South East Asian J. of Mathematics and Mathematical Sciences
Vol. 15, No. 2 (2019), pp. 97-104

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

UNSTEADY FLOW OF BLOOD THROUGH A STENOSED ARTERY UNDER THE INFLUENCE OF TRANSVERSE MAGNETIC FIELD

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(Received: June 27, 2019)

Abstract: The purpose of this work unsteady flow of blood through a stenosed artery under the influence of transverse magnetic field is studied analytically. The laminar, incompressible, fully developed, Newtonian flow of blood in an artery having mild stenosis and governing equations are solved analytically by using Bessel function. It is assumed that the surface roughness is cosine-shaped and the maximum height of the roughness is very small compared with the radius of the un constricted tube.

Keywords and Phrases: Slip Velocity, Transverse Magnetic field, Oscillatory flow, Hartmann number.

2010 Mathematics Subject Classification: 58D30.

1. Introduction

The study of blood flow through arteries are of considerable importance in many cardiovascular diseases particularly atherosclerosis. The normal blood flow is disturbed due to the formation of some constriction in the inner wall of the artery, which is called stenosis or atherosclerosis. Such constrictions in inner artery may cause many flow problems, like highly shear stress regions, improper flow rate, and it may cause reverse flow when the severity of the stenosis is large enough. Various mathematical models have been investigated by several researchers to explore the behavior of blood flow under the influence of transverse magnetic field.

Mishra et al., (2012) discussed a study of oscillatory blood flow through porous medium in a stenosed artery. Nubar (1967) studied effects of slip in the rheology of a composite fluid: Application to blood flow. Ponalgusamy (2007) investigated blood flow through an artery with mild stenosis: a two-layered model, different shapes of stenosis and slip velocity at the wall. Rathod and Tanveer (2009) considered pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and magnetic field. Rathod and Habeeb (2008) studied pulsatile inclined two layered flow under periodic body acceleration. Rathod and Hosurker (1998) discussed MHD flow of Rivlin-Ericksen fluid through an inclined channel. Srivastava and Saxena (1994) investigated two-layered model of Casson's fluid flow through stenotic blood vessels: application to cardiovascular system.

In this study a mathematical model is proposed to describe blood flow through stenosed artery with slip at wall when blood is represented as Newtonian fluid and a uniform magnetic field is applied on the flow. In the model non-linear equations have been solved analytically using bessel functions. The effects of various parameters such as the Reynolds number (R_e), Hartmann number (M) and Non-dimensional frequency parameter (β) on the flow velocity, volumetric flow rate, wall shear stress and resistive impedance are discussed through graphs.

2. Formulation of the Problem

The idealized geometry of stenosis is given by

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{\delta}{2R_0}[1 + \cos\frac{2\pi}{L_0}(z - d - \frac{L_0}{2})] & ;\text{when } \mathrm{d} \leq \mathrm{z} \leq \mathrm{d} + \mathrm{L_0} \\ 1 & ;\text{otherwise} \end{cases} \tag{1}$$

Where R(z) is the radius of the artery in the stenotic region, R_0 is the radius of the normal artery, L_0 the length of stenosis, d length of non-stenosis and δ the maximum height of stenosis such that $\frac{\delta}{R_0}\square 1$ (Fig. 1).

Let us consider the oscillatory flow of blood through an artery with mild construction. The flow is assumed to be laminar, Newtonian, viscous, incompressible, unsteady and axially symmetric. The density and viscosity are assumed to be constant. The artery is of constant radius preceding and following the stenosis. The governing equation may be written in the cylindrical coordinate system (r,θ,z) as,

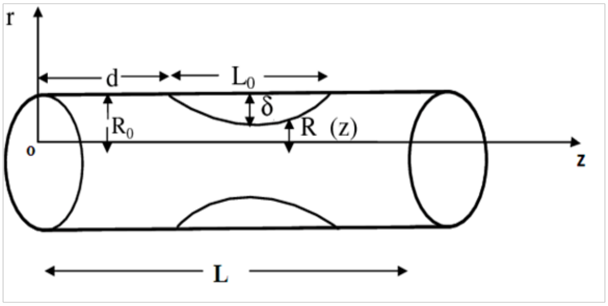


Figure 1

Equation of continuity

$$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r} = 0 \tag{2}$$

Equation of motion in radial direction

$$\rho(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}) = -\frac{\partial p}{\partial r} + \mu(\frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} - \frac{u}{r^2}) \tag{3}$$

Equation of motion in axial direction

$$\rho(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}) = -\frac{\partial p}{\partial r} + \mu(\frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial r^2}) - \sigma B_0^2 w \tag{4}$$

where r and u are the radial coordinates and velocity, z and w are the axial coordinate and velocity, B_0 is the applied magnetic field in r direction.

As axially-symmetric flow in a rigid circular tube of radius R_0 is considered, for which $u = 0$, $v = 0$, $w = w(r, z, t)$, p=(r,z,t). So the equation (2), (3) and (4) becomes respectively as

$$\frac{\partial p}{\partial r} = 0 \ and \ \frac{\partial w}{\partial z} = 0 \tag{5}$$

$$\rho(\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z}) = -\frac{\partial p}{\partial r} + \mu(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}) - \sigma B_0^2 w \tag{6}$$

Using (5) and (6) becomes

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}) - \frac{\sigma}{\rho}B_0^2 w \tag{7}$$

The boundary conditions are

$$\left.\begin{array}{lcl} Slip\ Condition & : & w = w_s\ at\ r = R(z) \\ Symmetry\ Condition & : & \frac{\partial w}{\partial r} = 0\ at\ r = 0 \end{array}\right\} \tag{8}$$

3. Mathematical Solution

It is convenient to write these equations in dimensionless from by means of the following transformation variables

$$r' = \frac{r}{R_0}, z' = \frac{z}{R_0}, R' = \frac{R}{R_0}, w' = \frac{w}{w_0}, t' = \frac{t}{t_0}, p' = \frac{p}{p_0}$$

where w_0, R_0, p, ρ, μ and B_0 are the average velocity, radius in the unobstructed tube, pressure, density, viscosity of blood and the transverse magnetic field in radial direction respectively. Let the solution for w and p be set in the forms

$$w(r,t) = w(r)e^{i\omega t}\ and\ -\frac{\partial p}{\partial z} = P(z)e^{i\omega t} \tag{9}$$

Substituting the dimensionless form of (9) in (7) we get

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \big(\frac{i\omega R_e}{R_t} - M^2\big)w = -p(z) \tag{10}$$

Where $R_t = \frac{t_0 w_0}{R_0}$, $R_\epsilon = \frac{\rho R_0 w_0}{\mu}$, $M = \sqrt{\frac{\sigma}{\mu}}B_0 R_0$.

On simplifying we have second order ordinary differential equation as follows,

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \beta^2 w = c \tag{11}$$

Where $\beta^2 = \frac{R_e i\omega}{R_t} - M^2$, $c = -p(z)$.

Using transformations $u = \frac{-c}{\beta^2} + v$, in equation (11), we have

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \beta^2 v = 0 \tag{12}$$

The equation (12) is solved by using bessel function we get $v = AJ_0(i\beta r)$
The expression for the velocity can be obtained as

$$w(r) = \frac{-c}{\beta^2} + AJ_0(i\beta r) \tag{13}$$

Using boundary conditions (8) in equation (13) we get

$$w(r) = \frac{-c}{\beta^2} + \frac{1}{J_0(i\beta.R(z))}\big(w_s + \frac{c}{\beta^2}\big)J_0(i\beta.r) \tag{14}$$

The velocity profile of the stenosed artery is

$$w(r,t) = \big[\frac{-c}{\beta^2} + \frac{1}{J_0(i\beta.R(z))}\big(w_s + \frac{c}{\beta^2}\big)J_0(i\beta.r)\big]e^{i\omega t} \tag{15}$$

The volumetric flow rate, Q of stenosed artery is

$$Q = 2\pi R_0^2 w_0 \int_{r=0}^{r=\frac{R(z)}{R_0}} rw dr$$

$$Q = 2\pi c R_0^2 w_0 \big[\frac{-R^2(z)}{2\beta^2} + \frac{R(z).J_1(i\beta.R(z))}{i\beta.J_0(i\beta.R(z))}\big(w_s + \frac{1}{\beta^2}\big)\big]e^{i\omega t} \tag{16}$$

The shear stress at the wall is defined as

$$\tau_R = -\mu\big(\frac{\partial w}{\partial r}\big)_{r=\frac{R(z)}{R_0}}$$

$$\tau_w = \frac{\mu w_0}{R_0}\big[\frac{J_1(i\beta.R(z))i\beta}{J_0(i\beta.R(z))}\big(w_s + \frac{c}{\beta^2}\big)\big]e^{i\omega t} \tag{17}$$

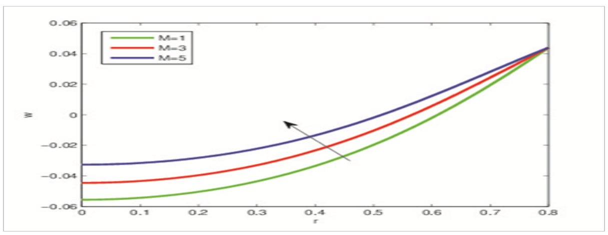
The resistive impedance to the flow is defined by

$$z = \frac{\frac{-\partial p}{\partial z}}{Q} \tag{18}$$

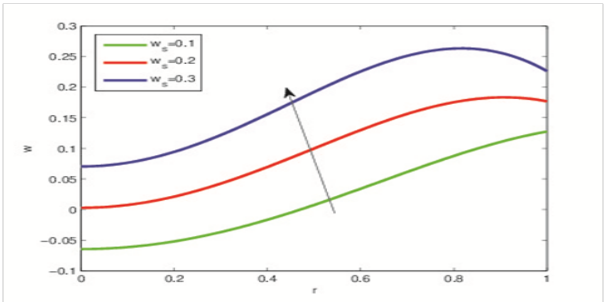
which the right hand side is known and can be obtained from equation (16).

4. Result and Discussion

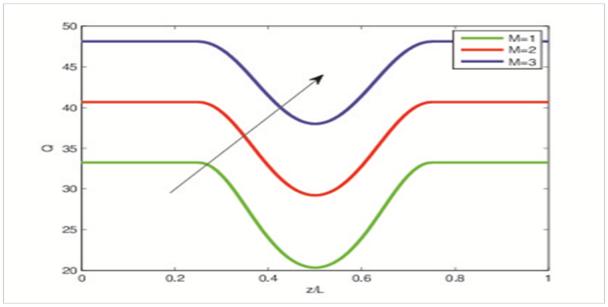
The theoretical study of Influence of transverse magnetic field and slip velocity on blood flow through a stensoed artery have been discussed. Numerical Values are Reynolds number (R_e)= 1, 2, 3, 4 and 5, Magnetic field (M) =1, 2, 3, 4 and 5, thickness of stenosis (δ) = 0.05, 0.10 and 0.15 and slip parameter (w_s) = 0.05, 0.1, 0.2 and 0.3.



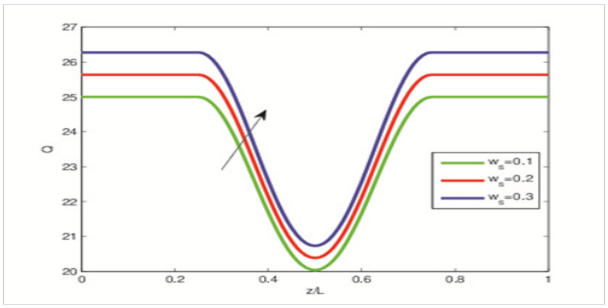
Effects of Magnetic field M on velocity profile w with radial direction r, where Re=3, ws=0.05



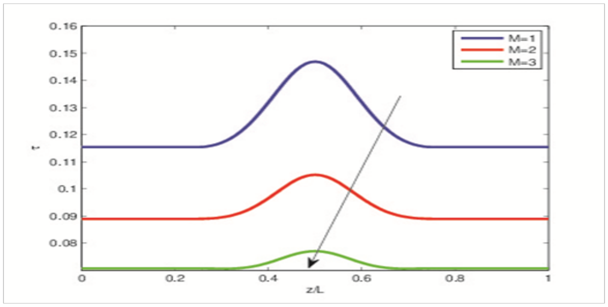
Effects of slip parameter ws on velocity profile w with radial direction r, where M=2.5, Re=3.



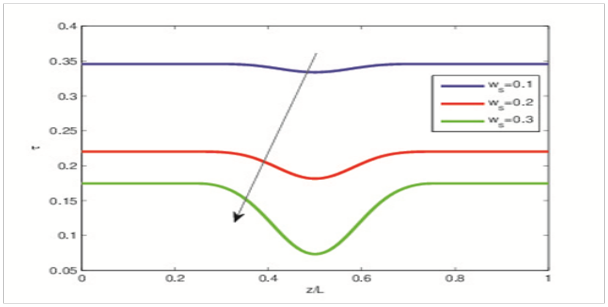
Effects of Magnetic field M on volumetric flow rate Q with z/L, where Re=3, ws=0.05



Effects of slip parameter ws on volumetric flow rate Q with z/L, where M=2.5, Re=3.



Effects of Magnetic field M on wall shear where Re=3, ws=0.05



Effects of Magnetic field M on velocity profile where M=2.5, Re=3

The expression for velocity profile $w(r,t)$ obtained in equation (15) has been depicted in Figure 2 and Figure 3 in presence of Magnetic field (M) and slip parameter (w_s). It is found that velocity increases with the increase of Magnetic field (M), increasing and slip parameter (w_s) . The variation of volumetric flow rate (Q) with length of the artery (z/L) for different parameters is presented in Figure 4 and Figure 5. Similarly it is seen that as Magnetic field (M) and slip parameter (w_s) increases as the increase of volumetric flow rate. The variation of wall shear stress (τ) with length of artery (z/L) for different parameters Magnetic field (M) and slip parameter (w_s) is shown in Figure 6 and Figure 7. It is noticed that wall shear stress decreases as the increase of Magnetic field (M) and slip parameter (w_s). It is important to notice that at higher Magnetic field wall shear stress so significantly that it may cause the stenosis to break off and the ultimate result is paralysis or sudden death.

5. Conclusion

In the analysis, we considered the effect of transverse Magnetic field on unsteady flow of blood through a stenosed artery and slip velocity at the boundary layer assuming that the flowing fluid blood is considered by a Newtonian fluid. The effect of various parameters to depict the graphs for axial velocity, volumetric flow rate, wall shear stress and resistive impedance. By applying transverse magnetic field attached with those instruments we may enhance their activities. By using an transverse magnetic field it is possible to control blood pressure and also it is effective for conditions such as poor circulation, travel sickness, headaches, pain, muscle sprains, strains and joint pain etc. Thus the mathematical expressions may help medical practioners to control the blood flow in the artery of a patient by applying transverse magnetic field.

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