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# A HYPERGEOMETRIC GENERATING FUNCTION INSPIRED BY THE WORK OF BEDIENT AND SHIVELY

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**Abstract:** In this article, a hypergeometric generating function with suitable convergence conditions in the form of Srivastava-Daoust triple hypergeometric function is derived by using series rearrangement technique. Some generating functions for Bedient's two polynomials and Shively pseudo-Laguerre polynomials are also obtained as special cases.

**Keywords and Phrases:** Hypergeometric functions; Series rearrangement technique; Pseudo-Laguerre polynomials; Bedient polynomials; Multiple series identity.

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## 1. Introduction and Preliminaries

### Pochhammer symbol

In our investigations, we shall use the following standard notations:
$\mathbb{N} := \{1, 2, 3, \cdots\}; \mathbb{N}_0 := \mathbb{N} \bigcup \{0\}; \mathbb{Z}_0^- := \mathbb{Z}^- \bigcup \{0\} = \{0, -1, -2, -3, \cdots\}$.
The symbols $\mathbb{C}$, $\mathbb{R}$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}^+$ and $\mathbb{R}^-$ denote the sets of complex numbers, real numbers, natural numbers, integers, positive and negative real numbers respectively. The Pochhammer symbol $(\alpha)_p$ $(\alpha, p \in \mathbb{C})$ [20, p.22, Eq.(1), p.32, Q.N.(8) and Q.N.(9), see also [29] p.23, Eq.(22) and Eq.(23)] is defined by

$$(\alpha)_p := \frac{\Gamma(\alpha + p)}{\Gamma(\alpha)} =$$

$$= \begin{cases} 1 & ;(p=0;\alpha\in\mathbb{C}\backslash\{0\}), \\ \alpha(\alpha+1)\cdots(\alpha+n-1) & ;(p=n\in\mathbb{N};\alpha\in\mathbb{C}), \\ \frac{(-1)^n k!}{(k-n)!} & ;(\alpha=-k;p=n;n,k\in\mathbb{N}_0;0\leq n\leq k), \\ 0 & ;(\alpha=-k;p=n;n,k\in\mathbb{N}_0;n>k), \\ \frac{(-1)^n}{(1-\alpha)_n} & ;(p=-n;n\in\mathbb{N};\alpha\in\mathbb{C}\backslash\mathbb{Z}). \end{cases}$$

It being understood conventionally that$(0)_0 = 1$ and assumed tacitly that the Gamma quotient exists.

**Generalized hypergeometric function of one variable**

A natural generalization of the Gaussian hypergeometric series ${}_2F_1[\alpha,\beta;\gamma;z]$ is accomplished by introducing any arbitrary number of numerator and denominator parameters. Thus, the resulting series

$${}_pF_q\left[\begin{array}{ccc}(\alpha_p); & & \\ & & z \\ (\beta_q); & & \end{array}\right] = {}_pF_q\left[\begin{array}{ccc}\alpha_1,\alpha_2,\ldots,\alpha_p; & & \\ & & z \\ \beta_1,\beta_2,\ldots,\beta_q; & & \end{array}\right] = \sum_{n=0}^{\infty}\frac{(\alpha_1)_n(\alpha_2)_n\ldots(\alpha_p)_n}{(\beta_1)_n(\beta_2)_n\ldots(\beta_q)_n}\frac{z^n}{n!}, \tag{1.1}$$

is known as the generalized hypergeometric series, or simply, the generalized hypergeometric function. Here $p$ and $q$ are positive integers or zero and we assume that the variable $z$, the numerator parameters $\alpha_1,\alpha_2,\ldots,\alpha_p$ and the denominator parameters $\beta_1,\beta_2,\ldots,\beta_q$ take on complex values, provided that

$$\beta_j \neq 0,-1,-2,\ldots \; ; \quad j=1,2,\ldots,q.$$

Supposing that none of the numerator and denominator parameters is zero or a negative integer, we note that the ${}_pF_q$ series defined by equation (1.1):

(i) converges for $|z|<\infty$, if $p\leq q$,

(ii) converges for $|z|<1$, if $p=q+1$,

(iii) diverges for all $z$, $z\neq 0$, if $p>q+1$,

(iv) converges absolutely for $|z|=1$, if $p=q+1$ and $\Re(\omega)>0$,

(v) converges conditionally for $|z|=1$ $(z\neq 1)$, if $p=q+1$ and $-1<\Re(\omega)\leqq 0$,

(vi) diverges for $|z|=1$, if $p=q+1$ and $\Re(\omega)\leqq -1$,

where, by convention, a product over an empty set is interpreted as 1 and

$$\omega := \sum_{j=1}^{q}\beta_j - \sum_{j=1}^{p}\alpha_j, \tag{1.2}$$

$\Re(\omega)$ being the real part of complex number $\omega$.

**Double hypergeometric function of Kampé de Fériet**

Just as the Gaussian ${}_2F_1$ function was generalized to ${}_pF_q$ by increasing the number of the numerator and denominator parameters, the Appell's four double hypergeometric functions $F_1, F_2, F_3, F_4$ [ 29, p.53, Eq.(4), Eq.(5), Eq.(6) and Eq.(7)] and their seven confluent forms $\Phi_1, \Phi_2, \Phi_3, \Psi_1, \Psi_2, \Xi_1, \Xi_2$ given by Humbert ([12, 14], see also [13, pp.75-76]) were unified and generalized by Kampé de Fériet [15] who defined a general hypergeometric function of two variables.

The notation introduced by Kampé de Fériet for his double hypergeometric function [5, p.150, Eq.(26)] of superior order was subsequently abbreviated by Burchnall and Chaundy [8, p.112]. We recall here the definition of a more general double hypergeometric function (than the one defined by Kampé de Fériet) in a slightly modified notation of Srivastava and Panda [30, p.423, Eq.(26)]:

$$F^{p:\ q;\ k}_{\ell:\ m;\ n}\left[\begin{matrix}(a_p)\ :\ (b_q)\ ;\ (c_k)\ ;\\ \\ (\alpha_\ell)\ :\ (\beta_m)\ ;\ (\gamma_n)\ ;\end{matrix}\ x,\ y\right]=\sum_{r,s=0}^{\infty}\frac{\prod_{j=1}^{p}(a_j)_{r+s}\prod_{j=1}^{q}(b_j)_r\prod_{j=1}^{k}(c_j)_s}{\prod_{j=1}^{\ell}(\alpha_j)_{r+s}\prod_{j=1}^{m}(\beta_j)_r\prod_{j=1}^{n}(\gamma_j)_s}\frac{x^r}{r!}\frac{y^s}{s!}, \tag{1.3}$$

where, for convergence [30, p.424, Eq.(27)]

$$(i)\ p+q<\ell+m+1,\quad p+k<\ell+n+1,\quad |x|<\infty,\quad |y|<\infty,\ \text{or} \tag{1.4}$$

$$(ii)\ p+q=\ell+m+1,\quad p+k=\ell+n+1\ \text{and} \tag{1.5}$$

$$\begin{cases}|x|^{1/(p-\ell)}+|y|^{1/(p-\ell)}<1, & \text{if } p>\ell,\\ \max\{|x|,|y|\}<1, & \text{if } p\leq\ell.\end{cases} \tag{1.6}$$

For absolutely and conditionally convergence of double series (1.3), we refer to a research paper of Hài et al. [11, pp.106-107, Th.(1), Th.(2) and Th.(3)].

**Multiple hypergeometric function of Srivastava-Daoust**

The following generalization of the hypergeometric function in several variables has been given by Srivastava and Daoust ([26, pp.199-200, Eq.(2.1),[28]]) which is referred to, in the literature as the generalized Lauricella function of several variables (see also [27, p.454, Eq.(4.1)]):

$$S^{A:\ B^{(1)};...;\ B^{(n)}}_{C:\ D^{(1)};...;\ D^{(n)}}\left(\begin{matrix}[(a_A):\ \vartheta^{(1)},...,\ \vartheta^{(n)}]:[(b^{(1)}_{B^{(1)}}):\varphi^{(1)}];...;[(b^{(n)}_{B^{(n)}}):\varphi^{(n)}];\\ \\ [(c_C)\ :\ \psi^{(1)},...,\ \psi^{(n)}]:[(d^{(1)}_{D^{(1)}}):\delta^{(1)}];...;[(d^{(n)}_{D^{(n)}}):\delta^{(n)}];\end{matrix}\ x_1,...,x_n\right)$$

$$= \sum_{m_1=0}^{\infty} \cdots \sum_{m_n=0}^{\infty} \frac{\prod_{j=1}^{A} \Gamma[a_j + \sum_{i=1}^{n} m_i \vartheta_j^{(i)}] \prod_{j=1}^{B^{(1)}} \Gamma[b_j^{(1)} + m_1 \varphi_j^{(1)}]}{\prod_{j=1}^{C} \Gamma[c_j + \sum_{i=1}^{n} m_i \psi_j^{(i)}] \prod_{j=1}^{D^{(1)}} \Gamma[d_j^{(1)} + m_1 \delta_j^{(1)}]} \times$$

$$\times \frac{\cdots \prod_{j=1}^{B^{(n)}} \Gamma[b_j^{(n)} + m_n \varphi_j^{(n)}] x_1^{m_1}}{\cdots \prod_{j=1}^{D^{(n)}} \Gamma[d_j^{(n)} + m_n \delta_j^{(n)}] m_1!} \cdots \frac{x_n^{m_n}}{m_n!} \tag{1.7}$$

$$= \frac{\prod_{j=1}^{A} \Gamma[a_j] \ \prod_{j=1}^{B^{(1)}} \Gamma[b_j^{(1)}] \ldots \prod_{j=1}^{B^{(n)}} \Gamma[b_j^{(n)}]}{\prod_{j=1}^{C} \Gamma[c_j] \ \prod_{j=1}^{D^{(1)}} \Gamma[d_j^{(1)}] \ldots \prod_{j=1}^{D^{(n)}} \Gamma[d_j^{(n)}]} \times$$

$$\times F^{A:\ B^{(1)};\ldots;\ B^{(n)}}_{C:\ D^{(1)};\ldots;\ D^{(n)}} \left( \begin{matrix} [(a_A):\ \vartheta^{(1)}, \ldots,\ \vartheta^{(n)}] : [(b^{(1)}_{B^{(1)}}) : \varphi^{(1)}]; \ldots; [(b^{(n)}_{B^{(n)}}) : \varphi^{(n)}]; \\ [(c_C)\ :\ \psi^{(1)}, \ldots,\ \psi^{(n)}] : [(d^{(1)}_{D^{(1)}}) : \delta^{(1)}]; \ldots; [(d^{(n)}_{D^{(n)}}) : \delta^{(n)}]; \end{matrix} \ x_1, \ldots, x_n \right) \tag{1.8}$$

$$= \frac{\prod_{j=1}^{A} \Gamma[a_j] \ \prod_{j=1}^{B^{(1)}} \Gamma[b_j^{(1)}] \ldots \prod_{j=1}^{B^{(n)}} \Gamma[b_j^{(n)}]}{\prod_{j=1}^{C} \Gamma[c_j] \ \prod_{j=1}^{D^{(1)}} \Gamma[d_j^{(1)}] \ldots \prod_{j=1}^{D^{(n)}} \Gamma[d_j^{(n)}]} \times$$

$$\times \sum_{m_1, m_2, \ldots, m_n = 0}^{\infty} \Omega(m_1, m_2, \ldots, m_n) \ \frac{x_1^{m_1}}{m_1!} \frac{x_2^{m_2}}{m_2!} \cdots \frac{x_n^{m_n}}{m_n!}, \tag{1.9}$$

where

$$\Omega(m_1, m_2, \ldots, m_n) := \frac{\prod_{j=1}^{A} (a_j)_{m_1 \vartheta_j^{(1)} + \cdots + m_n \vartheta_j^{(n)}} \prod_{j=1}^{B^{(1)}} (b_j^{(1)})_{m_1 \varphi_j^{(1)}} \cdots \ \prod_{j=1}^{B^{(n)}} (b_j^{(n)})_{m_n \varphi_j^{(n)}}}{\prod_{j=1}^{C} (c_j)_{m_1 \psi_j^{(1)} + \cdots + m_n \psi_j^{(n)}} \prod_{j=1}^{D^{(1)}} (d_j^{(1)})_{m_1 \delta_j^{(1)}} \cdots \ \prod_{j=1}^{D^{(n)}} (d_j^{(n)})_{m_n \delta_j^{(n)}}} \tag{1.10}$$

and the coefficients

$$\vartheta_j^{(i)}, j = 1, 2, \ldots, A; \ \ \varphi_j^{(i)}, j = 1, 2, \ldots, B^{(i)}; \ \ \psi_j^{(i)}, j = 1, 2, \ldots, C; \ \ \delta_j^{(i)}, j = 1, 2, \ldots, D^{(i)};$$

for all $i \in \{1, 2, \ldots, n\}$ are real and positive,
then, with the positive constants $\vartheta' s, \varphi' s, \psi' s$ and $\delta' s$ equated to one,
$F^{1:1;\ldots;1}_{0:1;\ldots;1}$ will correspond to Lauricella's $F_A^{(n)}$-function,

$F^{0:2;\ldots;2}_{1:0;\ldots;0}$ will correspond to Lauricella's $F_B^{(n)}$-function,

$F^{2:0;\ldots;0}_{0:1;\ldots;1}$ will correspond to Lauricella's $F_C^{(n)}$-function and

$F^{1:1;...;1}_{1:0;...;0}$ will correspond to Lauricella's fourth function $F^{(n)}_D$ [16, p.113]; while $F^{0:B^{(1)};...;B^{(n)}}_{0:D^{(1)};...;D^{(n)}}$ will yield the product

$$ {}_{B^{(1)}}F_{D^{(1)}}\left[\begin{array}{lr} (b^{(1)}_{B^{(1)}}); & \\ & x_1 \\ (d^{(1)}_{D^{(1)}}); & \end{array}\right] \ldots {}_{B^{(n)}}F_{D^{(n)}}\left[\begin{array}{lr} (b^{(n)}_{B^{(n)}}); & \\ & x_n \\ (d^{(n)}_{D^{(n)}}); & \end{array}\right] \tag{1.11} $$

of $n$ generalized hypergeometric functions with different arguments.
The multiple hypergeometric functions (1.7), (1.8) and (1.9) are the generalizations of Fox-Wright hypergeometric function of one variable ${}_p\Psi_q$ and ${}_p\Psi^*_q$ [31, 32].

Let

$$ E_i = \left( \mu_i^{1+\sum_{j=1}^{D^{(i)}} \delta^{(i)}_j - \sum_{j=1}^{B^{(i)}} \varphi^{(i)}_j} \right) \frac{\prod_{j=1}^{C} \left( \sum_{\ell=1}^{n} \mu_\ell \, \psi^{(\ell)}_j \right)^{\psi^{(i)}_j} \quad \prod_{j=1}^{D^{(i)}} (\delta^{(i)}_j)^{\delta^{(i)}_j}}{\prod_{j=1}^{A} \left( \sum_{\ell=1}^{n} \mu_\ell \, \vartheta^{(\ell)}_j \right)^{\vartheta^{(i)}_j} \quad \prod_{j=1}^{B^{(i)}} (\varphi^{(i)}_j)^{\varphi^{(i)}_j}}, \tag{1.12} $$

$$ \Delta_i = 1 + \sum_{j=1}^{C} \psi^{(i)}_j + \sum_{j=1}^{D^{(i)}} \delta^{(i)}_j - \sum_{j=1}^{A} \vartheta^{(i)}_j - \sum_{j=1}^{B^{(i)}} \varphi^{(i)}_j ; \ i = 1, 2, ..., n. \tag{1.13} $$

**Case I.** The multiple power series in (1.7) is convergent for all finite complex values or real values of $x_1, x_2, ..., x_n$, when $\Delta_i > 0$, $i = 1, 2, ..., n$.
**Case II.** The multiple power series in (1.7) is convergent when $\Delta_1 = \Delta_2 = ... = \Delta_n = 0$; $|x_1| < \varrho_1, |x_2| < \varrho_2, ..., |x_n| < \varrho_n$;
where

$$ \varrho_i = \min_{\mu_1,...,\mu_n > 0} \{E_i\}, \quad i = 1, 2, ..., n. \tag{1.14} $$

**Case III.** The multiple power series in (1.7) would diverge except when, trivially, $x_1 = x_2 = ... = x_n = 0$ when $\Delta_i < 0$, $i = 1, 2, ..., n$.

**Further analysis of Case II**

When

$$ \vartheta^{(1)}_j = \vartheta^{(2)}_j = ... = \vartheta^{(n)}_j = \vartheta_j, \ 1 \leq j \leq A, \tag{1.15} $$

$$ \psi^{(1)}_j = \psi^{(2)}_j = ... = \psi^{(n)}_j = \psi_j, \ 1 \leq j \leq C, \tag{1.16} $$

$$G_i = \frac{\prod_{j=1}^{C}(\psi_j^{(i)})^{\psi_j^{(i)}} \ \prod_{j=1}^{D^{(i)}}(\delta_j^{(i)})^{\delta_j^{(i)}}}{\prod_{j=1}^{A}(\vartheta_j^{(i)})^{\vartheta_j^{(i)}} \ \prod_{j=1}^{B^{(i)}}(\varphi_j^{(i)})^{\varphi_j^{(i)}}}, \quad i = 1, 2, ..., n, \tag{1.17}$$

$$\mho_i \equiv 1 + \sum_{j=1}^{C} \psi_j^{(i)} + \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} - \sum_{j=1}^{A} \vartheta_j^{(i)} - \sum_{j=1}^{B^{(i)}} \varphi_j^{(i)}, \quad i = 1, 2, ..., n \tag{1.18}$$

and

$$\Omega = \sum_{j=1}^{A} \vartheta_j - \sum_{j=1}^{C} \psi_j. \tag{1.19}$$

**Case II(a).** The multiple power series in (1.7) is convergent when $\mho_1 = \mho_2 = ... = \mho_n = 0$; $\Omega > 0$ and

$$\left(\frac{|x_1|}{G_1}\right)^{\frac{1}{\Omega}} + ... + \left(\frac{|x_n|}{G_n}\right)^{\frac{1}{\Omega}} < 1. \tag{1.20}$$

**Case II(b).** The multiple power series in (1.7) is convergent when $\mho_1 = \mho_2 = ... = \mho_n = 0$; $\Omega \leq 0$ and

$$\max\left(\frac{|x_1|}{G_1}, \frac{|x_2|}{G_2}, ..., \frac{|x_n|}{G_n}\right) < 1. \tag{1.21}$$

Series rearrangement technique is based upon certain interchanges of the order of a double (or multiple) summation. Several hypergeometric generating relations have been established using series rearrangement technique.

Here, we consider some well known results.

**Cauchy's double series identity** [29, p.100, Eq.(1), Eq.(3)]

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \Phi(m, n) = \sum_{m=0}^{\infty}\sum_{n=0}^{m} \Phi(m - n, n), \tag{1.22}$$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \Phi(m, n) = \sum_{m=0}^{\infty}\sum_{n=0}^{[\frac{m}{2}]} \Phi(m - 2n, n), \tag{1.23}$$

provided that the associated double series are absolutely convergent.

Series rearrangement technique for the multiple series [29, p.102, Eq.(16)] is given by:

$$\sum_{n=0}^{\infty}\sum_{k_1,...,k_r=0}^{\infty}\Phi(k_1,...,k_r;n)=\sum_{n=0}^{\infty}\sum_{k_1,...,k_r=0}^{m_1k_1,...,m_rk_r\leq n}\Phi(k_1,...,k_r;n-m_1k_1-...-m_rk_r), \tag{1.24}$$

where $m_1, m_2, ..., m_r$ are positive integers and the associated series are absolutely convergent.

**Srivastava's multiple series identity** [23, p.4, Eq.(12)]

$$\sum_{m=0}^{\infty}f(m)\frac{(x_1+x_2+\cdots+x_n)^m}{m!}=\sum_{m_1,m_2,\cdots,m_n=0}^{\infty}f(m_1+m_2+\cdots+m_n)\frac{{x_1}^{m_1}}{m_1!}\frac{{x_2}^{m_2}}{m_2!}\cdots\frac{{x_n}^{m_n}}{m_n!}, \tag{1.25}$$

provided that the multiple series involved are absolutely convergent.

**Shively's pseudo-Laguerre Polynomials**

In 1953, Shively defined pseudo-Laguerre polynomials $R_n(a,x)$ [22, p.54, see also [20], p.298, Eq.(1)] in the form

$$R_n(a,\ x)=\frac{2^{2n}(\frac{a}{2})_n(\frac{a+1}{2})_n}{n!(a)_n}\ {}_1F_1\left[\begin{array}{c}-n;\\ \\ a+n;\end{array}\ x\right]. \tag{1.26}$$

**Bedient's polynomials**

In 1958-59, Bedient's polynomials $R_m(\alpha,\beta;y)$ [7, p.15, Eq.(2.5), see also [20], p.297, Eq.(1)] are defined by

$$R_m(\alpha,\ \beta\ ;\ y)=\frac{(\alpha)_m(2y)^m}{m!}\ {}_3F_2\left[\begin{array}{c}\frac{-m}{2},\ \ \frac{-m+1}{2},\ \ \beta-\alpha;\\ \\ \beta,\ \ 1-\alpha-m\ ;\end{array}\ \frac{1}{y^2}\right], \tag{1.27}$$

other Bedient's polynomials $G_m(\gamma,\delta;y)$ [7, p.44, Eq.(3.4), see also [20], p.297, Eq.(2)] are defined by

$$G_m(\gamma,\ \delta\ ;\ y)=\frac{(\gamma)_m(\delta)_m(2y)^m}{(\gamma+\delta)_m\ m!}\ {}_3F_2\left[\begin{array}{c}\frac{-m}{2},\ \ \frac{-m+1}{2},\ \ 1-\gamma-\delta-m;\\ \\ 1-\gamma-m,\ \ 1-\delta-m\ ;\end{array}\ \frac{1}{y^2}\right]. \tag{1.28}$$

**Gauss' classical summation theorem**

Gauss' classical summation theorem [20, p.49, Th.(18)] is given by

$$ {}_2F_1\left[\begin{array}{ccc} \alpha, & \beta; & \\ & & 1 \\ & \gamma; & \end{array}\right] = \frac{\Gamma(\gamma)\,\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}, \tag{1.29} $$

where $\mathfrak{Re}(\gamma-\alpha-\beta) > 0$ and $\gamma \in \mathbb{C}\backslash\mathbb{Z}_0^-$.

A particular case of Gauss' classical summation theorem [20, p.49, Ex.] is given by

$$ {}_2F_1\left[\begin{array}{ccc} \frac{-\ell}{2}, & \frac{-\ell+1}{2}; & \\ & & 1 \\ & b+\frac{1}{2}; & \end{array}\right] = \frac{2^{\ell}(b)_{\ell}}{(2b)_{\ell}}, \tag{1.30} $$

where $\ell = 0, 1, 2, ...$ and $b + \frac{1}{2} \neq 0, -1, -2, ...$

Special functions has gained much importance in almost all fields of Science and Engineering. This branch of applied mathematics is rapidly developing with large number of applications in the real world. Many authors such as Agarwal [1, p.2316, Eq.(6), Eq.(10) and p.2317, Eq.(14)], Agarwal et al. [2, p.406, Eq.(2.1), Eq.(2.2), Eq.(2.3)], see also [3,4, p.3699, Eq.(21), Eq.(22), Eq.(23) and Eq.(24)], Baleanu and Agarwal [6, p.3, Eq.(12), Eq.(16)], Ruzhansky et al. [21], Srivastava and Agarwal [24, p.339, Eq.(2.13), Eq.(2.14) and p.340, Eq.(2.17), Eq.(2.18)] etc have studied this branch of applied mathematics and its applications. Likewise the concept of the generating functions was introduced by Laplace in 1812. Since, then the theory of generating functions has been developed in different directions and found wide applications in many branches of mathematics and mathematical physics.

**Linear generating function**

Two functions $F(x,t)$ and $G(x,t)$ of two independent variables $x$ and $t$ are called generating functions of the sets $\{f_n(x)\}$ and $\{g_n(x)\}$ respectively, if it is possible to represent $F(x,t)$ and $G(x,t)$ in the following series expansions of $t$

$$ F(x,t) = \sum_{n=0}^{\infty} b_n\ f_n(x)t^n; \quad t \neq 0, \tag{1.31} $$

$$ G(x,t) = \sum_{n=-\infty}^{+\infty} c_n\ g_n(x)t^n; \quad t \neq 0, \tag{1.32} $$

where the coefficients $b_n$ and $c_n$ are independent of $x$ and $t$ and may contain some parameters related with $f_n(x)$, $g_n(x)$ respectively.

Motivated by the work collected in beautiful monographs of Rainville [20, Ch.(8), pp.129-146], McBride [17, Ch.(1), pp.1-24; Ch.(5), pp.72-76], Erdélyi et al. [10, Ch.(19), pp.245-278], the papers of Srivastava et al. [25, p.350, Eq.(2.3) and p.351, Eq.(2.8)], Choi et al. [9, p.28, Eq.(2.1), Eq.(2.2) and p.29, Eq.(2.3), Eq.(2.4)] and the papers of Qureshi et al. [18, p.34, Eq.(2.1), Eq.(2.2), Eq.(2.3), p.35, Eq.(2.4), [19], p.63, Eq.(2.1), p.64, Eq.(2.2), Eq.(2.3) and Eq.(2.4)], we obtain a generating relation in this paper.

The present article is organized as follows. In section 2, we obtain a generating relation. In section 3, we have given the proof of hypergeometric generating relation using series rearrangement technique. In section 4, we discuss some applications.

## 2. Main Hypergeometric Generating Relations

When the values of numerator, denominator parameters and arguments leading to the results which do not make sense are tacitly excluded. Then

$$F^{A+D+G+K:Q;S;U}_{B+E+H+L:R;J;W}\left(\begin{array}{l}[(a_A):1,2,1],\ [(d_D):1,2,2],\ [(g_G):0,1,1],\ [(k_K):0,2,1]\textbf{:}\\[(b_B):1,2,1],\ [(e_E):1,2,2],\ [(h_H):0,1,1],\ [(\ell_L):0,2,1]\textbf{:}\end{array}\right.$$

$$\left.\begin{array}{l}[(q_Q):1];\ [(s_S):1];[(u_U):1];\\ [(r_R):1];\ [(j_J):1];[(w_W):1];\end{array}\ \lambda t,\ \mu t^2,\ yt\right)$$

$$=\sum_{m=0}^{\infty}\frac{\prod_{i=1}^{A}(a_i)_m\ \prod_{i=1}^{D}(d_i)_m\ \prod_{i=1}^{Q}(q_i)_m\ \lambda^m}{\prod_{i=1}^{B}(b_i)_m\ \prod_{i=1}^{E}(e_i)_m\ \prod_{i=1}^{R}(r_i)_m\ m!}\times$$

$$\times F^{G+K+R+1:S;D+U}_{L+Q+H\quad :J;E+W}\left(\begin{array}{r}[-m:2,1],[(k_K):2,1],[1-(r_R)-m:2,1],[(g_G):1,1]\textbf{:}\\ [(\ell_L):2,1],[1-(q_Q)-m:2,1],[(h_H):1,1]\textbf{:}\end{array}\right.$$

$$\left.\begin{array}{l}[(s_S):1]\ ;[(d_D)+m:1],[(u_U):1];\\ [(j_J):1]\ ;[(e_E)+m:1],[(w_W):1];\end{array}\ \tfrac{\mu}{\lambda^2},\ (-1)^{(Q+R+1)}\ \tfrac{y}{\lambda}\right)\ t^m, \tag{2.1}$$

where $\lambda\neq 0$,

$$=\sum_{p=0}^{\infty}\frac{\prod_{i=1}^{A}(a_i)_p\ \prod_{i=1}^{D}(\frac{d_i}{2})_p\ \prod_{i=1}^{D}(\frac{1+d_i}{2})_p\ \prod_{i=1}^{G}(g_i)_p\ \prod_{i=1}^{K}(k_i)_p\ \prod_{i=1}^{U}(u_i)_p\ 2^{(2D-2E)p}\ y^p}{\prod_{i=1}^{B}(b_i)_p\ \prod_{i=1}^{E}(\frac{e_i}{2})_p\ \prod_{i=1}^{E}(\frac{1+e_i}{2})_p\ \prod_{i=1}^{H}(h_i)_p\ \prod_{i=1}^{L}(\ell_i)_p\ \prod_{i=1}^{W}(w_i)_p\ p!}\times$$

$$\times\, F^{E+W+H+1:S;Q+L}_{D+U+G\quad :J;R+K}\left(\begin{array}{c}[-p:2,1],[1-(e_E)-2p:2,1],\\ [1-(d_D)-2p:2,1],\end{array}\right.$$

$$[1-(w_W)-p:2,1],[1-(h_H)-p:1,1]:[(s_S):1];[(q_Q):1],$$

$$[1-(u_U)-p:2,1],[1-(g_G)-p:1,1]:\ [(j_J):1];[(r_R):1],$$

$$\left.\begin{array}{c}[1-(\ell_L)-p:1];\\ [1-(k_K)-p:1];\end{array}\ \frac{\mu\ (-1)^{(G+H)}}{y^2},\ (-1)^{(D+E+G+H+K+L+U+W+1)}\ \frac{\lambda}{y}\right)\ t^p. \tag{2.2}$$

**Convergence conditions**
Suppose

$$\Delta_1 = 1+B+E+R-A-D-Q, \tag{2.3}$$

$$\Delta_2 = 1+2B+2E+H+2L+J-2A-2D-G-2K-S, \tag{2.4}$$

$$\Delta_3 = 1+B+2E+H+L+W-A-2D-G-K-U. \tag{2.5}$$

(i) When $\Delta_1>0,\ \Delta_2>0, \Delta_3>0$, then the triple series in left hand side of equations (2.1) and (2.2) is convergent for all finite (real and complex) values of $\lambda, \mu, y$ and $t$.

(ii) When $\Delta_1=\Delta_2=\Delta_3=0$, then the triple series in left hand side of equations (2.1) and (2.2) is convergent for appropriately constrained values of $|\lambda t|, |\mu t^2|$ and $|yt|$,

provided that in each hypergeometric function, denominator parameters are neither zero nor negative integers.

## 3. Proof of Main Generating Relations

Let

$$\Psi = F^{A+D+G+K:Q;S;U}_{B+E+H+L:R;J;W}\left(\begin{array}{c}[(a_A):1,2,1],\ [(d_D):1,2,2],\ [(g_G):0,1,1],\ [(k_K):0,2,1]:\\ [(b_B):1,2,1],\ [(e_E):1,2,2],\ [(h_H):0,1,1],\ [(\ell_L):0,2,1]:\end{array}\right.$$

$$\left.\begin{array}{l}[(q_Q):1];\ [(s_S):1];[(u_U):1];\\ \\ [(r_R):1];\ [(j_J):1];[(w_W):1];\end{array}\ \lambda t,\ \mu t^2,\ yt\right) \tag{3.1}$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\frac{[(a_A)]_{m+2n+p}[(d_D)]_{m+2n+2p}[(g_G)]_{n+p}[(k_K)]_{2n+p}[(q_Q)]_m[(s_S)]_n}{[(b_B)]_{m+2n+p}[(e_E)]_{m+2n+2p}[(h_H)]_{n+p}[(\ell_L)]_{2n+p}[(r_R)]_m[(j_J)]_n}\times$$

$$\times\frac{[(u_U)]_p(\lambda t)^m(\mu t^2)^n(yt)^p}{[(w_W)]_p\ m!\ n!\ p!}, \tag{3.2}$$

where $[(a_A)]_m=(a_1)_m(a_2)_m...(a_A)_m\ =\ \prod_{i=1}^{A}(a_i)_m$.
Then

$$\Psi=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\frac{(a_1)_{m+2n+p}...(a_A)_{m+2n+p}(d_1)_{m+2n+2p}...(d_D)_{m+2n+2p}(g_1)_{n+p}...(g_G)_{n+p}}{(b_1)_{m+2n+p}...(b_B)_{m+2n+p}(e_1)_{m+2n+2p}...(e_E)_{m+2n+2p}(h_1)_{n+p}...(h_H)_{n+p}}\times$$

$$\times\frac{(k_1)_{2n+p}...(k_K)_{2n+p}(q_1)_m...(q_Q)_m(s_1)_n...(s_S)_n(u_1)_p...(u_U)_p\ \lambda^m\mu^n y^p\ t^{m+2n+p}}{(\ell_1)_{2n+p}...(\ell_L)_{2n+p}(r_1)_m...(r_R)_m(j_1)_n...(j_J)_n(w_1)_p...(w_W)_p\ m!\ n!\ p!}. \tag{3.3}$$

Replacing $m$ by $m-2n-p$ in equation (3.3) and applying series identity (1.24), we get

$$\Psi=\sum_{m=0}^{\infty}\sum_{n,p=0}^{2n+p\le m}\frac{(a_1)_m...(a_A)_m(d_1)_{m+p}...(d_D)_{m+p}(g_1)_{n+p}...(g_G)_{n+p}(k_1)_{2n+p}...(k_K)_{2n+p}}{(b_1)_m...(b_B)_m(e_1)_{m+p}...(e_E)_{m+p}(h_1)_{n+p}...(h_H)_{n+p}(\ell_1)_{2n+p}...(\ell_L)_{2n+p}}\times$$

$$\times\frac{(q_1)_{m-2n-p}...(q_Q)_{m-2n-p}(s_1)_n...(s_S)_n(u_1)_p...(u_U)_p(-m)_{2n+p}\ \lambda^{m-2n-p}\mu^n y^p\ t^m}{(r_1)_{m-2n-p}...(r_R)_{m-2n-p}(j_1)_n...(j_J)_n(w_1)_p...(w_W)_p(-1)^p\ m!\ n!\ p!}$$

$$=\sum_{m=0}^{\infty}\frac{(a_1)_m...(a_A)_m(d_1)_m...(d_D)_m(q_1)_m...(q_Q)_m\ \lambda^m}{(b_1)_m...(b_B)_m(e_1)_m...(e_E)_m(r_1)_m...(r_R)_m m!}\times$$

$$\times\left(\sum_{n,p=0}^{2n+p\le m}\frac{(d_1+m)_p...(d_D+m)_p(g_1)_{n+p}...(g_G)_{n+p}(k_1)_{2n+p}...(k_K)_{2n+p}}{(e_1+m)_p...(e_E+m)_p(h_1)_{n+p}...(h_H)_{n+p}(\ell_1)_{2n+p}...(\ell_L)_{2n+p}}\times\right.$$

$$\left.\times\ \frac{(q_1+m)_{-(2n+p)}...(q_Q+m)_{-(2n+p)}(s_1)_n...(s_S)_n(u_1)_p...(u_U)_p(-m)_{2n+p}\ \mu^n y^p(-1)^p}{(r_1+m)_{-(2n+p)}...(r_R+m)_{-(2n+p)}(j_1)_n...(j_J)_n(w_1)_p...(w_W)_p\ \lambda^{2n+p}\ n!\ p!}\right)\ t^m$$

$$=\sum_{m=0}^{\infty}\frac{\prod_{i=1}^{A}(a_i)_m\ \prod_{i=1}^{D}(d_i)_m\ \prod_{i=1}^{Q}(q_i)_m\ \lambda^m}{\prod_{i=1}^{B}(b_i)_m\ \prod_{i=1}^{E}(e_i)_m\ \prod_{i=1}^{R}(r_i)_m\ m!}\times$$

$$\times\Bigg(\sum_{n,p=0}^{2n+p\le m}\frac{(-m)_{2n+p}\ (d_1+m)_p...(d_D+m)_p(g_1)_{n+p}...(g_G)_{n+p}(k_1)_{2n+p}...(k_K)_{2n+p}}{(e_1+m)_p...(e_E+m)_p(h_1)_{n+p}...(h_H)_{n+p}(\ell_1)_{2n+p}...(\ell_L)_{2n+p}}\times$$
$$\times\frac{(1-r_1-m)_{(2n+p)}...(1-r_R-m)_{(2n+p)}(s_1)_n...(s_S)_n}{(1-q_1-m)_{(2n+p)}...(1-q_Q-m)_{(2n+p)}(j_1)_n...(j_J)_n}\times$$
$$\times\frac{(u_1)_p...(u_U)_p\ \mu^n y^p(-1)^{(Q+R+1)p}}{(w_1)_p...(w_W)_p\ \lambda^{2n+p}\ n!\ p!}\Bigg)\ t^m. \tag{3.4}$$

Now using the definition (1.8) of hypergeometric function of Srivastava-Daoust, we get the generating relation (2.1).
Similarly, when we replace $p$ by $p-m-2n$ in equation (3.3) and after simplification, we get right hand side of (2.2).

## 4. Some Applications

In generating relation (2.2), put $\lambda=0,\ \mu=-1,\ A=B=0,\ y=2x$ and after simplification, we get

$$F^{D+G+K:S;U}_{E+H+L:J;W}\left(\begin{matrix}[d_D):2,2],[(k_K):2,1],[(g_G):1,1]:[(s_S):1];[(u_U):1];\\ \\ [(e_E):2,2],[(\ell_L):2,1],[(h_H):1,1]:[(j_J):1];[(w_W):1];\end{matrix}\ -t^2,\ 2xt\right)$$
$$=\sum_{p=0}^{\infty}\frac{\prod_{i=1}^{D}(\frac{d_i}{2})_p\ \prod_{i=1}^{D}(\frac{1+d_i}{2})_p\ \prod_{i=1}^{G}(g_i)_p\ \prod_{i=1}^{K}(k_i)_p\ \prod_{i=1}^{U}(u_i)_p}{\prod_{i=1}^{E}(\frac{e_i}{2})_p\ \prod_{i=1}^{E}(\frac{1+e_i}{2})_p\ \prod_{i=1}^{H}(h_i)_p\ \prod_{i=1}^{L}(\ell_i)_p\ \prod_{i=1}^{W}(w_i)_p\ p!}\ (2^{(2D-2E+1)}x)^p\times$$
$$\times\ {}_\rho F_\upsilon\left[\begin{matrix}\frac{-p}{2},\ \frac{-p+1}{2},\ \frac{1-(e_E)-2p}{2},\ \frac{2-(e_E)-2p}{2},\ 1-(h_H)-p,\ (s_S),\ \frac{1-(w_W)-p}{2},\ \frac{2-(w_W)-p}{2};\\ \\ \frac{1-(d_D)-2p}{2},\ \frac{2-(d_D)-2p}{2},\ 1-(g_G)-p,\ (j_J),\ \frac{1-(u_U)-p}{2},\ \frac{2-(u_U)-p}{2};\end{matrix}\ \frac{(-1)^{(G+H+1)}4^{(E-D+W-U)}}{x^2}\right]t^p, \tag{4.1}$$

where $\rho=2E+2W+H+S+2$ , $\upsilon=2D+2U+G+J$ and $(b_B)=b_1,b_2,...,b_B$.

In generating relation (4.1), put $D=E=K=L=H=U=W=0,\ G=1,\ g_1=\beta,\ S=1,\ s_1=\gamma-\beta,\ J=1,\ j_1=\gamma$ and applying Binomial theorem and using the definition of Bedient polynomials $R_p(\beta,\gamma;x)$ (1.27), we get a known result [20, p.297, Eq.(4)]

$$(1-2xt)^{-\beta}\ {}_2F_1\left[\begin{matrix}\beta\ ,\ \ \gamma-\beta\ ;\\ \\ \gamma\ ;\end{matrix}\ \frac{-t^2}{1-2xt}\right]=\sum_{p=0}^{\infty}R_p(\beta,\ \gamma;\ x)\ t^p. \tag{4.2}$$

In generating relation (4.1), put $D = E = K = L = S = J = U = W = 0$, $G = 2$, $g_1 = \alpha$, $g_2 = \beta$, $H = 1$, $h_1 = \alpha + \beta$ and using the definition of Bedient polynomials $G_p(\alpha, \beta; x)$ (1.28) and multiple series identity of Srivastava (1.25), we get a known result [20, p.298, Eq.(6)]

$$ {}_2F_1\left[\begin{matrix} \alpha\ ,\ \beta\ ; \\ \alpha+\beta; \end{matrix}\ 2xt - t^2\right] = \sum_{p=0}^{\infty} G_p(\alpha,\ \beta;\ x)\ t^p. \tag{4.3} $$

In generating relation (4.1), put $D = E = K = L = 0$ and after simplification, we get

$$ F^{G:S;U}_{H:J;W}\left[\begin{matrix} (g_G): (s_S);\ (u_U); \\ (h_H): (j_J);\ (w_W); \end{matrix}\ -t^2,\ 2xt\right] = \sum_{p=0}^{\infty} \frac{\prod_{i=1}^{G}(g_i)_p\ \prod_{i=1}^{U}(u_i)_p}{\prod_{i=1}^{H}(h_i)_p\ \prod_{i=1}^{W}(w_i)_p}\ \frac{(2x)^p}{p!} \times $$

$$ \times\ {}_\eta F_\theta\left[\begin{matrix} \frac{-p}{2},\ \frac{-p+1}{2},\ 1-(h_H)-p,\ \frac{1-(w_W)-p}{2},\ \frac{2-(w_W)-p}{2}, (s_S); \\ 1-(g_G)-p,\ \frac{1-(u_U)-p}{2},\ \frac{2-(u_U)-p}{2}, (j_J); \end{matrix}\ \frac{(-1)^{(G+H+1)}4^{(W-U)}}{x^2}\right] t^p, \tag{4.4} $$

where $\eta = H + S + 2W + 2$ and $\theta = G + J + 2U$.

In generating relation (2.1), put $\lambda = 2, \mu = 1, y = -x$, $A = B = D = E = G = J = K = L = Q = R = S = U = W = 0$, $H = 1$, $h_1 = \frac{1+a}{2}$, we get

$$ \sum_{m=0}^{\infty} \frac{(2t)^m}{m!} \sum_{n,p=0}^{\infty} \frac{t^{2n}(-xt)^p}{(\frac{1+a}{2})_{n+p} n!\ p!} = \sum_{m=0}^{\infty} \frac{2^m}{m!} \left( \sum_{n,p=0}^{\infty} \frac{(-m)_{2n+p}\ x^p}{(\frac{1+a}{2})_{n+p}\ n!\ p!\ 4^n\ 2^p} \right) t^m. \tag{4.5} $$

Now using Srivastava's multiple series identity (1.25) in the left hand side of equation (4.5), we get

$$ e^{2t}\ {}_0F_1\left[\begin{matrix} -\ ; \\ \frac{1+a}{2}; \end{matrix}\ t^2 - xt\right] = \sum_{m=0}^{\infty} \frac{(2t)^m}{m!} \sum_{p=0}^{\infty} \frac{(-m)_p\ x^p}{(\frac{1+a}{2})_p\ 2^p\ p!}\ {}_2F_1\left[\begin{matrix} \frac{-m+p}{2}, \frac{-m+p+1}{2}; \\ \frac{1+a}{2} + p; \end{matrix}\ 1\right]. \tag{4.6} $$

Using summation theorem (1.30) and the definition of pseudo-Laguerre polynomials $R_m(a, x)$ defined by Shively (1.26) in equation (4.6) and after simplification, we get the known result [20, p.298, Eq.(4)]

$$ e^{2t}\ {}_0F_1\left[\begin{matrix} -\ ; \\ \frac{1+a}{2}; \end{matrix}\ t^2 - xt\right] = \sum_{m=0}^{\infty} \frac{R_m(a,\ x)}{(\frac{1+a}{2})_m}\ t^m. \tag{4.7} $$

## 5. Conclusion

We conclude our present investigation by observing that the generating relation deduced above is quite significant and can lead to yield numerous generating relations and generating functions involving various special functions by suitable specializations of arbitrary parameters. Moreover, presented generating relation is expected to find some applications in probability theory, quantum physics, multivariate statistics, number theory. It may also be potentially useful to non-specialists who are interested in Applied Mathematics or Mathematical Physics.

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