{2\delta k\theta^r}{r+2k} \end{aligned}$$

therefore

$$E(X^r) = \frac{k\theta^r(r + 2k - \delta r)}{(r+k)(r+2k)} \tag{3.1}$$

Then, putting $r = 1, 2$ in the above r^{th} moment, one can get the mean and second moment which are,

$$E(X) = \frac{k\theta(1 + 2k - \delta)}{(1+k)(1+2k)} \tag{3.2}$$

Then the second moment is given by

$$E(X^2) = \frac{k\theta^2(2+2k-2\delta)}{(2+k)(2+2k)} = \frac{k\theta^2(1+k-\delta)}{(2+k)(1+k)} \tag{3.3}$$

Therefore, from equations (3.2) and (3.3), the variance $V(X)$ is given by

$$V(X) = E(X^2) - [E(X)]^2$$

therefore

$$V(X) = \frac{k\theta^2(1+k-\delta)}{(2+k)(1+k)} - \left[\frac{k\theta(1+2k-\delta)}{(1+k)(1+2k)}\right]^2 \tag{3.4}$$

4. Control Limits using the Transmuted Mukherjee Islam distribution

This type of control chart has several values and control limit sets. Therefore, when the process is under control, almost all points are within the upper control (UCL) and lower (LCL) limits [Duncan (1986) and Montgomery (2012)]. Therefore, from equations (3.2) and (3.4), the control limits are given by

$$\text{Upper Control Limit } (UCL) = \frac{k\theta(1+2k-\delta)}{(1+k)(1+2k)} + 3\sqrt{\frac{k\theta^2(1+k-\delta)}{(2+k)(1+k)} - \left[\frac{k\theta(1+2k-\delta)}{(1+k)(1+2k)}\right]^2}$$

$$\text{Central Line } (CL) = \frac{k\theta(1+2k-\delta)}{(1+k)(1+2k)}$$

$$\text{Lower Control Limit } (LCL) = \frac{k\theta(1+2k-\delta)}{(1+k)(1+2k)} - 3\sqrt{\frac{k\theta^2(1+k-\delta)}{(2+k)(1+k)} - \left[\frac{k\theta(1+2k-\delta)}{(1+k)(1+2k)}\right]^2}$$

where $0 < x < \theta, \quad k > 0, \quad \theta > 0; \quad |\delta| \leq 1$

5. Numerical Illustration

An example regarding the construction of control limits is considered for illustrating the applications of the proposed method. The control limits of the Transmuted Mukherjee-Islam distribution are obtained for simulated data set. For parameters $k = 1, \theta$ and δ being random variables the table 1 is constructed. All the generated samples are reported in Table 1.

It is observed that from table 1, for the fixed value of the parameter θ, the control limits increase whenever the parameter δ increases. As well as for the increasing of parameter θ, the control limits increase whenever the parameter δ is fixed. The Transmuted Mukherjee Islam (TMI) Control Chart is shown in Fig. 1 for $\theta = 5$, $\delta = -0.8$.

θ	δ	CL	LCL	UCL
5	-0.8	3.17	0	7
10	-0.9	6.5	0	13.90
15	0	7.5	0	20.49
	0.2	7	0	19.90
	0.3	6.75	0	19.54
	0.4	6.5	0	19.14
	0.5	6.25	0	18.69
	0.6	6	0	18.19
	0.7	5.75	0	17.63
	0.8	5.5	0	17.02
	0.9	5.25	0	16.35
20	0	10	0	27.32
	0.2	9.33	0	26.54
	0.3	9	0	26.06
	0.4	8.67	0	25.52
	0.5	8.33	0	24.92
	0.6	8	0	24.25
	0.7	7.67	0	23.51
	0.8	7.33	0	22.70
	0.9	7	0	21.80
25	0	12.5	0	34.15
	0.2	11.67	0	33.17
	0.3	11.25	0	32.57
	0.4	10.83	0	31.90
	0.5	10.42	0	31.15
	0.6	10	0	30.31
	0.7	9.58	0	29.39
	0.8	9.17	0	28.37
	0.9	8.75	0	27.25

Table 1: Control limits using Transmuted Mukherjee Islam Distribution

It is also pointed out that, the observations of the process control must be $0 < x < \theta, \quad k > 0, \quad \theta > 0; \quad |\delta| \leq 1$, otherwise the process will be out of control. That is, depends on the manufacturing products, the manufacturing engineers should fix the parameters value based on what type of data they are working with. As we can see on Table 1, the more the parameters values increase, the higher control limits.

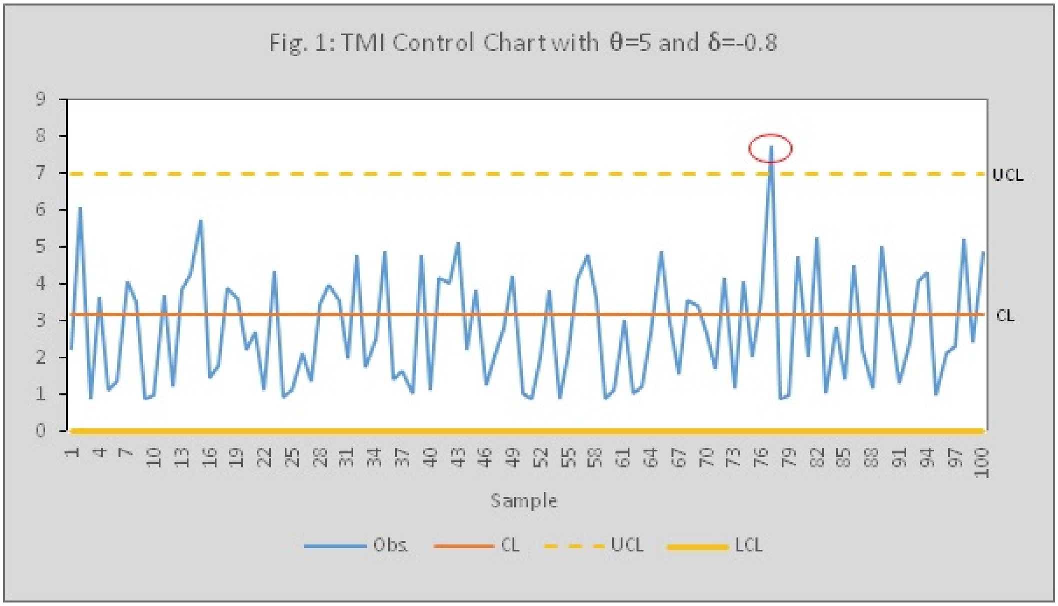

Fig. 1: TMI Control Chart with θ=5 and δ=-0.8

6. Conclusion

In this article, the process control has been developed using a Transmuted Mukherjee Islam Distribution. A novel algorithm is given for sentencing the process while manufacturing. The Control limits are given using the Transmuted Mukherjee Islam Distribution with different values of parameters θ and δ. Table is constructed to help to the selection of the parameters based on the type of data the manufacturing engineer is faced with. The control chart is drawn by considering the parameters $\theta = 5$ and $\delta = -0.8$ where the 77^{th} observation is out of control because it is greater than the value of parameter θ. Hence it is advisable to keep θ greater than the observed process data.

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