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AN ASYMPTOTIC STABILITY ANALYSIS OF NEUTRAL TIME DELAYS SYSTEM WITH NONLINEAR UNCERTAINTY

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Abstract: In this paper, the asymptotic stability for neutral delay differential system with nonlinear Uncertainties is investigated. Many works have been reported using a variety of methods. However, more focus on the use of the Lyapunov-Krasovskii theory to derive sufficient stability conditions in the form of linear matrix inequalities. These stability conditions are formulated as linear matrix inequalities (LMIs) which can be easily solved by various convex optimization algorithms. Here we present the basic concepts involved in stability and also we reported and developed to analyze the asymptotic stability of Neutral Time Delay-differential systems.

Keywords and Phrases: Stability, Delay-dependent stability, Linear Matrix Inequality, Lyapunov-Krasovskii functional, Time-varying delay.

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1. Introduction

Dynamical systems with time delays have been of considerable interest for the fast few decades. In particular, the interest in stability analysis of various neutral differential systems has been growing rapidly due to their successful applications in practical fields such as circuit theory, bio engineering, population dynamics, automatic control and so on. Current efforts on the problem of stability of time delay systems of neutral type can be divided into two categories, namely delay independent criteria and delay dependent criteria. A number of delay independent sufficient conditions for the asymptotic stability of neutral delay differential systems have been presented by various researchers [1]-[8]. The Lyapunov functional technique combined with matrix inequality technique and a new operator are used in to investigate the problem of robust stability for neutral systems and time delay systems. By using descriptor model transformation and decomposition technique, some delay-dependent stability criteria are obtained. In the stability conditions are developed by descriptor model transformation technique, and the nonlinear uncertainties are handled by the S-procedure. However, these results are only concerned with the asymptotic stability, without providing any conditions for exponential stability and any information about the decay rates.

Throughout this paper, the notation $*$ represents the elements below the main diagonal of a symmetric matrix. A^T means the transpose of A. We say $X > T$ if X-Y is positive definite, where X and Y are symmetric matrices of same dimensions. $\|\cdot\|$ refers to the Euclidean norm of vectors.

To know about the formation of LMI and its stability, consider the linear system

$$\dot{x}(t) = Ax(t), \tag{1}$$

where $A \in R^{n*n}$ and $x(t) \in R^n$. Assume that (1) has equilibrium X=0.

2. System Description and Preliminaries

Consider a neutral delay differential system of the form:

$$\begin{aligned} \dot{x}(t) =& Ax(t) + Bx(t-h(t)) + C\dot{x}(t-h(t)) + f_1(t, x(t)) \\ x(s) =& \phi(s), \quad \dot{x}(s) = \varphi(s), s \in [-h, 0], \end{aligned} \tag{2}$$

$x(t) \in R^n$ is the state and $\phi(\cdot)$ and $\varphi(\cdot)$ are continuous vector valued initial functions, A, B and C are real constant matrices and h(t) denotes time varying delay and it is assumed to satisfy $0 \le h(t) \le h_M$ and $0 \le \dot{h}(t) \le d \le 1$, where h_M and d are positive constants, where $f_1(t, x(t))$ in the non linear uncertainity which satisfies the condition

$$\|f_1(t, x(t))\| < \alpha_1 \|x(t)\|.$$

Definition 2.1. Stability: *Let X(t) be a solution of the functional differential equation $f(t, x_t)$. The stability of the solution concerns the systems behavior when the system trajectory X(t) deviates from y(t). Let us assume that the functional differential equation x=f(t, x(t)) admits the solution. X(t)=0, which will be referred to as the trivial solution. For the system, $\dot{x} = f(t, x_t)$ the trivial solution X(t)=0 is said to be stable. If for any $t_0 \in R$ and $\delta > 0$, there exists a $\delta = \delta(t_0, \varepsilon) > 0$ such that $\|X(t_0)\|_c < \delta$ implies $\|X(t)\| < \varepsilon$ for $t \geq t_0$.*

Definition 2.2. Asymptotically stable: *Let X=0 be an equilibrium point of $\dot{X} = f(X)$, let $V : R^n \to R$ be a continuously differentiable function such that:*

$$
\begin{aligned}
&(i) \quad V(0) = 0,\\
&(ii) \quad V(X(t)) > 0,\\
&(iii) \quad \dot{V}(X(t)) < 0.
\end{aligned}
$$

This leads to the celebrated theorem of Lyapunov of (1).

Lemma 2.3. (Schur complement [1]) *Let M, P, Q be given matrices such that $Q > 0$, then*

$$\begin{pmatrix} P & M^T \\ M & -Q \end{pmatrix} < 0 \leftrightarrow P + M^T Q^{-1} M < 0.$$

Lemma 2.4. *For any vectors $a, b \in R^n$ and scalar $\varepsilon > 0$, we have*

$$2a^T b \leq \varepsilon a^T a + \varepsilon^{-1} b^T b. \tag{3}$$

Theorem 2.5.(Lyapunov second Theorem) *Given system (1) with equilibrium X=0, if there exists an Lyapunov function V, then X=0 is Lyapunov stable. Furthermore, if $\dot{V}(X(t)) < 0$, then X=0 is asymptotically stable.*
The power of Theorem 1.3 is that one can make conclusions about trajectories of a system (1) without actually solving the differential equation. For the system (1), a common choice of Lyapunov function candidate is the quadratic form.
By choosing Lyapunov function $V(X) = X^T PX, P > 0$, where X=x(t).
Then derivative analysis is

$$
\begin{aligned}
\dot{V} =& \dot{X}^T PX + X^T P\dot{X}\\
=& X^T A^T PX + X^T PAX\\
=& X^T (A^T P + PA)X.
\end{aligned} \tag{4}
$$

The quadratic form of this derivative proves, if the central quantity satisfies

$$A^T P + PA < 0 \tag{5}$$
$$\dot{V}(X) < 0. \tag{6}$$

Which is asymptotically stable.

Example 2.1. Consider the Linear system $\dot{X}(t) = \begin{bmatrix} -1 & 4 \\ 0 & -3 \end{bmatrix} x(t)$.
Using MATLAB LMI Control toolbox the above LMI $A^T P + PA < 0$, one can get the following Positive definite matrix $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{5}{6} \end{bmatrix} > 0$.
For this P,

$$A^T P + PA = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{7}$$

Which is negative definite, this implies that the system is asymptotically stable in the sense of Lyapunov. It is well known that time-delay is usually a cause of instability and oscillations of recurrent neural networks (RNNs).
Therefore, the problem of stability with time-delay is of importance in both theory and practical applications with the help of the LMI approach.
By inspired of this we continued here by short noted the results of the researchers who worked on the delay differential system of stability by using Lyapunov-Krasovsii function to frame LMI's.

Here we can prove the stability of the system by help of this above stability concept.

3. Global Stability Results

In this section, we will perform stability analysis of neutral system with time varying delay described by (1). We can rewrite system (1) to the following descriptor system:

$$\dot{x}(t) = y(t),$$
$$y(t) = Ax(t) + Bx(t - h(t)) + Cy(t - h(t)) + f_1(t, x(t)).$$

Theorem 3.1. *System (1) is asymptotically stable with convergence rate $\alpha > 0$ if there exist some positive definite matrices $P_i > 0$; i=1, 2, 3, 4 and real matrices $N_j > 0$; j=1, 2, 3 such that the following LMI condition is satisfied,*

$$\Xi_1 = \begin{bmatrix} \Omega & \pi \\ * & \Delta \end{bmatrix} < 0,$$

where

$$\Omega = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} \\ * & \varphi_{22} & \varphi_{23} & \varphi_{24} \\ * & * & \varphi_{33} & \varphi_{34} \\ * & * & * & \varphi_{44} \end{bmatrix}, \pi = \begin{bmatrix} \pi_{11} & \pi_{22} & 0 & 0 \\ 0 & 0 & \pi_{33} & 0 \\ 0 & 0 & 0 & \pi_{44} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{and}$$

$$\Delta = \begin{bmatrix} \Delta_{11} & 0 & 0 & 0 \\ 0 & \Delta & 0 & 0 \\ 0 & 0 & \Delta_{33} & 0 \\ 0 & 0 & 0 & \Delta_{44} \end{bmatrix},$$

are the coefficient matrix of $[x^T(t)x^T(t-h)y^T(t)]$.

Proof: This theorem can be prove by considering the Lyapunov functions are $V = V_1 + V_2 + V_3 + V_4$ are as follows.
We define Lyapunov functions as follows

$$V_1(t) = x^T P_1 x(t),\ V_2(t) = \int_{t-h}^{t} x^T(t+s)P_2 x(t+s)ds,$$
$$V_3(t) = \int_{t-h}^{t} Y^T(t+s)P_3 Y(t+s)ds,\ V_4(t) = h_M \int_{-h_M}^{0}\int_{t+\beta}^{0} \dot{x}^T(t+s)P_4\dot{x}(t+s)d\beta.$$

Here we using lemma 2.4, we have

$$\begin{aligned}\dot{V} =& \big[x^T(t)(AP_1 + A^T P_1 + 2\alpha P_1 + P_2 + P_5) + 2x^T P_1 Bx(t-h) + 2x^T P_1 Cy(t-h) \\ &+ 2x^T Pf_1(t, x(t)) - (1-h)x^T(t-h)P_2 x(t-h) - y^T(t)By(t) + h_M^2 y^T(t)P_4 y(t) \\ &- (1-\dot{h})y^T(t-h)P_3 y(t-h) - x^T P_4 x(t) + 2x^T P_4 x(t-h) \\ &- x^T(t-h)P_4 x(t-h)\big].\end{aligned}$$

By previous lemma we have

$$h_M \int_{t-h_M}^{t} \dot{x}^T(s)P_4 x(s)ds \leq [x(t) - x(t-h)]^T P_4[x(t) - x(t-h)].$$

From the above system we have

$$2[y^T N_1^T + x^T N_2^T + x^T(t-h)N_3^T] \times [-y(t) + A + B + Cx(t-h) + f_1] = 0.$$

We have

$$\begin{aligned}\dot{V} =&[x^T(t)(AP_1 + A^TP_1 + 2\alpha P_1 + P_2 + P_5) + 2x^TP_1Bx(t-h)\\&+ 2x^TP_1Cy(t-h) + 2x^TPf_1(t, x(t)) - (1-h)x^T(t-h)P_2x(t-h)\\&- y^T(t)By(t) + h_M^2y^T(t)P_4y(t) - (1-\dot{h})y^T(t-h)P_3y(t-h)\\&- x^TP_4x(t) + 2x^TP_4x(t-h) - x^T(t-h)P_4x(t-h) - 2y^TN_1^Ty(t)\\&+ 2y^TN_1^Ty(t)Ax(t) + 2y^TN_1^Ty(t)Bx(t-h) + 2y^TN_1^TCx(t-h) + y^TN_1^Ty(t)\\&+ 2x^TN_2^TAx(t) + 2x^TN_2^TBx(t-h) + 2x^TN_2^TCx(t-h)\\&+ \alpha_1^2x^Tx(t)\{\beta_1 + \beta_2 + \beta_3 + \beta_4)],\end{aligned}$$

where $\xi^T = [x^T(t)\ x^T(t-h)\ y^T]$.
Then

$$\dot{V} \leq \xi^T\ \Omega\ \xi,$$

and

$$\Omega = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} \\ * & \varphi_{22} & \varphi_{23} & \varphi_{24} \\ * & * & \varphi_{33} & \varphi_{34} \\ * & * & * & \varphi_{44} \end{bmatrix} < 0. \tag{8}$$

By applying Lemma 2.3 in R with some effort, we get $\Xi_1 < 0$. Therefore, by Lyapunov-Kraspvskii stability theorem $\dot{V}(X) < 0$.
Hence we concluded that the system is asymptotic stable.

4. Conclusion

We have presented a sufficient condition to guarantee the asymptotic stability for neutral delay differential system with uncertainties. Based on the Lyapunov-Krasovskii functional theory, the delay dependent criterion have been derived to guarantee the asymptotic stability of neutral delay differential system with uncertainties.

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